

A C O M P L E A T

Body of Arithmetick,

In Four B O O K S, *Viz* :

Book I.	Part
Part I. <i>Integers.</i>	3. <i>Logarithmes.</i>
2. <i>Fractions.</i>	4. <i>Cosicks.</i>
Book II.	5. <i>Surds.</i>
1. <i>Geodæticals.</i>	6. <i>Algebra.</i>
2. <i>Figurals.</i>	Book IV.
Book III.	1. <i>Ratio's.</i>
1. <i>Decimals.</i>	2. <i>Proportions disjunct.</i>
2. <i>Astronomicals.</i>	3. <i>Proportions continued.</i>
	4. <i>Æquations, &c.</i>

W H E R E I N T H E

Whole Nature of N U M B E R S, with their Simple and Comparative Elements in all the Parts of A R I T H M E T I C K, are plainly Declared, and fully Handled.

Every P A R T Explain'd by

Necessary Rules, Cases, Theorems, Questions, Observations, and Variety of Operations : Illustrated by fundry Tables, Diagrams, and very many Examples. Together with divers Etymologies, Symboles, Characters, and Abbreviations for Artificial Terms, Words, Names, and Denominations : The Whole digested into a succinct, and orderly Method ; and delivered in a familiar Style.

To every P A R T is added,

Excellent R U L E S of P R A C T I C E ; which makes it very Useful to Merchants, and all that would understand A C C O U N T S, or the Mathematical and Mechanical A R T S and S C I E N C E S.

W I T H

Many Large and Exact T A B L E S of all Sorts of Coins, Weights, Measures, &c. Ancient and Modern, in most Countries, reduced to our own : The Best that has yet been extant.

By S A M U E L J E A K E, Merchant.

Non partis Studiis agimur, &c.

L O N D O N :

Printed for *Tho. Newborough*, at the *Golden Ball*, in *St. Paul's Church-yard* ; and *John Nicholson*, at the *King's Arms*, in *Little-Britain*. MDCCI.

To the Honourable
Sir *ROBERT SOUTHWELL*, K^{nt.}
P R E S I D E N T
O F T H E
R O Y A L S O C I E T Y.

S I R,

ALthough it was the Author's Pleasure to prefix my Name to the En-
fuing Treatise ; which I am therefore
necessitated to insert : lest I should be
accused of altering that Work, the Im-
preffion whereof was recommended to
me ; and whereto I am obliged without
any respect to Profit. Yet as I thereby
received a Right to dispose of the De-
dication : So I could not otherwise ac-
quiesce, than in the Resignation of my
Interest therein, to some greater Name ;
b esteeming

The Epistle Dedicatory.

esteeming my own too minute to stand before it. And because I am not under any particular Obligation to whom I should present it : I have adventured to shroud it under your Honourable Patronage. Presuming that where no Personal Advantages are expected ; no imputation of Flattery can be charged : But that the Motives were alone that Merit and Generosity which must always be venerated by

S I R,

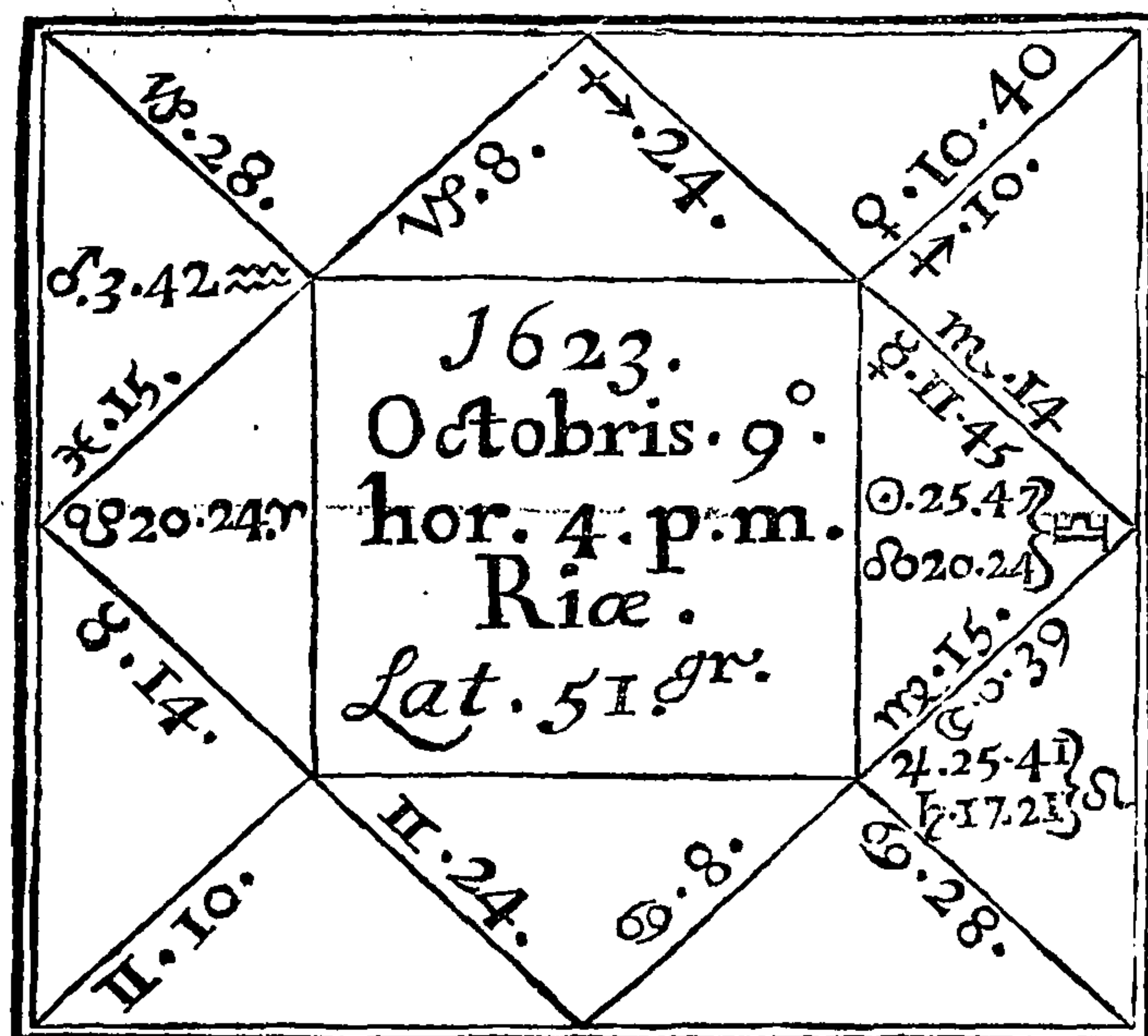
Your most

Humble Servant,

S. 7 E A K E

Defectum

Defectum Iconismi Placuit supplere Themate Natalitio Authoris.



NO Planets here by Exaltation proud :
None by a Rest Supine, in House bestow'd.
But congruous Heav'n at this Birth dispos'd,
T'inspirit a clear Soul in Flesh enclos'd.
The mildest Dodecatemorie springs
In beauteous Orient : the encircling Rings
Of her Cœrulean Lord's Quaternion,
By Starry Regulus in Triumph shone.
That bright Superior's Domination fixt
In Heav'n's Culmen. Gen'rous Aspects mixt :
His Fiery Partil Trine to actuate
The Active House to a more Active Fate.
Nor was it vain : the happy Site of this
Æthereal Ruler of the Genesis,

A Judg-

A Judgment firmly form'd ; whose Adjutant
Mnemonick pow'r, did by Cœlestial grant
Of Saturn's seminated Beams ensue,
In Platique Synod, with Proportion due.
As when the skilful Artist to compose
His mighty Theriaque ; Weighs the Critick Dose
Of Theban Opium ; which with Virtue full
Quickens that Brain, it's least Excess would dull.
The Wit's Dictator from the brighter Scale
Suits his harmonious Trine, whose Rays may fall
On th' Eastern Point : Whilst the Hesperian Face
Resplendent Venus doth the Ninth House grace.

S. J. Authoris Filius.

THE

T H E

Authors Epistle.

T O

His Well-Beloved Son,

SAMUEL JEAKE.

My Son,

PEradventure the Reader (if ever the ensuing Piece be made Publick) may expect here, more than will be found ; and yet find more than he expects : But sure I am, Thou wilt not find here all I could wish thee, nor yet all I intend thee. What thou findest, was mine before thine, and though thine is mine notwithstanding. The Gift thereof may enrich thee, but cannot improve me. And the surest way to make it thine is, to make it others too by Publication. By which though others, it will be nevertheless secured to thee, as being then incapable of perishing in Private Papers. And because thy Right to inherit what is mine is indubitable, and thy Duty to defend what I leave thee, (though but a small Patrimony :) I have sought no other Patron, nor (seeing *Vino Vendibili non opus est bederâ*) do I want any, or shroud it under thy Patronage, thereby to gain the more respect or honour to my self or it, (the great cause of Dedications) if Art will not Patronize it, I am content to bear the accruing blame, whatever it be.

But seeing how good soever the Wine of the Book be (to make it the more Vendible) it is now grown Customary to hang out the Ivy-Bush of an Epistle as an Apology for the Author, or his Work, or both : Or lest it should be thought so useless or unprofitable,

B

that

The Author's Epistle.

that nothing can be said to commend it or its Publication : I shall add a few things, and but a few on that Accompt.

Among the many Authors of Arithmetick that have come to my hands, I have ever observed that each pleaseth himself with his own Method, as I my self have done with mine. But this I must needs say, that I never met with any Single Piece but left me dissatisfied in some or other parts of Arithmetick. Some handling only the Operations in *Whole Numbers* and *Extraction of Roots*, Others *Whole Numbers* and *Fractions*, Some all these with some *Rules in Proportions*, Others together with them have taught *Decimals* ; Some have dealt only with *Logarithmes*, Others with *Cosicks* and *Algebraical Notes*, &c. So as none I have yet seen gives a Compleat Accompt of some necessities thereto. And besides, the Accompt given by several, is so disordered and imperfect, as the Art hath been but a little beholding to them for presenting her to the World in so rural a Dress. Wherefore if the labour of a Complete Collection of the Cream of other Authors may be acceptable to any ; or the Foundation of a Method large enough to bear all the parts of the Building whereon may be fastned, and from whence may be drawn, the Resolution of any Question concerned in Arithmetick ; this Piece may as well as others, that want both, be welcome to the Press, and crowd in for a place among the multitude of Books now Printed, wherein I hope 'twill neither shame the Author, nor be ashamed of the Title of *Logisticologia*, seeing all the Concerns and Appurtenances of Arithmetick are therein discoursed of, and largely Surveyed and Reviewed.

Perhaps some may think, it is but to light a Candle to the Sun, since so many already have wrote on the Subject ; as if *Nihil dictum quod non dictum prius*. To which I may plead with the Lawyers, *Non modo & forma*, and put the Issue on the Countrey to try.

True it is, most new Models are but the Light that sometime shined in anothers Lamp, with an addition of fresh Oyl out of a new Vessel, *Et facile est inventis addere*. But he that is sensible of the charge of buying, and trouble of turning over many Books to learn some one thing, will I doubt not excuse my further plea herein, and plead for me ; especially if he knew that I speak not without Experience, of no little time and trouble to glean so many Fields for one Grist, having pickt up the knowledge of *Integers*, *Fractions*, *Figurals*, *Cosicks*, and *Surdes* principally from *Record*, *Decimals* from *Johnsen*, *Astronomicals* from *Blundevile*, *Logarithmes* from *Briggs*, *Species* and *Æquations* from *Oughtred*, with a conference of many others. It follows therefore that each may have his due, what is here may be accompted anothers, yet is it all my own, and some things therein so far my own, as will be found in none extant that I know of. And because this
may

The Author's Epistle.

may prove Beneficial to some, unless such prove remiss to themselves in the perusal ; it may serve for a second encouragement to the Publication.

Besides some Pieces are wrote in *Latine*, and of those in *English*, some too short in their Rules, or too long or dark in their Examples, or in the Reason and Ground thereof, and worse than that have Examples instead of Rules, or Rules which will not hold generally, nor answer to several Cases, so as setting aside all the Errors of the Press, (a great Mischief to a Learner of these things by Book, where the Sence is not guided by the Antecedent or Subsequent Matter) all the Learner reapes after the Expence of his Time, Cost, and Travel, expecting with the Mower to fill his Lap ; is but a handful of the Grass of the House-top, to wit, the Resolution of some few Questions. And therefore the spreading this Table with such Varieties of Rules for almost all Cases, and fitting Examples to them (and not the Rules to the Examples) and over and above the Explanation of both in very many places, may I suppose pass for a further full and sufficient plea for it's *Imprimatur*.

As to the Work it self, the occasion of it's first penning was to help an Imperfect Memory, not once then thinking it should ever have seen the Sun. Most of whose rough drawn, and unpolisht Papers have layn by me above Twenty Years, in which time there have not wanted the often pressures of Friends for a Transcription, which to them yet on this side the Grave will I know be grateful. If others undeservedly flight it, it will but give occasion for the deserved flighting of their own Opinions, and not at all hurt me or it. *Alij quidem nesciunt, neque quidem curant scire, & quia nesciunt nolunt scire. Nihil enim desiderabile est dum ignotum, nec amatum, nisi cognitum.*

Nevertheless to take away occasion from such as seek occasion, (it being too common for some to seek in Books for advantages against their Authors ;) and to obviate seeming Objections, as also that all causeless Scruples and Calumnies intended to blurr both Author and Work may be wiped off (if possible) and utterly to rase the foundation thereof, and absterge such rubbish, I shall add,

1. Wherein I dissent from others in the Method, placing Continued Proportions after disjunct, and both after Figural Numbers, it is sufficient there is a necessity for it, because without the knowledge of Figural Numbers, and Extraction of their Roots, *Progression* and several of the Dependants thereof cannot be Learned, nor Doubled nor Tripled *Disjunct Proportions* wrought, as the new-born *Sciolist* will easily see : And whether it be not preposterous to teach the more hard and Sublime parts of any Science, before the Introductory ; let any *Tyro* judge.

2dly, *Fractions*

The Author's Epistle.

2^{ly}. *Fractions* upon the same account of necessity are here made to precede them all ; for otherwise it can never be understood, how to Square or Cube, &c. a *Fraction*, nor how to value the Remain upon any Division in the *Golden Rule*, and other *Proportional Operations* ; if it be not known, what a *Fraction* is, and how to work therewith.

3^{ly}. For the same reason, *Æquations* are set after *Surdes*, and *Surdes* before *Species*, seeing as in *Species* there happen *Surdes* ; so both *Surdes* and *Species* arise in *Æquations*. But if no such necessity were for these disagreements ; yet the symmetry and suitable agreement of the several parts with the whole in the Method pursued, and their concatenate concurrence or dependance one with, or upon another, with the reasons here and there in places apt for the purpose rendered, may put a stop to any further dispute concerning the same.

4^{ly}. Neither may the placing of *Reduction in Geodæticals* before *Addition*, &c. be any Stone of Stumbling, considering *Geodæticals* are now acknowledged to be *Contract Numbers*, as limited to *Denominations*. And in *Contract Numbers* it is no strange thing to see *Reduction* an Ortive part of *Numeration* put before the Original. And how helpful *Geodætical Reduction* is to other the Prime parts of *Geodætical Numeration*, nothing can so well discover as the Survey it self, where it will easily be perceived without that Order, especially *Multiplication* and *Division* of those Numbers if placed before, must have been imperfect and disorderly.

5^{ly}. If some of the Characters in *Cossicks* and *Surdes* anciently used are changed for others : I think it Apology enough to say, they are arbitrary, and if for expedition (the reason of their alteration) other distinct Notes may be found, the Practitioner is at liberty to change all the rest.

6^{ly}. Varieties of working a Question, if objected as troublesome, are added as a benefit, like multiplicity of words in a Language for the same thing ; if one be not hit upon, the other may ; and where the one is dark, the other helps to Illustrate, and each serve to prove the other.

7^{ly}. Many of the Examples may be thought needless ; but it may be remembered, that *Profunda lustrare absque exemplis arduum* ; and oftentimes one Example is not enough to shew the sufficiency of a Rule, wherefore several Examples are added, where the Rule is dark, or the Work difficult, and several of them explained, lest I should seem to walk in the Clouds, accompting it for a *Maxime*, That nothing is worth the Writing, which cannot be understood when wrote. And surely plurality of Examples of so general Approbation and Practice with others, cannot come under dislike here, and if it be a fault, must needs be one pardonable.

8^{ly}. When

The Author's Epistle.

8ly. When Termes are used promiscuously, as *Mixt Numbers*, for *Numbers* and *Fractions*, and sometimes for Numbers made up of *Digits* and *Articles*, or if *Denominator* and *Denomination* be used either for other, or *Ratio* and *Proportion*, or such like; Yet will the place where, and Matter about which they are so used, easily discover to the Observant Reader, which Term is properly intended without further help of an Expofitor. And thus in the Common Elements of *Addition*, *Subtraction*, *Multiplication* and *Division*, if any of them be mentioned when *Integers* are in hand, it shall be taken for *Addition*, *Subtraction*, *Multiplication*, or *Division of Integers*: But when *Fractions* or other *Numbers* are in hand, then those *Elements* shall accordingly be understood to be used, and wrought after the manner of *Fractions*, &c. respectively, without annexing to every repetition words at length to direct it.

9ly. Where ever any Term or Phrase hath escaped Exposition, and may seem discrepant from the more common Road of acceptance in other Writings, yet will not this widen the difference, if such Termes or Phrases be proper to *Arithmetick*: For all things here are to be taken in congruity thereto. Wherefore a Perfect Number shall not be Chymically, but Arithmetically so; for with the *Chymists* 10 is a Perfect Number. *Chymical Collections*, p. 92. but seeing the Aliquot parts of 10 will make but 8, Arithmeticians count it Imperfect and Defective.

10ly. It's more than probable, That in the First Chapter of *Geodaticals* some of the Divisions of Foreign or Domestick Denominations may have lost, or in time may lose much of their Propriety to the present State of Affairs, or new Accompts of such things as there declared, they having been adapted to the Elder Laws, Customs, and Usages of Kingdoms and Countries, yet mutable and alterable as Reasons of State or other Contingencies in every Kingdom, or Countrey shall or may enforce. Wherefore (this Exception, if not provided against by sufficient Caution and warning thereof given in the Chapter it self) let allowances be made as occasion shall require, either by taking them in the *Præterperfect-Tense*, or making all not certainly known, but Suppositions, which nevertheless will not prejudice the truth of the Conclusions where such Suppositions are but Conditionals, since it is not necessary in the Resolution of a Question, that the Suppositions be true, but that the Conclusion be true according to such a Conditional Supposition. For if 120 be counted for an Hundred, then 2 Hundred must be 240, but it will not follow that the Hundred must alwaies be reckoned at that rate.

11ly. No Man may stumble at the use of the Word Infinite or endless, it being intended only beyond the Power or Skill of Man to count or cast up, not Infinite in a proper or abstract Sence, for so nothing is

The Author's Epistle.

Infinite but God, and it is impossible there should be two Infinites.

And lastly, if besides all this, the Lawyers will favourably overlook the unintentional misquoting or representing any Statute or Law and Construction thereof, (if any such be,) And Merchants, Goldsmiths, and other Artificers will bear with the Termes in Questions of their particular Concerns, as possibly not so proper to their Professions as others best known to themselves, And Grammarians add their usual Grains of Allowance (due on the Score of humane frailty) for misplacing of Letters, misspelling of Syllables, mispointing of Sentences, Omission of Points, Parentheses, &c. There cannot be much left to need excuse; but if there be, it must now stand or fall to it's own Master.

Whilst we all live in the Atmosphære, no doubt but in a clear day some Motes may be seen; and considering the Imperfections of the Penman, discomposures and disadvantages under which, most was wrote or transcribed (enough to have distracted a more accurate Accomptant and abler Pen:) 'tis well, if there be no more faults, than the unprejudiced Reader in common Charity can or ought to remit or pass by. And inasmuch as none that may be found are wilful or intentional, I will not be so uncharitable as to think I shall want their Charity or candid Construction thereof.

I never thought *Humanum est errare*, was, or ever will be a warrant or plea sufficient to commit or excuse the commission of Wilfull Transgressions; but for involuntary Errors, Mistakes, and Aberrations, it hath always been accompted sufficient to cover them when committed.

To please or displease, is an inconsiderable and unprofitable end in Writing, and they that aim at the former commonly miss. It was said of old, *Ne Jupiter quidem omnibus placet*. I write neither to praise or dispraise, flatter or bespatter any. Some who being dead yet speak by their Writings I have here and there as worthy of Commendation valued, and am yet of Opinion this Nation stands much obliged among others to *Record* of old, and *Oughtred* of late, for their labours in this Art; the former for his plainness and clearness in those things he hath handled, and the latter for his piercing apprehension into the more lofty and mysterious parts of the Mathematicks: But none dead or living have I any where wrongfully charged, or mentioned without due reverence to their Persons and respect to their works. So far as I might not contract a guilt of leading or misleading others into a dark unpleasant and perplexed Path where one more plain was nearer hand. And many of their Examples (whether corruptly wrought or printed I cannot say) I have purposely chosen to correct the Errors found therein, that if this
Impression

The Author's Epistle.

Impression escape the *Errata* will need no further amendment.

However whether their Lines or these be faulty, it is not so much material, if no blemish thereby be imputed to the Art it self. *Arithmetick* is a Noble and high born Science, useful and profitable in several things both divine and humane, lends to many, but borrows of few, and hath midwifed into the World divers excellent Atchievements, which without her aid and assistance would have been Still-born, and slept an Everlasting Sleep. And because a fit Encomium thereof deserves a most clear and unclouded capacity I shall desist. Nor shall I undertake to exhibit a Narrative of the rise, growth, production or propagation of *Arithmetick*, nor yet to catalogize so much as the *English* Authors thereof; all I have further to say is

Whoever would profit hereby, must begin at the beginning, and go through every part gradually, but (if like most Scholars a little will content) the Two First Books, and the Second Part of the Fourth, will furnish the Learner for Merchants Affairs Trade and ordinary Commerce in a good measure; the other is written *Non cuivis*, but for those who shall not think their labour lost in such Speculations, to whom though *Studiorum radices amaræ*, yet *fructus erunt sapidi*.

Thus (my Son) as in some Projections of *Geometry* and *Astronomy* my thoughts have out-done me, so in this of *Arithmetick* I have out-done my thoughts in the length both of the Epistle and Book; to shorten which I have forborne to exemplifie the Prints of the several Coines mentioned in the First Chapter of *Geodeticals*, and to pursue my intentions as well about the Addition and Substraction of *Surdes*, especially of Universals, as the Division of *Surdes* and *Species*, wherein somethings might not have been unprofitably added, as likewise in *Progression* of both kinds by Instances in *Fractions* and *Decimals*: Nevertheless what is written may be sufficient to the ingenious, and therefore I shall say no more but conclude, The Lord bless thee and it, and make both instrumental to his glory, which is and will be the hearty and earnest Prayer of

Rye, March 25th.

1674.

Thy Loving Father

S A. J E A K E.

Books

Books principally consulted with in the ensuing Treatise ,
besides the Sacred Scriptures, divers Histories, Lexicons,
Dictionaries, &c.

Authors.	Titles.		Impressions.
John Henry	Alsted.	Encyclopædia.	Lugd. 1649.
Richard	Balam.	Algebra.	Lond. 1653.
Isaac	Barrow.	Euclide's Elements.	Lond. 1660.
Thomas	Blundevile.	Exercises.	
Henry	Brigges.	Logarithmica Arithmetica.	Lond. 1624.
		Logarithmical Arithmetick.	Lond. 1631.
Michael	Dalton.	The Countrey Justice.	Lond. 1643.
Michael	Dary.	Dary's Diary.	Lond. 1650.
John	Dee.	Mathematical Preface, &c.	Lond. 1651.
Richard	Delamain.	Grammelogia.	Lond. 1630.
Thomas	Digges.	Pantometria.	Lond. 1591.
		Stratoticos.	Lond. 1579.
William	Eldred.	The Gunners Glaſs.	Lond. 1646.
Henry Van	Etten.	Mathematical Recreations.	Lond. 1633.
Giles	Fletcher.	History of Russia.	Lond. 1657.
Thomas	Fuller.	Pisgah Sight of Palestine.	Lond. 1650.
Edward	Grimstone.	Eſtates &c. of the World.	Lond. 1615.
Edmund	Gunter.	Works, &c. of the Sector, &c.	Lond. 1653.
Peter	Heylin.	Cosmography.	Lond. 1657.
Nicolas	Hunt.	Handmaid to Arithmetick.	Lond. 1633.
John	Johnson.	Johnson's Arithmetick.	Lond. 1657.
Gerard	Malines.	Lex Mercatoria.	Lond. 1636.
Jonas	Moore.	Moore's Arithmetick.	Lond. 1650.
John	Nepair.	Rabdologia.	Lugd. 1626.
William	Oughtred.	Clavis Mathematicæ limata.	Oxon. 1652.
		Circles of Proportion.	Oxford 1660.
Seth	Partridge.	Rabdologia.	Lond. 1648.
Ferdinando	Pulton.	Collection of Statutes.	Lond. 1640.
Peter	Ramus.	Way to Geometry.	Lond. 1636.
William	Raſtal.	Collection of Statutes.	Lond. 1572.
Robert	Record.	Ground of Arts.	Lond. 1636.
		Whetstone of Wit.	Lond. 1557.
John	Speidel.	Treatiſe of Sphærical Triangles.	Lond. 1627.
Christian	Viſtitiuſ.	Elementa Arithmeticæ.	Baſil 1602.
William	Webſter.	The Principles of Arithmetick.	Lond. 1634.
Vincent	Wing.	Harmonicon Cœleſte.	Lond. 1651.
		Aſtronomia Inſtaurata.	Lond. 1656.
Richard	Wit.	Arithmetical Queſtions.	Lond. 1613.
Edward	Wright.	Deſcription, &c. of Logarithmes	Lond. 1618.
		The Pathway to knowledge.	Lond. 1596.

&c.

The

The Contents of the Chapters.

The First Book.

Part 1. Chap.	1. <i>The Introduction.</i>	Page 1
	2. <i>Nature of Numbers.</i>	4
	3. <i>Elements of Numbers.</i>	9
	4. <i>Of Integers.</i>	14
	5. <i>Addition of Integers.</i>	16
	6. <i>Subtraction of Integers.</i>	18
	7. <i>Multiplication of Integers.</i>	21
	8. <i>Division of Integers.</i>	29
Part 2. Chap.	1. <i>Of Fractions.</i>	41
	2. <i>Reduction of Fractions.</i>	44
	3. <i>Addition of Fractions.</i>	50
	4. <i>Subtraction of Fractions.</i>	52
	5. <i>Multiplication of Fractions.</i>	55
	6. <i>Division of Fractions.</i>	57

The Second Book.

Part 1. Chap.	1. <i>Of Geodæticals.</i>	Page 61
	2. <i>Reduction of Geodæticals.</i>	152
	3. <i>Addition of Geodæticals.</i>	160
	4. <i>Subtraction of Geodæticals.</i>	162
	5. <i>Multiplication of Geodæticals.</i>	165
	6. <i>Division of Geodæticals.</i>	169
Part 2. Chap.	1. <i>Of Figurals.</i>	173
	2. <i>Production of Figurals.</i>	179
	3. <i>Extraction of Roots.</i>	192
	4. <i>Figurate Fractions.</i>	206

The Third Book.

Part 1. Chap.	1. <i>Of Decimals.</i>	Page 207
	2. <i>Reduction of Decimals.</i>	210
	3. <i>Indices Added and Subtracted.</i>	216
	4. <i>Addition of Decimals.</i>	217
	5. <i>Subtraction of Decimals.</i>	218
	6. <i>Multiplication of Decimals.</i>	219
	7. <i>Division of Decimals.</i>	222
	8. <i>Figuration of Decimals.</i>	225
Part 2. Chap.	1. <i>Of Astronomicals.</i>	Page 229
	2. <i>Reduction of Astronomicals.</i>	231
	3. <i>Addition of Astronomicals.</i>	236
	4. <i>Subtraction of Astronomicals.</i>	237
	5. <i>Multiplication of Astronomicals.</i>	239
	6. <i>Division of Astronomicals.</i>	240
	7. <i>Figuration of Astronomicals.</i>	242
	8. <i>Of the Sexagenary Table.</i>	246

The Contents of the Chapters.

Part 3. Chap.	1. Of Logarithmes.	Page 249
	2. Reduction of Logarithmes.	257
	3. Addition of Logarithmes.	261
	4. Substraction of Logarithmes.	264
	5. Multiplication of Logarithmes.	266
	6. Division of Logarithmes.	268
Part 4. Chap.	1. Of Cossicks.	Page 271
	2. Addition of Whole Cossicks.	274
	3. Substraction of Whole Cossicks.	275
	4. Multiplication of Whole Cossicks.	277
	5. Division of Whole Cossicks.	278
	6. Of Broken Cossicks.	279
	7. Reduction of Broken Cossicks.	280
	8. Addition of Broken Cossicks.	282
	9. Substraction of Broken Cossicks.	284
	10. Multiplication of Broken Cossicks.	286
	11. Division of Broken Cossicks.	287
	12. Figuration of Cossicks.	289
Part 5. Chap.	1. Of Surdes.	Page 293
	2. Reduction of Surdes.	296
	3. Addition of Simple Surdes.	298
	4. Substraction of Simple Surdes.	300
	5. Multiplication of Simple Surdes.	304
	6. Division of Simple Surdes.	306
	7. Addition of Compound Surdes.	308
	8. Substraction of Compound Surdes.	314
	9. Multiplication of Compound Surdes.	319
	10. Division of Compound Surdes.	322
	11. Of Fractionary Surdes.	329
	12. Figuration of Surdes.	330
Part 6. Chap.	1. Of Species.	Page 334
	2. Addition of Integral and Rational Species.	338
	3. Substraction of Integral and Rational Species.	339
	4. Multiplication of Integral and Rational Species.	340
	5. Division of Integral and Rational Species.	342
	6. Reduction of Fracted and Rational Species.	343
	7. Addition of Fracted and Rational Species.	346
	8. Substraction of Fracted and Rational Species.	348
	9. Multiplication of Fracted and Rational Species.	351
	10. Division of Fracted and Rational Species.	352
	11. Figuration of Rational Species.	354
	12. Reduction of Irrational Species.	360
	13. Addition of Simple Irrational Species.	361
	14. Substraction of Simple Irrational Species.	362
		Chap. 15.

The Contents of the Chapters.

15.	<i>Multiplication of Simple Irrational Species.</i>	Page 363
16.	<i>Division of Simple Irrational Species.</i>	364
17.	<i>Addition of Compound Irrational Species.</i>	365
18.	<i>Subtraction of Compound Irrational Species.</i>	367
19.	<i>Multiplication of Compound Irrational Species.</i>	369
20.	<i>Division of Compound Irrational Species.</i>	371
21.	<i>Figuration of Irrational Species.</i>	374

The Fourth Book.

Part 1. Chap.	1.	<i>Of Ratio's.</i>	Page 405
	2.	<i>Reduction of Ratio's.</i>	411
	3.	<i>Addition of Ratio's.</i>	412
	4.	<i>Subtraction of Ratio's.</i>	414
	5.	<i>Multiplication of Ratio's.</i>	416
	6.	<i>Division of Ratio's.</i>	417
Part 2. Chap.	1.	<i>Of Proportions Disjunct.</i>	Page 418
	2.	<i>The Direct Rule of Three.</i>	421
	3.	<i>The Indirect Rule of Three.</i>	425
	4.	<i>Practice.</i>	427
	5.	<i>Specificks.</i>	436
	6.	<i>The Rule of Five Numbers Direct.</i>	449
	7.	<i>The Rule of Five Numbers Indirect.</i>	452
	8.	<i>Fellowship.</i>	456
	9.	<i>Alligation.</i>	467
	10.	<i>Barter and Exchange.</i>	479
	11.	<i>Loss and Gain.</i>	486
	12.	<i>Æquation of Payment.</i>	490
	13.	<i>Factorship.</i>	494
	14.	<i>Falshood or Position.</i>	496
	15.	<i>Proportions Doubled.</i>	505
	16.	<i>Proportions Tripled, &c.</i>	515
Part 3. Chap.	1.	<i>Of Proportions Continued.</i>	Page 529
	2.	<i>Progression Arithmetical.</i>	530
	3.	<i>Transposition.</i>	540
	4.	<i>Technologie.</i>	542
	5.	<i>Progression Geometrical.</i>	560
	6.	<i>Transmutation.</i>	576
	7.	<i>Anatocism.</i>	578
Part 4. Chap.	1.	<i>Of Æquations.</i>	Page 613
	2.	<i>Invention of Æquations.</i>	616
	3.	<i>Reduction of Æquations.</i>	620
	4.	<i>Resolution of Æquations.</i>	622

An Appendix.

Properties of some Numbers.

662

Arithmetick

ARITHMETICK.

The First BOOK,

CONCERNING
Numbers absolutely abstract;

In Two PARTS.

WHEREIN
INTEGERS } are { Examined,
FRACTIONS } { Explained,
AND THEIR
SIMPLE ELEMENTS.

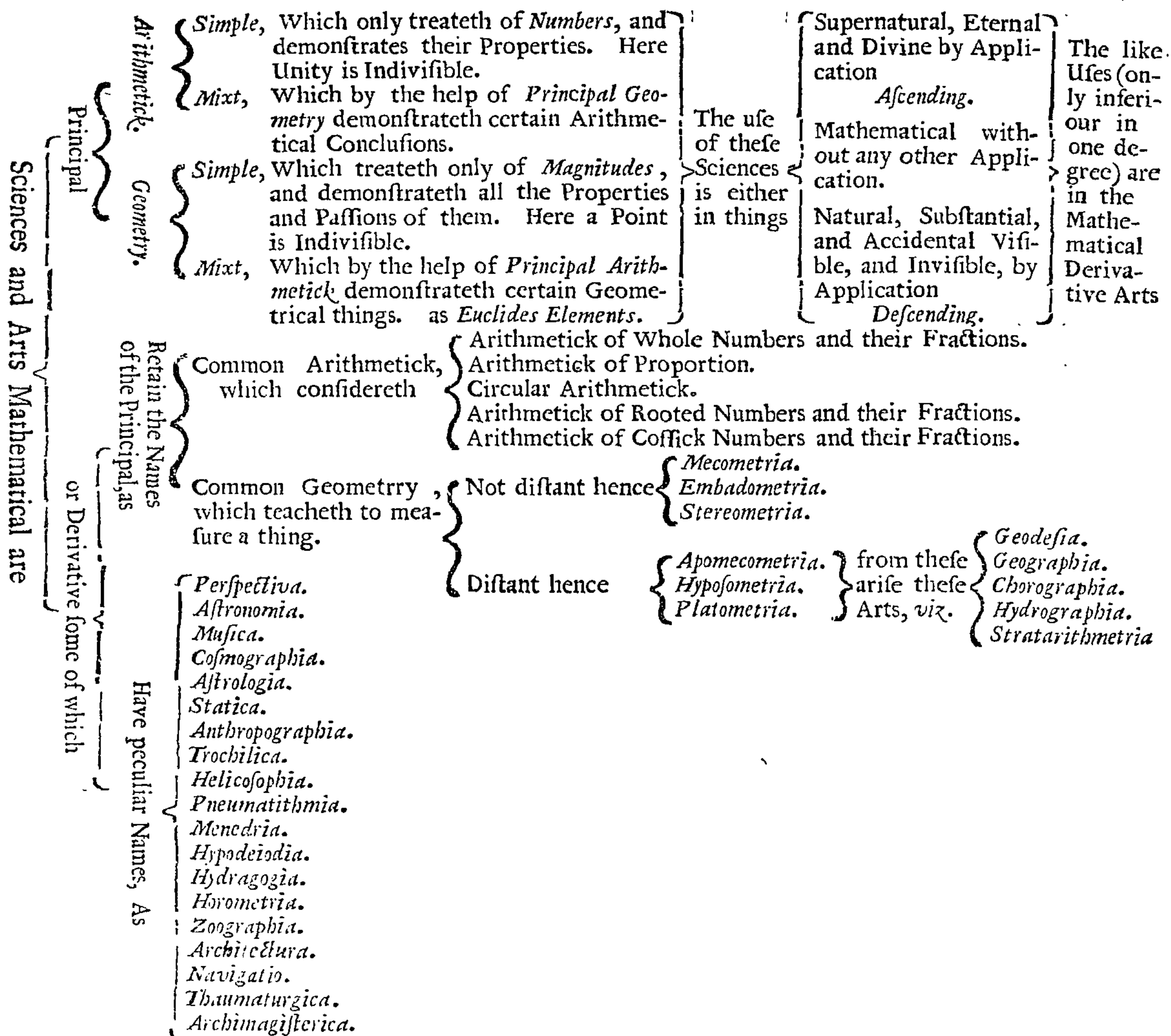
CHAP. I.

A Brief Introduction to the Parts of Arithmetick hereafter handled.

THE Noble Science of *Arithmetick* is the very Foundation of the *Mathematicks*, whose *Roots* are *Number* and *Magnitude*. The Original of the one an *Unit*, of the other a *Point*; but the *Products* of either are Infinite. The *Mathematical Science* specifically appropriate to *Number* is *Arithmetick*; to *Magnitude*, *Geometry*. From these as Two Principal Springs *Mathematical* considered simply or *per se*, and mixt or *inter se* are derived many other Excellent Arts and Mysteries; useful not only in Natural, Substantial, Accidental, Visible, and Invisible things, but also in Supernatural and Divine, as the Learned *John Dee* affirmeth in his *Mathematical Table* inserted in *John-Henry Alsted* his *Encyclopædia*, lib. 2. p. 70. which I have thought fit here to Translate.

Arithmetick
what the
ground of.
The Original an
Unit.

Dr. Dee his
Mathematical
Table.



Number helpful to search out the Creature Vertues, Natures, &c. Number of Universal Use.

Without Numbers, no Knowledge of the Mathematicks.

Threefold Consideration of Number.

Here, under the Third Consideration to be viewed in Four Books.

1.

2.

And the same Doctor *Dee* in his Mathematical Preface to the first Six Books of *Euclides Elements* further assures us, That *Number* seems to be so Immaterial and Pure, that thereby we may wind our selves into the deep Search and View of all Creatures distinct Vertues, Natures, Properties, Forms, &c. The Universal Use of *Number* is Witnessed to also, by the Noble *Picus*, Earl of *Mirandula*, who had set up in *Rome* Nine Hundred Conclusions in all kinds of *Sciences* openly to be disputed of; and in the Eleventh Conclusion saith, By *Numbers*, a way is had to the Searching out, and Understanding of every thing able to be known.

Boetius and others are not far behind them in Commending *Number*; but had it none of their *Encomiums*, yet certain it is, That whosoever would fit himself for the *Mathematicks*, unless he begin with the Science of *Numbers*, will quickly find himself in a Labyrinth, from which he can never escape, nor deliver himself from many Inextricable Doubts, without the Assistance thereof.

Great *Mathematicians* and *Philosophers* have considered *Number*: First, In Respect of the Creator, Simple, Pure, and Immixt. Secondly, In Reference to Spiritual and Angelical Minds, Including the Soul of Man: And, Thirdly, In relation to every Creature, and their compleat Constitution: And in the First and Second respect, term it *Number* numbring; but in the Third *Number*, numbred. In the sense of the Two former I shall not intermeddle to treat of *Numbers*, referring it to a more Able and Divine Pen. But in the latter respect, and that in the lowest and most gross consideration of *Numbers*, *Viz.* In Conference and Coincidence to Visible, Material, and Corporeal things, I have shadowed out this Science of *Numbers* in the Ensuing Treatise, under the Name of *Arithmetick Surveyed and Reviewed*: Containing Four Books The First Treating of *General Arithmetick*, Examining the *Simple Elements* of *Abstract Numbers*, in Two Parts, *viz.* *Integers* in the First Part, and *Fractions* in the other Part; although this latter after a Sort may be called *Contract*, in respect to their *Denominators*. The Second and Third Books handle *Special Arithmetick*, Explaining the *Simple Elements*

ments of *Contract Numbers*, more generally as of *Geodeticals* and *Figurals*, in the Second Book, in Two Parts : Those more Specially *Contract*, are dealt with in the Third Book, in Six Parts ; viz. *Decimals*, *Astronomicals*, *Logarithmes*, *Cossicks*, *Surdes*, and *Species*, in each Part distinctly. In the Fourth Book is taught the Whole Doctrine of Proportions , *Disjunct*, *Continual*, and *Aequated*. And in one or other of these, (especially as to the Practick Part of *Arithmetick*) is included what Others otherwise have divided and delivered concerning *Numbers* ; under that Name of *Arithmetick*.

Arithmetick derives its *English* Name from the Latine *Arithmetica* ; and this again from the Greek *Ἀριθμός*, signifying as *Keckerman* saith, *Phys. Lib. 1.* both to Number and Measure ; and so may truly include *Arithmetick* mixt with *Geometry*, as before hinted ; but with us is generally taken for the Art of Numbring Restrictively, intending only its Essentiall Consistence in *Numbers*, and but Collateral Converse with *Magnitudes* : In which Notion I am contented likewise to take it, reserving not only a Liberty to understand it in a higher Note, when Occasion serves ; but also to accept it under the Vulgar Terms of *Casting Accompt* and *Cyphering*, till they that so Abase this high born Science shall see it worthy of a more Excellent Name than those.

Arithmetick,
whence the
Name.

Is the Science
of Numbring
generally taken.

Cyphering too
vulgar a Name.

The Antient Hebrews and Greeks Numbred by their Letters ; as Ensueth in these Tables.

Hebrew Account.

Sec. Masorethas.	א	Two Hundred.	ל	Thirty.	יא	Eleven.	א	One.
	ב	Three Hundred.	מ	Forty.	יב	Twelve.	ב	Two.
	ג	Four Hundred.	נ	Fifty.	יג	Thirteen.	ג	Three.
	ד	Five Hundred.	ס	Sixty.	יד	Fourteen.	ד	Four.
	ה	Six Hundred.	ע	Seventy.	טו	Fifteen.	ה	Five.
	ו	Seven Hundred.	פ	Eighty.	טז	Sixteen.	ו	Six.
	ז	Eight Hundred.	צ	Ninety.	יז	Seventeen.	ז	Seven.
	ח	Nine Hundred.	ק	One Hundred.	יח	Eighteen.	ח	Eight.
	ט	One Thousand.		dred.	יט	Nineteen.	ט	Nine.
	י				כ	Twenty.	י	Ten.

Greek Account.

I α'. One.	ΔΠΙΙ. ιζ'. Seventeen.	Ϟ Η. χ'. Six Hundred.	ΜΜΜ. μ. Forty Thousand.
II β'. Two.	ΔΠΙΙΙ. ιη'. Eighteen.	ϞΗ. ψ'. Seven Hundred.	ΙΜΙ. ν. Fifty Thousand.
III γ'. Three.	ΔΠΙΙΙΙ. ιθ'. Nineteen.	ϞΗΗ. ω'. Eight Hundred.	ΙΜΙΜ. ξ. Sixty Thousand.
IIII δ'. Four.	ΔΔ. κ'. Twenty.	ϞΗΗΗ. π'. Nine Hundred.	ΙΜΙΜΜ. ρ. Seventy Thousand
Π ε'. Five.	ΔΔΔ. λ'. Thirty.	Χ. α. One Thousand.	ΙΜΙΜΜΜ. π. Eighty Thousand
ΠΙ ς'. Six.	ΔΔΔΔ. μ'. Forty.	XX. β. Two Thousand.	ΙΜΙΜΜΜΜ. ς. Ninety Thousand.
ΠΙΙ ζ'. Seven.	Ε. ν'. Fifty.	XXX. γ. Three Thousand.	ρ. One Hundred Thousand.
ΠΙΙΙ η'. Eight.	Ϟ Δ. ξ'. Sixty.	XXXX. δ. Four Thousand.	ς. Two Hundred Thousand.
ΠΙΙΙΙ θ'. Nine.	Ϟ ΔΔ. ς. Seventy.	Ϟ. ς. Five Thousand.	τ. Three Hundred Thousand.
Δ. ι. Ten.	Ϟ ΔΔΔ. π'. Eighty.	ϞΧ. ζ. Six Thousand.	υ Four Hundred Thousand.
ΔΙ ια'. Eleven.	Ϟ ΔΔΔΔ. ς. Ninety.	ϞXX. ζ. Seven Thousand.	φ. Five Hundred Thousand.
ΔΙΙ ιβ'. Twelve.	Η. ρ'. One Hundred.	ϞXXX. η. Eight Thousand.	χ. Six Hundred Thousand.
ΔΙΙΙ ιγ'. Thirteen.	ΗΗ. σ'. Two Hundred.	ϞXXXX. θ. Nine Thousand.	ψ. Seven Hundred Thousand.
ΔΙΙΙΙ ιδ'. Fourteen.	ΗΗΗ. τ'. Three Hundred.	Μ. ι. Ten Thousand.	ω. Eight Hundred Thousand.
ΔΠ. ιε'. Fifteen.	ΗΗΗΗ. υ'. Four Hundred.	ΜΜ. κ. Twenty Thousand.	π. Nine Hundred Thousand.
ΔΠΙ. ις'. Sixteen.	Ϟ. φ'. Five Hundred.	ΜΜΜ. λ. Thirty Thousand.	Ϟc.

The Latines made use only of Seven of their Letters : Viz. C. D. I. L. M. V. X.

Latine Account.

I.	One.	XII.	Twelve.	L.	Fifty.	DCC.	Seven Hundred.
II.	Two.	XIII.	Thirteen.	LX.	Sixty.	DCCC.	Eight Hundred.
III.	Three.	XIIII. XIV.	Fourteen.	LXX.	Seventy.	DCCCC.	Nine Hundred.
IIII. IV.	Four.	XV.	Fifteen.	LXXX.	Eighty.	M. c. l. c. c.	One Thousand.
V.	Five.	XVI.	Sixteen.	XC.	Ninety.	V. v. c. c. l. c. c.	Five Thousand.
VI.	Six.	XVII.	Seventeen.	C.	One Hundred.	X. c. c. l. c. c. m. c.	Ten Thousand.
VII.	Seven.	XXII. XIX.	Eighteen.	CC. c.	Two Hundred.	L. l. c. c. l. c. c.	Fifty Thousand.
VIII.	Eight.	XIX.	Nineteen.	CCC.	Three Hundred.	C. c. m. c. c. c. c. l. c. c.	One hundred Thousand.
VIIII. IX.	Nine.	XX.	Twenty.	CCCC.	Four Hundred.	D. l. c. p. c. c. l. c. c. c.	Five hundred Thousand.
X.	Ten.	XXX.	Thirty.	D. l. c.	Five Hundred.	c. c. c. c. l. c. c. c.	One Million.
XI.	Eleven.	XL.	Forty.	DC.	Six Hundred.		Ϟc.

The

Numeral Let.
of the English.

The *English* following the *Latine* made up their Antient Accompts by the same Seven Letters: Accompting I. one. V. five. X. ten. L. fifty. C. a hundred. D. five hundred. and M. a Thousand, &c.

Arithmetick
best performed
by Figures.

The Art of *Numbers* being best performed by the Pen with the *Arabick* Notes, or Characters, commonly called Figures. I shall altogether wave the Practise thereof by Counters, and Letters, and all other Instrumental Arithmetick (save only a little touch of *Nepairs* Bones) and remit those desirous to learn such kind of Operations to the several Treatises of the respective Authours who have dealt therewith.

CHAP. II.

Of the Nature of Numbers.

What Arith-
metick is.

IN pursuance of what hath been said, and hereafter followeth *Arithmetick* may be defined, The Science of *Numbring* well. In *Arithmetick* two things are principally to be considered; The Subject, in which the Nature of *Numbers* is declared: And the Affection of the Subject, wherein the Elements of *Numbers* are exercised. The Subject of *Arithmetick* is *Number*, all the Precepts and Doctrines of the whole Art, having a particular Relation thereunto. *Number* is first taken restrictively, and so *Number* is considered only according to it's multitude or capacity of Units; hence *Euclide*, lib. 7. def. 2. will have *Number* to be a multitude of Units. An Unite is indeed the Original of *Number*, but being in the Prædicament of Quantity, and in *potentia*, may well pass for a *Number*. And *Euclide* himself in the 18, 19, and 20. Definitions of the same Book, cannot be excused from accompting an Unite for a *Number*. Secondly, *Number* is taken more largely for the Quantity according to which any thing is Numbred, or Accompted; So according to an Unite any singular thing is said to be One; as one Man, one House, and such like; and two things, as Stars, Ships, Men, &c. are said to be two, according to that *Number*: And in this Metaphysical Sense, not only the Unit it self, but the parts of an Unite, as one half of a Day, two third parts of a Pound, &c. are reckoned for *Numbers*.

The Subject
thereof.

Number taken
restrictively.

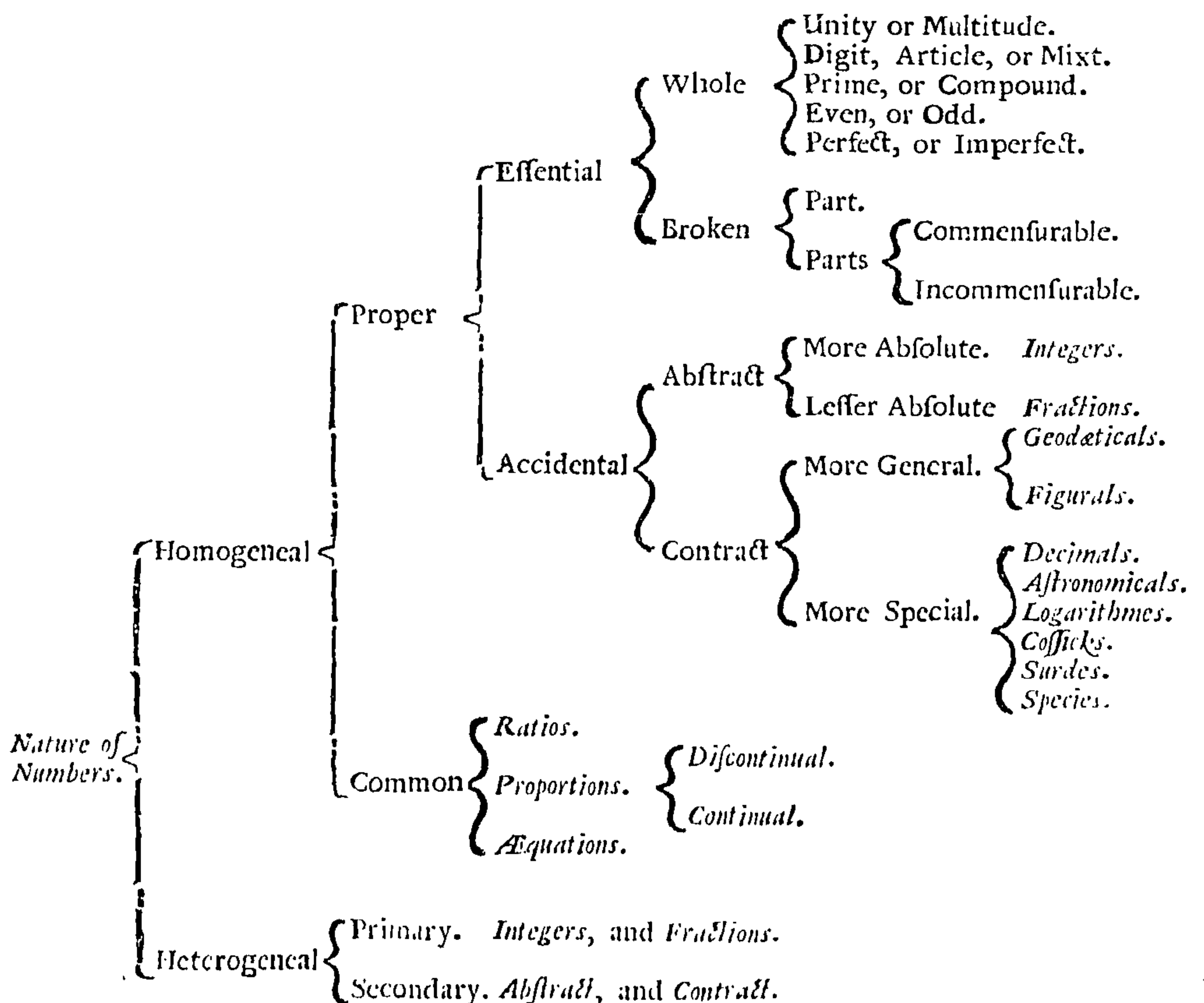
How an Unit
is a Number.

Number taken
largely.

Parts of an U-
nite how Num-
bers.

Numbers therefore largely taken, may be better discern'd in their proper Natures, as they are capable of being Divided: A brief *Synopsis* whereof followeth.

A Synopsis of
the Nature of
Numbers.



For

For the Explanation whereof observe *Numbers* according to their Nature have some peculiar properties to themselves, and something common with others; and are of two sorts, *Homogeneous*, and *Heterogeneous*; *Homogeneous* are of the same Nature or Kind, whether Proper, or Common. Essential, or Accidental. Whole, or Broken, Abstract, or Contract: *Numbers Heterogeneous* are mixt *Numbers* of Whole and Broken, Abstract and Contract.

Whole Numbers are called *Integers* from the *Latine*, *Integrum*, and *Broken Numbers*, *Fractions*, from the *Latine*, *Fractio*.

A proper *Whole Number* according to his Essence, passeth under a five fold Consideration, First, Whither it be Unite or Multitude. Secondly, Digit, Article, or Mixt, Thirdly, Prime, or Compound. Fourthly, Even, or Odd: And Fifthly, Perfect, or Imperfect.

An Unite is the beginning of *Multitude*, and foundation of *Number*; never more than one. *Multitude* is the Collection of Unites, as one, one, one, are equal to three, being Collected together, &c.

Secondly, *Integers* are again divided into Digits, Articles, and Mixt Numbers. A Digit is any *Integer* under Ten, as One, Two, Three, Four, Five, Six, Seven, Eight, Nine; Digits are sometimes called *Monades*. An Article is Ten, with all Whole Numbers that may be justly divided into Ten parts, as Twenty, Thirty, Forty, &c. These are called Round Numbers, and sometimes *Decades*. A Mixt Number is compound of an Article, and a Digit, as Eleven, Twelve, &c. Twenty one, Twenty-two, &c.

Thirdly, A Number, Prime, called also Simple, and sometime Uncompound, is a Number made only by *Addition*, or Collection of Units, and not by *Multiplication*, so an Unit only can measure it; as Two, Three, Five, &c. Compound Numbers are such as are made by *Multiplication* of two Numbers together, and not by *Addition*, though they may seem to be made of both: Yet because they may be measured only by the Numbers of which they are compounded, they are not to be accounted Prime, as Six made by *Multiplication* of Two and Three, is Compound, because by either of them it may be measured, or divided; and though it may be made by *Addition* of Five and One, yet shall it not be Prime; for that neither Five nor One, can equally measure Six.

Fourthly, *Numbers* are again considered as they are, Even or Odd; Even may be divided into Two Equal Parts, Odd cannot; Even *Numbers* are distinguished into Three Sorts. First, Even *Numbers* Evenly, these continually may be parted into halves, till you come to an Unite, as Sixteen, into Eight, Four, into Two, One, &c. Secondly, Even *Numbers* Oddly, these may be parted into equal halves, but the halves will be odd *Numbers*, as Ten into Five, and Five, &c. Thirdly, Even *Numbers*, Evenly and Oddly, as those which may a while be parted into even halves; but before you come to one, the halves will be odd *Numbers*, as Twelve into Six, Three, &c.

Fifthly, *Integers* are differenced as they are Perfect or Imperfect. Perfect *Numbers* are such whose aliquot or even parts joyned together will exactly return the whole *Number*, as Six, Twenty-Eight, &c. for of Six the half is Three, the Third part Two, and the Sixth part One, which added together, make Six; and it hath no more aliquot parts in whole *Numbers*, for the fourth part, is one and an half, and the fifth part, one and a fifth. So Twenty-Eight, whose parts being Fourteen, Seven, Four, Two, and One, exactly return Twenty-Eight, which therefore appears to be a perfect *Number*. Perfect *Numbers* are almost as rare as perfect Men; for between One, and One Million of Million there are but Ten; and the Twentieth perfect *Number* exceeds the value of Hundred Thousand Million of Million of Million. *Mathemat. Recreations*, p. 92. Imperfect *Numbers* on the Contrary, are those whose even parts added together, will not return the Primary *Number*, whose parts they be: And these are either *Abundant*, or *Defective*. *Abundant* called also superfluous, whose parts added together make more than the whole *Number*; as Twelve, whose parts being one, two, three, four, and six, together make sixteen; So the parts of Twenty make Twenty two, &c. *Defective* are such, whose parts added together make less than the *Integer*; as Eight, whose parts being one, two, and four, make but seven; likewise the parts of Sixteen make but Fifteen, and of Forty Five make but Thirty Three.

Fractions or *Broken Numbers* arise from the Division of the Unite into parts, and therefore are properly parts of *Numbers*, because every proper *Fraction* is less than an Unite; as when a Unite signifieth any Denominate Quantity, in respect to that Quantity, it may be divided into lesser parts than that one Quantity; So one Pound parted into Four Equal parts, then shall one half, or three Quarters thereof, be less than one whole Pound, and these, and such parts be called *Fractions* or *Broken Numbers*. These are considered as they contain, either one part of the whole, or more parts; as one half,

Numbers Homogeneous are proper and common. Proper Essential, or Accidental.

Integers and Fractions, whence the Names.

Integers considered five ways Essentially.

1. *One or Many.*

2. *Digits, Articles, and mixt. Digits called Monades.*

Articles called Decades.

3. *Prime or Compound.*

4. *Even or Odd. Even of Three sorts.*

1.

2.

3.

5. *Perfect or Imperfect.*

Perfect Numbers very rare.

Imperfect of two sorts.

Abundant.

Defective

Fractions, whence they arise.

Doubtly considered five ways Essentially.

then shall the whole be divided but into two parts, but three quarters denote the Unite to be parted into four parts, and the Fraction to contain three of them : These parts are either Commensurable, or Incommensurable.

The latter sort
either Commensurable.

or,

Incommensurable.

Commensurable, when the two Termes of the *Fraction* have any common part, that will equally divide them both ; as Three being a part of Twelve and Fifteen will divide them both equally: therefore Twelve Fifteenths being the Termes of a Fraction, are parts Commensurable, and may be measured by Three, &c.

Incommensurable on the contrary have no such parts for a common Divisor, as Eighteen Twenty Fiftths, for Twenty Five can be equally divided by no Number but Five, and Five cannot divide Eighteen.

Homogeneous
Numbers considered.

Accidentally as
Abstract and
Contract.

Abstract more
or less Absolute.

More Integers.

The Nature of *Numbers Homogeneous* are further discerned in some properties accidental to them singly, or amongst themselves, and that is to be Abstract, or Contract : That called numbring, or formal ; this numbred, or material.

Abstract are such as have no Denomination annexed to them ; These are more or less Absolute : The more Absolute are *Integers*, as One, Two, Three, or any other whole Number, without any denomination, or surname, relation, or comparison, &c. there- to belonging, but are free to value Men, Money, Years, or any other quantity. Of *Integers* and their Affections, *Vide plus lib. 1. par. 1.*

Less Fractions.

The less Absolute are *Fractions*, as one quarter, one half, two thirds, &c. though Absolute without respecting Denominations, and therefore left free to be parts of Weight, Measure, Time, or any other quantity, or thing whatsoever ; yet must be considered in respect of the one number to the other. So that *Fractions* are not so Absolute as *Integers*, but in a sort, as I said before, may be accounted for a sort of Denominate Numbers, relation being had between the Numerator, and Denominator. Of these *Vide plus lib. 1. par. 2.*

The same Numbers Abstract, are again to be consider'd, as they may be Contract.

Contract.

Contract, called also *Concrete Numbers*, proceed from Abstract, and are such as are refringed by the Annexion of some or other Denomination, as Two Groats, Ten Shillings, Two Thirds of a Pound, &c. where the Numbers are restrained from the liberty of valuing any thing else, but Groats, Shillings, &c. according to the annexed Denomination. Some *Contract Numbers* are contracted more generally, as *Geodaticals* and *Figurals* : Others more specially, in respect to the several sorts of Denominations, by which they are contracted.

Generally.

Geodaticals.

These pass for
vulgar denomi-
nate Numbers.

Numbers Geodatical, which are considered according to those Vulgar Names or Denominations, by which Money, Weights, Measures, &c. are generally known, or particularly divided by the Laws and Customs of several Nations ; these spring from Denominate *Fractions*, and by omitting the Denominators, when the parts are commonly known, and annexing names Artificial or Inartificial, making some Mark or Sign to distinguish them, they pass into the Catalogue of vulgar Denominate Integral *Numbers* ; As because the twentieth part of a pound *sterling* is well known by the name of a Shilling, we use to say Three, Four or Five Shillings, &c. and not Three Twentieths, Four Twentieths, and Five Twentieths of a Pound. The like is done also in the Twelve parts of a Shilling ; as for One Twelfth, Two Twelfths, Three Twelfths, &c. we say One penny, Two pence, Three pence, &c. The same also may be understood of the parts of Weights, Measures, Time, Motion, &c. Of these more particularly may be seen, in the 1st Part of the 2^d Book.

Figurals.

These of 3 sorts.

The second sort of more Generally *Contract Numbers* are *Figural*, so called because they do or may represent some Figure Geometrical ; in relation to which they are ever considered, and thereupon by some are reckoned absolutely among Denominate *Numbers*. *Figural Numbers* are either *Lineary*, *Superficial*, or *Solid*.

1. Lineary.

Lineary, which have relation to length only, and are considered Universally or Restrictively. Universally, and so though most aptly it be referred to such a Number as will make no other formduely, yet may it also be applied to any Number Abstract. Restrictively, and so *Numbers Lineary* are taken for the sides of all *Æquilateral Figures*, or *Forms*, and Metaphorically these are called *Roots*.

2. Superficial.

Superficial, Called also *Flat Numbers*, are considered in the Formes they make by Progression or Multiplication, whereof there be as many varieties as there be like Figures in Geometry ; as *Numbers Triangular*, *Quadrangular*, *Quinquangular*, &c. *Numbers Circular*, *Diametral*, *Oblong*, &c. all which have length, and breadth, but no depth.

Solids

Solids, or *Sound Numbers*; otherwise termed *Bodily*, or *Cubical*, as by the first Multiplication they take length, and breadth like *Flat Numbers*, so by the next Multiplication they take depth, or thickness; which thickness or solidity is increased according to the Number of Multiplications, and accordingly from thence do they take their Names; as Cubes, Squared Squares, Surfolids, &c. *ad Infinitum*. These take up the 2^d Part of the 2^d Book.

2. *Solids.*

Again, *Numbers Specially Contract* are considerable, as first *Decimals*, which arise from the Abbreviation of *Fractions* before mentioned, or rather from the Conversion of one kind of *Fraction* into another; For let the *Integer* be what it will, it shall be broken but into Ten parts, and one of these Ten parts, into other Ten, and so infinitely decreasing by Ten only, whereas in other *Fractions* the Denominators might be any other Number; and because the Denominators here are still known to be One with Cyphers more by One place than the Figures of the Numerator, the Denominator is always omitted as needless; and the Numerator only set down. As is further declared, *Book 3^d Part 1st*.

Numbers Specially Contract, of six sorts.

1. *Decimals.*

Secondly, *Numbers Astronomical*, which are conversant about *Astronomy*, in Time, and Motion, have their Denominators Sixty certain; and so also omitted and ordered very like *Decimals*. As appeareth *Book 3^d Part 2^d*.

2. *Astronomical.*

Thirdly, *Numbers Logarithmical*, or *Rational*, which have the same foundation with *Decimal* and *Astronomical Arithmetick*, and like them have their Denominators with the Numbers they come of, also omitted. These are of such singular use, as they will for ever Renown the Honourable Lord *Nepair* who Invented them, and may be further viewed in *Book 3^d Part 3^d*.

3. *Logarithmes.*

Fourthly, *Numbers Cossical*, which proceed from *Figural Numbers Abstract*, and may be accompted *Rational-Contract-Compound-Figural-Numbers*, As *Two Roots*, *Three Squares*, *Three Cubes*, and such like. *Cossicks* are Simple, or Compound, and of either sort, Whole, and Broken, as is demonstrated in *Book 3^d Part 4th*.

4. *Cossicks.*

Fifthly, *Numbers Surde*, or *Irrational*, which are such Numbers set for *Roots*, as cannot be expressed by any other Number Absolute, arising from Lines, or Figures Inequilateral, whose measure is a whole Number and a Fraction, and in a sort are to *Figurals* like *Fractions* to *Integers*. These are also Simple, and Compound; and take up the fifth Part of the 3^d Book.

5. *Surdes.*

Sixthly, Because the Characters used in many Denominations are Arbitrary, and marked with Letters, also in Geometrical Figures for brevity and distinction take one line is noted with one Letter, and another with another; hence the Alphabetical Notes are called *Species*, and are ordered in Arithmetical form in the 6th Part of the 3^d Book; And are duely inserted among the rest of *Contract Numbers*, because every Magnitude, or Number, &c. is at pleasure denominate A. B. C. &c. or by any such other Letter, or Mark.

6. *Species.*

Homogeneal Numbers, whether Abstract, or Contract, are considerable in their Common Nature that is Relative; as when Five is compared to Ten, it is but one half; to Eighteen is tripple to Six; here the Numbers being compared together, are fitly termed Relative. This Relation is taken differently, as when two Numbers only; or more than two are compared together: That called *Ratio*; this *Proportion*, or *Analogy*.

Homogeneal Numbers considered in Common.

Related two ways. Ratio.

Ratio hath a property common with *Fractions*, to be Commensurable, or Incommensurable; and peculiar to it self is bisected into Equality, or Inequality. *Ratio* of Equality, is when two Equal Numbers are compared together, as Two to Two, &c. *Ratio* of Inequality, when one Number is compared to another different from him, viz. Greater, or Lesser. Greater, as Six to Two. Lesser, as Two to Six. The Greater is of two kinds; Prime, or Simple, Conjunct, or Compound. The Simple are again divided into two sorts. First, When the Greater Number containeth the Lesser, once and a part more, whether one half, one third, &c. as Six to Five, &c. Secondly, When the Greater containeth the Lesser with some parts of him, as Five to Three; which containeth the Lesser once, and two Thirds more, &c. *Ratio* of the Greater Inequality Compound, are distinguished into three sorts. First, When the Greater Number containeth the Lesser divers times, whether twice, four times, or such like, as Four to Two, Nine to Three, &c. Secondly, When the Greater containeth the Lesser many times, and a part of him besides, as Five to Two, Sixteen to Five, &c. Thirdly, When the Greater containeth the lesser many times, and also many parts thereof, as Eight to Three, Eleven to Three, &c. *Ratio* of the Lesser Inequality, like *Fractions*, either contain a part of the Number, as one Third, one Fourth, &c. So Two to Six, Two to Eight, &c. Or else many parts, as Three Quarters, Two Thirds, &c. So Four to Six is Two Thirds, and Nine to Twelve, Three Quarters, &c. These are further treated of in the 1st Part of the 4th Book.

This two-fold Equality and Inequality. Inequal of two sorts.

Greater is Simple, or Compound.

Simple 2. fold. Compound 3. fold.

Lesser is 2. fold

Proportion
2. fold.

Proportion or *Analogy* when more Numbers than two are compared together, then there may be a conference of the former several *Ratios* in their several termes, and this is different either in *Discontinual*, or *Continual Proportion*.

Discontinual,
Direct, and
Indirect.

Discontinual, Is when in four termes the first, and second, third, and fourth termes are compared together, but not the second and third; so Five to Fifteen, is as Six to Eighteen: Where the second term Fifteen, and the third term Six are not compared together. *Discontinual Proportion* is again divided into *Direct* and *Indirect Proportion*; and either of these double. *Indirect*, if the Greater requireth a Lesser, or the Lesser, a Greater to be found. *Direct*, when sometimes the Lesser requireth the Lesser, and sometime the Greater requireth the Greater to be found out.

Continual
Arithmetical,
Geometrical.

Continual Proportion, is when three or more Numbers bear like Difference, or Proportion in their Progression. This is double, *First*, When between every two Numbers the Difference, or Excess is Equal, as between Three, Six, Nine, Twelve, &c. the Difference is Three. *Secondly*, When the *Ratio* is equally alike, as Four, Eight, Sixteen, &c. the *Ratio* is Two. These are dealt with in the 2^d and 3^d Parts of the 4th Book.

Equations are
Compound Pro-
portions or
Numbers in the
Ratio of Equa-
lity.

Again *Homogeneousal Numbers* may hold community in Special Denominations, being considered in the *Ratio*, or *Proportion of Equality*; and such are called *Equations*, and by some *Algebra*; So Numbers *Æquational* are Numbers equal one to the other, though the Denominations of such Numbers are different, as one Square shall be Equal to Three Roots, if the Root be Three; wherefore *Equations* may be rightly placed amongst Numbers Denominate, the Operations of *Algebra* annexing, and of necessity requiring *Cossical*, *Speciosal*, or other Denominations unto Absolute and Undenominated Numbers, without which they must still remain uncouth and indeterminable. The Mysteries of these are unveiled in the 4th Part of the 4th Book.

Numbers Hete-
rogeneal.

Enough in this place hath been said of *Homogeneousal Numbers*, the other sort are *Heterogeneousal*, called also Mixt Numbers, so that Mixt Numbers admit of a double acceptance as well in difference to Digits and Articles (as before noted) as in opposition to *Homogeneousal Numbers* which are immixt, or of one sort.

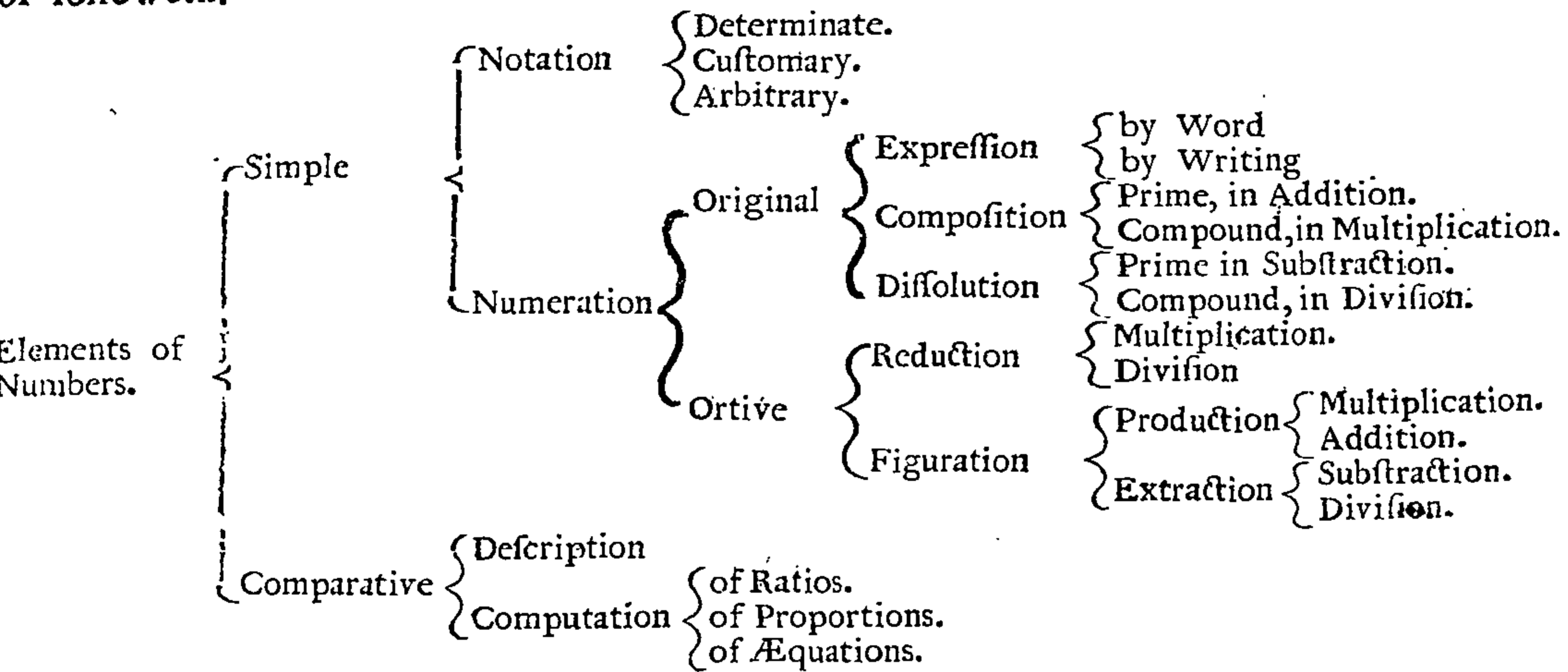
Integers and
Fractions.
Abstract and
Contract.

Briefly, to conclude, Numbers are *Heterogeneousal*, or of divers kinds when they are mixed primarily, or secundarily. The First are made up of *Integers* and *Fractions*, as One and a half, Two, and Three quarters, &c. The other are Mixt of Abstract, and Contract Numbers, as Three Squares, and Two Thirds, Thirteen Roots, and Four Integers, &c. The latter sort of them may be observed in the Three Last Books, Generally. The Former particularly Book 1st Part 2^d, and Occasionally throughout the whole Volume.

C H A P. III.

Of the Elements of Numbers.

WHAT hath not been seen of the Affections of *Numbers* in the former Chapter of their *Nature*, may be further discovered in their *Elements* : A Type whereof followeth.



Affections of Numbers further to be seen in their Elements.
A Type of the Elements of Numbers.

In this Table the *Elements of Numbers* appear double, viz. Simple, and Comparative. Simple consisteth either in Notation, or Numeration.

Simple Elements, 2. fold.

Notation, which in some Authours passeth for Numeration teacheth the Notes, Marks, and Characters whereby Numbers, Quantities, Denominations, &c. are described, and accompted as valuable as expressed in words at length. And these are either Certain, and Determinate; or Uncertain and Arbitrary; or else Customary, nevertheless variable, being partly certain, and partly uncertain.

1. Notation and this.

Certain Notation I call the most excellent invention of expressing all Cardinal Numbers by those Ten Characters; viz. 1. 2. 3. 4. 5. 6. 7. 8. 9. 0. Generally called Figures, the Tenth is called properly in Latine, *Circulus*, or *Ciphra*, in English a Circle, or Cypher, and vulgarly Nought, and of it self signifieth nothing, but being joyned to the right hand of the other, helpeth to increase their value by accident; the other Nine are called signifying Figures, as 1. doth signifie One, 2. Two. 3. Three. 4. Four. 5. Five. 6. Six. 7. Seven. 8. Eight. 9. Nine. All Ordinal Numbers are expressed by the same Figures, only at top inclining to the right hand, is placed one, or two of the final Letters of the Word, as the First, 1st. Second, 2^d. Fourth, 4th. &c.

Certain in the use of Figures.

Notation Uncertain, and Arbitrary is either in form, or in quantity. In Form, and so in Cossical, and Surde Denominations some Authours use to write them in one form, and others in another; as some write thus, Z. for Zenzike, others. Q. and sometime, q. for Quadrate: Some thus, √. and others thus, √. at pleasure. Likewise in Surds this √. standeth for a Cube Root in one, for a Squared Square Root in another. Characters both in Cossicks, and Surdes used in this Treatise may be seen, Book 3^d. Part 4th. and 5th. And so from each Authour is to be expected an Explanation of their own Marks, alwayes minding that every man in his private practise useth those he hath most accustomed, or best fancieth, and may change them for others sooner made. Secondly, some Notes are uncertain in their Quantities, as in Species; for though any Quantity, or Number, whole, or broken, for the time present (or during the work of a question) may be noted by any Letter of the Alphabet, Greek, or Roman, small or Capital; as A. B. Γ. Δ. &c. α. β. γ. δ. &c. A. B. C. D. &c. a. b. c. d. &c. yet when the Question is ended, and another in working, the same Letters shall signifie other Magnitudes, or Numbers greater or lesser, at the will of the Arithmetician, as is evident by the practise of the 6th Part of the 3^d Book.

Arbitrary in several Marks and Characters

Customary Notation is certain in respect of the General use, yet uncertain in respect of the end for which used; for if any form of better representation, may be more commodiously made, they may be altered for others. And such are all those Symbols, or Characters used for Vulgar Denominations with many that are frequented, to avoid prolixity, and the tedious, and often rescription of some Words, and termes of Art: Some of the most certain, and chiefly in use the following Table will easily demonstrate.

Customary in several Symbols and Characters of common use.

*Vulgar Denominations marked, Words abbreviated, and
Termes of Art Characterised.*

		Latine.	English.	Characters.
Geodæticks.	Money	Libræ	Pounds, Liures, Gilders	l. t. s. d. s.
		Solidi	Shillings; Solx, Stivers	s. f. d. s. s.
		Denarij	Pence, Deniers	d. d.
		Oboli	Half-pence	ob. ob.
		Quadrantes	Farthings	qd. q.
		Coronati	Crownes	Δ. ω.
		Centum	Hundreds	c. d.
		Quadrantes	Quarters	q. s. g. s.
		Libræ	Pounds	lb. lb.
		Unciæ	Ounces	z. z.
	Weight	Pondo-denarij	Penny-weights	p. pd. pw. ^{ts}
		Drachmæ	Drams	z. z.
		Scrupuli	Scruples	z.
		Graines	Graines	Gr. γκ.
		Caractæ	Karacts	K. Kκ.
		Per Centum	By the Hundred	℥.
		Continens	Containing	ton.
		Superpondium	Tare	ts.
		Abſque Superpondio	Without Tare	netto.
		Anni	Years	℞.
	Time and Motion	Anno Domini	In the Year of Our Lord	An. ° D.
		Anno Chriſti	In the Year of Chriſt	An. ° C.
		Dies	Daies	D. d.
		Horæ	Hours	H. h.
		Signa	Signs	S.
		Gradus	Degrees	G. γ. d. deg.
		Minuta, Secunda, &c.	Minutes, Seconds, &c.	l. ll. &c.
		Ante Meridiem	Before Noon	a. m.
		Poſt Meridiem	After Noon	p. m.
	Sexagenæ	Primæ, Secundæ, Tertix, &c.	Primes, Seconds, Thirds, &c.	℥. ℥. ℥. vide plus
	Sexageſimæ	Primæ, Secundæ, Tertix, &c.	Primes, Seconds, Thirds, &c.	l. ll. ll. lib. 2. par. 2
	Signes	Aries	Ram	Head and Face
		Taurus	Bull	Neck and Throat
		Gemini	Twinn	Arms and Shoulders
		Cancer	Crab	Breaſt and Stomack
		Leo	Lion	Heart and Back
		Virgo	Virgin	Bowels and Belly
		Libra	Ballance	Reins and Loins
		Scorpio	Scorpion	Secrets
		Sagittarius	Archer	Thighs
		Capricornus	Goat	Knees
Aſtronomicks.	Planets and Mettals.	Aquarius	Waterman	Leggs
		Piſces	Fiſhes	Feet
		Saturnus	Saturn	Lead
		Jupiter	Jupiter	Tinn
		Mars	Mars	Iron
		Sol	Sun	Gold
		Venus	Venus	Braſs, Copper
		Mercurius	Mercury	Quick-Silver
		Luna	Moon	Silver
		Caput Draconis	Dragons head	
	Nodes	Cauda Draconis	Dragons tail	
		Pars Fortunæ	Part of Fortune	
		Antifcia	Antifcions	
		Contrantifcia	Contrantifcions	
		Conjunctio	Conjunction	
		Vigintilis	Vigintil	
		Quindecilis	Quindecil	
				Aspects

A	Aspects	B Semisextilis	Semisextil	SS.V.
		Semiquintilis	Semiquintil, or Decil	Dc. Σ .
		Semiquartilis	Semiquartil	Sq. Σ .
		Sextilis	Sextil	*. Σ .
		Quintilis	Quartil	Q. Σ .
		Quartilis	Quartil	Q. Σ .
		Sesquiquintilis	Sesquiquintil or Tredecil	U. Σ .
		Trigonus	Trine	Td. Σ .
		Sesquiquartilis	Sesquiquartil	Δ .
		Biquintilis	Byquartil	SSQ. Σ .
		Quincunx	Quincunx	Bq. Σ .
		Oppositio	Opposition	Vc. Σ .
		Horoscopus, Ascendens	The Angle of the East, or 1 st house	Σ .
		Medium Cœli	Midheaven or 10 th house	Hor. Ascen.
		Angulus Occidentis	West Angle, or 7 th house	Mc. Med. C.
		Sinum Cœli	The Angle of the Earth, or 4 th house	Ang. Occ.
		Septentrio	North	IC.
		Oriens	East	N.
		Meridies	South	E.
		Occidens	West	S.
		Longitudo	Longitude, or Length	W.
		Latitudo	Latitude, or Breadth	Long.
		Altitudo	Altitude, or Height	Lat.
		Recta Ascensio	Right Ascension	Alt.
		Recta Descensio	Right Descension	R. Asc.
		Obliqua Ascensio	Oblique Ascension	R. Desc.
		Obliqua Descensio	Oblique Descension	Ob. Asc.
		Logarithmus	Logarithme	Ob. Desc.
		Sinus	Sine	Log.
		Sinûs Complementum	Cosine	S.
		Tangens	Tangent	cos.
		Tangentis Complementum	Cotangent	t.
		Secans	Secant	cot.
		Secantis Complementum	Cosecant	sec.
		Radix	Root	cosec.
		Radix	Root	Σ .
		Radix Universalis	Universal Root	V.
		Radix, five Latus	Root, or Side	V.
		Quadratus	Square	p.
		Cubus	Cube	q.
		Diameter	Diameter	c.
		Diametri Quadratus	Square of the Diameter	D.
		Diametri Cubus	Cube of the Diameter	Dq.
		Radius	Semidiameter, or Radius	Dc.
		Radij Quadratus	Square of the Radius	R. Rad.
		Radij Cubus	Cube of the Radius	Rq.
		Peripheria	Circumference, or Periphery	Rc.
		Peripheriæ Quadratus	Square of the Periphery	P.
		Radix Binomii	Binomial Root	Pq.
		Radix Residui	Residual Root	\sqrt{b} .
		Radix Supposititia	Supposititious Root	Vr.
		Duo Numeri	Two Numbers	A.
		Major	Greater	AE.
		Minor	Lesser	A.
		Summa	Sum	E.
		Differentia	Difference	Z.
		Rectangulum	Rectangle, or Product	X.
		Summæ Quadratus	Square of the Sum	Æ. AE. P.
		Differentiæ Quadratus	Square of the Difference	Zq.
		Summæ Cubus	Cube of the Sum	Xq.
		Differentiæ Cubus	Cube of the Difference	Zc.
				Xc.

Summa Quadratorum	Sum of the Squares	Σ .
Summa Cuborum	Sum of the Cubes	Σ^3 .
Differentia Quadratorum	Difference of the Squares	Δ .
Differentia Cuborum	Difference of the Cubes	Δ^3 .
Tres continuè proportionales	Three Numb. contin. proportional	A, M, E.
Quatuor continuè proportionales	Four Numb. continually proportional	A, M, N, E.
<i>Arithmetical Progression.</i>		
Primus Terminus minimus	The First Term, or least	a .
Ultimus maximus	The Last Term, or greatest	ω .
Numerus Terminorum	The Number of Termes	T.
Differentia communis	The Difference, or Excess common	X.
Summa omnium Terminorum	The Sum of all the Termes	Z.
<i>Geometrical Progression.</i>		
Terminus primus	The first Terme	a .
Terminus secundus	The second Terme	β .
Terminus tertius	The third Terme	$\frac{\beta^2}{a}$.
Terminus quartus	The fourth Terme	$\frac{\beta^3}{a^2}$.
$\&c.$	$\&c.$	a^q .

+ Commonly called St. *Georg's* Cross, is the sign of *Addition*, and being set before a Number, or quantity, signifieth *More*, *More by*, *and*, or any such word that may shew the same Number, or Quantity to be affirmative.

- A *Right Line* is the sign of *Subtraction*, and set before a number or quantity, imports *less*, *less by*, *lacking*, *wanting*, or any such word that may declare the same number or quantity to be negative.

X Commonly called St. *Andrew's* Cross, is the sign of *Multiplication*, and set between two Numb. or Quant. implies multiplyed *by*, *in*, *into*, *times*, or any such word, that may denote the numb. or quant. multiplyed one into the other.

) The *Lunular Increscent* is the sign of *Division*, and set between two numbers or quantities, signifyes dividing, and declares the right hand number or quantity of the *Two*, to be divided by the left.

(The *Lunular Decrescent* is the sign of the *Quotient* of any *Division*, and the Number there standing demonstrates how often the *Divisor* is contained in the *Dividend*.

.... *Four Points* in a *Right Line*, is used sometime in *Reduction of Fractions*, to separate the old Numerators from the new, and the old Denominators from their least termes.

— A *Right Line* between two numbers, or quantities, one standing above, and the other beneath, commonly denotes a *Fraction*, and that the upper number, or quantity is to be divided by the neather. And sometimes is used to separate one Number from another.

L The *Rectangle* is the *Seperatrix* between *Integers*, and *Decimals*, of which and other Distinctions thereof, *Vide plus*, lib. 3. par. 1. & 2. Sometime also is instead of the *Decrescent Lunular* in *Division*.

, The *Comma*, is the Distinction also between *Integers* and *Decimals*; and between the *Logarithme* and his *Characteristique*.

: The *Colon*, Including *Numbers* or *Quantities*, is a Note of the *Universal Root* of such Number, or Quantity.

. The *Period*, is often used for Distinction-sake, but in disjunct Proportions, between two Numbers or Quantities, understands the word, *To*.

: *Three Pricks* or *Points*, in the middle of four numbers, or quantities, are sometimes used in Disjunct Arithmetical Proportions, for the words, *is as*

:: *Four Points* thus, in the middle of four numbers or quantities is the sign of *Analogy*, and frequently used in Disjunct Geometrical Proportions for the words, *is as*; or *so is*.

= *Parallels*, are the sign of *Equality*, and implies the numbers or quantities of the one side thereof, are equal to the numbers or quantities of the other side thereof.

≧ Greater. **≧.** Next Greater. **≦**. Lesser. **≦.** Next Lesser.

≧ Not Greater. **≦**. Not Lesser. **≧**. Equal or Less. **≧**. Equal or Greater.

≧ Between

Signal Sym-^{bol}
boles for Terms
of Art.

c	Between two Numbers, or Quantities, the one above, and the other below, is the sign of a <i>Ratio</i> .
•	Greater Ratio. \div Lesser Ratio. \div Continual Proportionals.
∴	Commensurable. \nexists Incommensurable
∝	Commensurable in Power. \nexists Incommensurable in Power.
∝	Rational. \nexists Irrational. \nexists Medial.
∝	A Line cut according to extreme, and mean Proportion.
∝	The greater Portion thereof. τ The lesser Portion thereof.
∝	A mean Proportional. \propto Divided by.
∝	The Difference of the two Magnitudes.

Common Termes.	By.	Other words not having any proper Symbole are
	lib.	abbreviated when occasion requires by two, or three
	Dr.	of the first Letters of the word, as <i>Duc.</i> for <i>Ducats</i> .
	foB.	<i>Mon.</i> for <i>Moneths</i> , &c. <i>D.</i> and <i>Q.</i> besides what as be-
	n ^o . n ^o .	fore they stand for Notes of, may sometime peradvent-
	ps.	ture be found for <i>Data</i> , and <i>Quasita</i> , which termes
	Rd.	import the One such <i>Numbers</i> or <i>Magnitudes</i> , &c. as
	Ec.	are given for the Resolution of a Question, the other
		the <i>Numbers</i> or <i>Magnitudes</i> sought.

Nothing need be added for demonstration of the Table, but that some Difference may be observed between the Characters used in Printed Books, and those practised by the Pen, according as the written, and Printed Alphabets differ.

Where denominations have two or more marks any one of them will serve, as may be further observed in the parts of the Treatise to which they belong.

The next part of the *Simple Elements* of Numbers is *Numeration*, which is both *Original* and *Ortive*. *Original* is of double use; for it serveth either rightly to Express, and Accompt the Value of Numbers by their Notes, Symboles, Characters, Places, &c. or else to find out, and procure Numbers valuable Greater, or Lesser. This last is sometime called *Algorithm*, though *Algorithm* more properly is *Cosical Arithmetick*.

2. *Simple Element of Numbers is Numeration, and this either Original, which is of 3 sorts.*

The first kind teacheth the Order, which *Arithmeticians* do observe, in the usual expressing, and valuing of Numbers, Quantities, &c. either by Word or Writing; and with several Authours passeth for the only Numeration: This for *Integers* is to be sought in the next Chapter, and for *Fractions*, and all the other sorts of Numbers, in the first Chapter of every Part of the other Books.

The second sort of *Original Numeration*, sheweth the Method of Increasing Numbers. The Increasing of Numbers is called Composition, or the Genesis of Numbers; This is Simple in *Addition*, and Compound in *Multiplication*.

The third sort of *Numeration Original*, is the Diminishing of Numbers, or their Dissolution, which likewise is Simple in *Subtraction*, and Compound in *Division*, and both these may be called the *Analysis* of Numbers. So that *Addition* and *Subtraction*, are both the *Prime*, or *Simple* parts of *Numeration*; and therefore shall in order precede *Multiplication*, and *Division*, which are the *Conjunct* parts thereof: The several Chapters whereof are to be seen for each sort of Numbers distinctly, in the Parts where they are handled.

Numeration Ortive, ariseth from the former Species of *Numeration Original*, and consisteth in two things, *Reduction*, and *Figuration*. *Reduction* is useful in *Fractions*, and *Contract Numbers*, and performed by *Multiplication*, or *Division*: As will sufficiently be evident in the second part of this Book, and elsewhere in *Contract Numbers*.

Ortive, which is 2 fold.
Reduction.

Figuration, serveth for Figural Numbers, to produce them, or extract their Roots. *Production*, or their *Genesis*, is effected by *Multiplication*, and *Addition*. The Extraction of their Roots, or *Analysis* is made up principally of *Subtraction*, and *Division*, occasionally using *Multiplication* and *Addition*; which being further set forth in the second Part of the Book may here be spared.

Figuration.

The *Comparative Elements* of Numbers, belong to the Work of *Proportions*; The Description, and Computation whereof, both *Arithmetical*, or *Numeral*, and *Geometrical*, or *Mensural*, and of each kind *Disjunct*, and *Continued*, together with *Aequations*, are reserved for the Fourth Book, and therefore omitted here.

Comparative Elements to be seen in the 4th. Book.

C H A P. I V.

O f I N T E G E R S.

*Integers first
proceeded with.*

TH E *Nature*, and *Elements* of *Numbers* seen in general, it is now requisite to descend into particulars, and to proceed in Order according to the foregoing Method. The first sort of Numbers that present themselves, are proper, and homogeneous Numbers, called *Integers*, Considered abstractively.

*All Numbers
expressed by
9. Figures and
the Cypher.*

The Notes, or Characters, frequented both in *Integers*, and *Fractions* absolute, or of common Denomination, and generally in all Arithmetical Operations, are the forementioned Figures, 1. 2. 3. 4. 5. 6. 7. 8. 9. 0. the which thereby that Arithmeticians may express all Numbers, they have ordered them into certain places, and periods, proceeding alwayes from the right hand towards the left in a Decimal progression, so as the aforesaid Figures may express greater Numbers, or lesser as necessity requireth.

*Place of a
Figure, what.*

A Place is the Seat or Room that a Figure standeth in, and so many Figures and Cyphers as there are in one Number or Sum, so many places hath that whole Number.

*No certain
Number of
Places.*

Of these Places there is no certain Number, but that is called the first place that is next to the right hand, and reckoning in order towards the left hand, the next is the second, then the third, and so *ad infinitum*, as a. b. c. d. thus standing, d. thus standeth in the first place, c. in the second, b. in the third, and a. in the fourth, &c.

*Every Place
hath some
Denomination.*

Every place hath a certain denomination properly belonging to it. Whereby a figure according to its standing comes to be valued many times more than the Figure standing single would import. For 1. 2. 3. &c. shall not only signifie so many entire Unites, or Ones, according to their formes, but may also signifie 1. 2. 3. &c. Tens Hundreds, Thousands, &c. according to the place the figure occupieth, for by so much as any figure inclineth towards the left hand, by so much is the value thereof increased.

Unites Place

The first place, hath the denomination of Units, and doth signifie that every figure standing there, betokeneth his own simple value according to his form, as the figure 1. to signifie but One, the figure 2. but Two, and so of the rest.

Tens

The second place, toward the left hand hath the denomination of Tens, and every Figure here standing shall betoken his own certain value Ten Times; as 1. if it stands in the second place shall be one Ten. 2. Twice Ten, or Twenty. 3. Three times Ten, or Thirty, &c.

Hundreds.

The third place, to the left hand hath the denomination of Hundreds, and so every Figure there standing shall betoken his own value a hundred times, as 6. standing there denotes six hundred, &c.

Thousands, &c.

In the fourth place, every figure standing, signifieth his value a Thousand times, In the fifth place, Ten thousand times. In the sixth place, one hundred thousand times. In the seventh place, one Thousand Thousand times, or one Million (which is called by some the first great Thousand.) In the Eighth place, Ten Millions. In the Ninth place, one hundred Millions. In the Tenth place, one Thousand Millions, (called by some the second great Thousand.) In the Eleventh place, Ten Thousand Millions. In the Twelfth place, One Hundred Thousand Millions. In the Thirteenth place One Million of Millions, (or the third great Thousand.) So infinitely Names may be given to every place, each succeeding place exceeding the former Ten times; though in ordinary practise we seldom need thirteen places, yet if any list to exceed, it is but doubling the Millions to begin as at the Eight place, for the fourteenth place is Ten Million of Millions, The Fifteenth like the Ninth is Hundred Million of Millions, and thus proceeding till the Nineteenth place, where tripling the Millions, go on as before to the Twenty fifth place, and then quadruple the Millions, and so as before *ad Infinitum*.

Others instead of doubling the Millions in the thirteenth place, call it Billion; the nineteenth place Trillion, instead of Millions of Millions of Millions; the twenty fifth place Quadrillion, and the Thirtieth place Quintillian, &c.

*Denomination
of the Places
respect, only
Quantity.*

Note here, That a Number though thus denominate according to his place, yet is accompted Abstract, without another denomination, this denomination only respecting the quantity, number, or multitude of the thing or things propounded, and that denomination which truly decyphers a Number Contract, respects the quality, or nature of the thing numbred; yet notwithstanding it bears some similitude thereto, for as the denomination

denomination of quantity is the last recited in the verbal expression of any Number, as to say Two Hundred, where Two is the Numerator, or Number, and Hundred the denomination of quantity which is last repeated. So is the denomination of quality repeated after the quantity; As in saying Two Hundred Pounds, after the Sum of Two Hundred the quantity, is last of all named Pounds the denomination of quality; likewise Thirty Men, here Men is the Contract Denominator, and Thirty the Numerator, or Valuer, &c.

Has different to the denomination of Quantity.

These places are distinguished into Degrees, and Periods. Degrees are three; Once, Ten times, a Hundred times. A Period is a comprehension of Degrees, and is Simple, or Compound. Simple is made up of one Ternary of Degrees, containing three places, as 123. Compound is double, as 12345. Or treble, as 12345678, or fourfold, &c. A further view whereof may be had in the following Table, which may be set in several formes.

Places are distinguished into Periods.

A. z. y. x.			w. u. t.			f. r. q.			p. o. n.			m. l. k.			i. h. g.			f. e. d.			c. b. a			
5.	6.	7.	8.	9.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	C	X	I
M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	C	X		
M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	C	X		
M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	C	X		
M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	C	X		
M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	C	X		
M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	C	X		
M	C	X																						
The eight Period of Thousands of Millions of Millions of Millions.			The seventh Period of Millions of Millions of Millions.			The sixth Period of Thousands of Millions of Millions.			The Fifth Period of Millions of Millions.			The Fourth Period of Thousands of Millions.			The Third Period of Millions.			The Second Period of Thousands.			The First Period of Units.			
The eight Period of Thousands of Millions of Millions of Millions.			The seventh Period of Millions of Millions of Millions.			The sixth Period of Thousands of Millions of Millions.			The Fifth Period of Millions of Millions.			The Fourth Period of Thousands of Millions.			The Third Period of Millions.			The Second Period of Thousands.			The First Period of Units.			

Numeration Table.

Here the order of the places is noted by the Roman Letters at top. The Value or denomination by the Numeral Capital Letters. The Periodical Division shews the thousandth place of the Number.

These things observed it will not be difficult to express the true content of any Number Integral by word or writing: For the former consider the figures and the places they stand in, and call their values such as the figure is augmented by his standing; wherefore if I see 98. standing, and would express the right value thereof, I know the first figure is but Eight, for it beareth that form, and standing in the Units place, and therefore can signifie himself but once; but the 9. must be Nine times Ten, because it standeth in the second place, and so is the value, or content of that quantity Ninety Eight, as if it should stand thus 90. 8. and be expressed Nine times Ten and Eight. And this 398. Three hundred ninety eight, as if it stood thus 300. 90. 8. and after this manner are all quantities valued that are beneath a Thousand. If the quantity exceed, for the better remembring the periods, and so thereby the more readily to express great Sums, or quantities, mark the periodical Division of the Table on the given Number with a prick above or below the figure standing in that place, beginning at the right hand set one prick over the first Figure, another over the fourth, &c. as to numerate or express the value of this Sum 3479841234. The pricks are placed thus 3479841234. then beginning at the left hand read as it were every pricked parcel apart, with the denomination of the place to it: So at last you shall have the whole value; as first One Thousand, Two hundred thirty four; Then, Nine Million, eight hundred forty one thousand, two hundred thirty four; Lastly, Three Thousand four hundred seventy nine million, eight hundred forty one thousand, two hundred thirty four. And so much is the value, or quantity of that Number; the like is to be done with others.

To express a Number by word.

To express in Writing, any propounded Integer, remember all numbers from one to ten, are expressed by a digit in the Units place; All numbers above ten, and under a hundred with two Figures, or one Figure, and one Cipher, to wit Tens in the

To express a Number by word.

place

Void places to be supplied with Cyphers.

place of tens, and Units or Cyphers in the place of Units, according to the Numbers to be expressed ; likewise all Numbers under a thousand, and above a hundred are expressed by three Figures, or two Figures and one Cipher, &c. Then observe the places as before, and begin at the right hand , put down the quantity of Unites pronounced, and so proceed unto the left. If the given Number have any rooms void, they are alwayes to be supplied with Cyphers, as to write down one hundred the places of Units and tens being void are filled up with Cyphers, and one is set in the hundreds place thus 100 So one thousand thus 1000. and nine thousand and seventy million ten thousand one hundred and one, thus 9070010101. where Cyphers occupy several places instead of Figures , because neither Tens, Thousands, Hundred Thousands, Millions, nor Hundred Millions were given in the Number.

C H A P. V.

Addition of Integers.

HAVING seen the due ordering and placing of the Notes for the Expressing of Numbers by them, it remaineth to declare the other parts of Numeration , and first the Prime Part of

Addition, what it is.

Addition is that part of *Arithmetick* whereby divers Numbers are collected, and added together into one total Sum.

Addends, what. Total, what.

Addition of *Integers*, hath respect to *Collocation*, *Operation*, and *Probation*. *Collocation*, is the due placing the Numbers given to be added, called the *Addends*, and distinguishing them from the Number found out by *Addition*, called the Sum Total, or Aggregate.

How to place the Addends.

Place the *Addends* in rank and file one directly under another, beginning at the right hand ; it matters not whether the greatest or least be uppermost, so that Numbers of one quantity, may stand under Numbers of the same quantity, that is to say, Units under Units, Tens under Tens, &c. then draw a right line under them, to distinguish them from the Total.

Induction what it is, called Simple Addition.

Operation, hath *Induction* ; and *Consummation*, or *Perfection*. *Induction* , teacheth the Sum that any two digits added together make, called sometime *Simple Addition* ; and had need be ready in Memory with the Accomptant ; This is declared in the following Table.

Addition

0	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

Table.

The use of the Table.

By this Table the sum of any two digits is easily had, for entring with one at the head, and the other at the left hand side, the Common Angle is the Sum, as 3. and 7. make 10. and so much is found over against 7. in the side, and 3. in the head, or against seven in the head and three in the side, if the Table be quadrangular as this is ; but for that

that the lower part of the Table beneath the black scale is sufficient, it may be set in that Triangular form, for 2. and 1. and 1. and 2. are all alike, &c.

By the help of these Digits added, the Sum of the Decades or Articles are also known, because their signifying figures are but digits, as 20. and 20. make 40. because 2. and 2. are 4. the Cyphers being only reserved to keep place as before, but increase not the Sum, for 1. and 0. is but 1.

Perfect Operation, sometime called *Compound Addition* finisheth the Addition of Mixt Numbers thus; begin at the right hand, and take all the figures or digits in the right hand file, as they stand one over another, and putting them together consider what the result or total thereof is, and if it be a digit write it under the right line directly in the same file, and so do in every of the files; but if any file being cast up amount to an Article write the Cypher under the file, and reserve in mind the figure of the Article to be added in the next place, and when the Sum of the next file is found add that reserved Article, to wit, for every ten reserved, one; as two for 20. three for 30. &c. and this result or total subscribe accordingly; and if the sum or total of any file with or without such reserved Article, if any be, amount to a mixt number, then set down the digit of that mixt Number, and reserve the Article thereof, as before.

Perfect Operation called Compound Addition how it is wrought.

Example. There are two Numbers propounded, whereof the one was 234. and the other was 342. What is the Sum of both? The numbers set as at A. in the right hand file are found 2. and 4. which put together make 6. to be set under the file because a digit, as is represented by the work standing at B. Then going forward to the left hand in the next file are 4. and 3. which make 7. to be subscribed as at C. Lastly in the third place are 3. and 2. which make 5. and the whole work stands as at D. where the Total appears to be 576.

Example 1.

Rank			
A	$\begin{array}{r} 234 \\ 342 \\ \hline \end{array}$	B	$\begin{array}{r} 234 \\ 342 \\ \hline 6 \end{array}$
C	$\begin{array}{r} 234 \\ 342 \\ \hline 76 \end{array}$	D	$\begin{array}{r} 234 \\ 342 \\ \hline 576 \end{array}$
Total.			

Second Example. Suppose Two Numbers whereof the Sum is desired be 40235: and 34973. then are they set as before, and in the right hand file 3. and 5 make 8. to be there subscribed. Then 7. and 3. make 10. an Article, the Cypher therefore is set down under the line in the second file, and the Article reserved. Then 1. carried in mind, and 9. is 10, and 2. is 12. which being a mixt number, two the digit thereof is subscribed, and the Article reserved as before. Again 1. reserved and 4. is 5. to be set down under the 4. not regarding the 0. because the number is not augmented thereby in Addition. Lastly adding 3. and 4. together they make 7. which is set in the last place, and the work standeth as at K. where the total 75208. is the number desired. The several other Paragraphs stand as followeth.

Example 2.

E	F	G	H	I	K
$\begin{array}{r} 40235 \\ 34973 \\ \hline \end{array}$	$\begin{array}{r} 40235 \\ 34973 \\ \hline 8 \end{array}$	$\begin{array}{r} 40235 \\ 34973 \\ \hline 08 \end{array}$	$\begin{array}{r} 40235 \\ 34973 \\ \hline 208 \end{array}$	$\begin{array}{r} 40235 \\ 34973 \\ \hline 5208 \end{array}$	$\begin{array}{r} 40235 \\ 34973 \\ \hline 75208 \end{array}$

Probation, is the *Examen* or *Demonstration* whereby the Operation may be proved true. The vulgar proof is by casting all the Nines that can be had both out of the Aggregate, and Numbers added, which done will leave like figures if the work be right. To do this begin with the Total, or the Addends, at the left or right hand, not regarding the places of the Numbers, but as though they were all Units add them together, and as the Numbers increase above 9. reject 9. and go forward with the rest, and what remains when all the Numbers are gone over, set down at the one side or end of a right line; Then do so with the other part of the Addition, and this remain place at the other side or end of the same line: As to instance in the first Example 4. and 2. make 6 in the right hand file, then 6. and 4. make 10. coming to the next file, from which 9. rejected there resteth 1. that 1. and 3. make 4. which 4. and 3. make 7. and 2. is 9. which cast away there remaineth 0. to be set on the line as at M. Then in the

Proof of Addition. Common.

Total 6. and 7. is 13. from which 9. cast there remaineth 4. which 4. and 5. are 9. to be again cast away, and there resteth 0. also to be set under the line as at N.

L

234

342

576

M

0

N

0

0

or thus 0—0

This Proof by 9. uncertain.

Although if the work be right, it will never vary from leaving equal remains, yet by reason some digit or other may happen through inadvertency to be misplaced, this kind of proof is uncertain, for it is evident, that if the places of 7. and 6. in the Total should be unhappily changed, or of 5. and 7. yet would the remains by rejection of nines be as before, though the Sum much altered in the value. For both 567. and 576. also 756. and 765. likewise 657. and 675. after the nines are cast away leave 0. yet is their value greatly different: Which thing may fall out in every other Number.

Best Proof by Subtraction.

Therefore the best Proof is by *Subtraction*, which may be seen in the next Chapter.

CHAP. VI.

Subtraction of Integers.

Subduction or Subtraction, what it is.

THE first part of the Genesis of Numbers unfolded , The prime part of their Analysis followeth, which is *Subtraction*, called also *Subduction*.

Subtraction, is that numerative part of *Arithmetick*, which teacheth how to deduct one Number from another, and to shew what remaineth.

Subtraction of Integers, like *Addition*, respecteth *Collocation* , *Operation* , and *Probation*.

How to place the Numbers.

Collocation, placeth in the uppermost Rank the greater number, (or number from which *Subtraction* is to be made,) for in *Integers* no greater number can be taken from a lesser; and underneath the same (with or without an interjacent line) the number to be subtracted or deducted called the *Subtrahend*, so orderly that every figure may stand under his like, as Units under Units, Tens under Tens, &c. Then with an interjacent right line seperate these two given Numbers from the Number found out by *Subtraction*, which is called the Remainder, Remain, Rest, Difference, or Excess.

Subtrahend what.

Remainder what and how called.

Operation hath *Induction* and *Perfection*.

Induction called Simple Sub- straction.

Induction, sometime called *Simple Subtraction* sheweth the Difference between any two digits subtracted one from the other, and is requisite to be remembred by the Practitioner, according to the following Table.

Subtraction

0	1	2	3	4	5	6	7	8	9
1	0	1	2	3	4	5	6	7	8
2	1	0	1	2	3	4	5	6	7
3	2	1	0	1	2	3	4	5	6
4	3	2	1	0	1	2	3	4	5
5	4	3	2	1	0	1	2	3	4
6	5	4	3	2	1	0	1	2	3
7	6	5	4	3	2	1	0	1	2
8	7	6	5	4	3	2	1	0	1
9	8	7	6	5	4	3	2	1	0

Table.

This

This Table would be sufficient, if only the upper part above the black Scale be used. The difference between two Digits is found thus; enter with the lesser or subtracting digit at the left side, and the other at the head, and in the Common Angle is the difference found, as 2 from 9 leave 7. which stands just against 2 in the side, and 9 in the head; the like also will be found against 2 in the head, and 9 in the left side of the Table, if the Table be quadrangular as this is.

By help of the difference of these *Monades* the difference of the Articles also are had, as 30 from 40 leaves 10, because 3 from 4 leaves 1. the Cypher, only keeping their place as before.

Consummate Subtraction sometime called *Compound* perfecteth the work with mixt Numbers in this manner. It beginneth at the right hand, and withdraweth or abateth the lower Numbers or Figures, particularly one after another, out of the upper standing over them, and subscribeth the Number remaining, if any be, but if nothing remain, 0. except it happen to be the last file, and then the Cypher need not be set down.

Example. If 241. were to be taken from 343. and it were desired to know the Remain. The numbers being placed as at A. then 1. out of 3. leave 2. to be subscribed under the Line in the right hand file, as at B. then 4. from 4. there resteth 0. as at C. and 2. from 3. there remaineth 1. and the compleat work stands as at D. So the whole difference between the two given Numbers is found to be 102.

Rank							
A	343	File.	B	343	C	343	D 343 Greater Number.
	241			241		241	241 Subtrahend.
				2		02	102 Remain.

If it happen that the neather figure be greater than the upper, so that *Subtraction* cannot be made, then in imagination borrow Ten, and adding it to the upper Figure make *Subtraction* from both, and for that borrowed ten account one back in the next file, either by counting the next figure to be subtracted one more than it is, or the next Figure to be subtracted from one less than it is.

Example. Suppose the difference between 30971. and 12381. be required. The numbers are set as at E. and 1. from 1. taken, 0. is left, as at F. but 8. from 7. cannot be taken without borrowing 10. which put to 7. makes 17. from which 8. taken leaves 9. to be subscribed in the second file, as at G. for which 10. reckoning the next 3. for 4. or the 9. but 8. and taking the lesser from the greater, that is either 4. from 9. or 3. from 8. the remain is 5. subscribed as at H. then 2. from 0. cannot be subtracted, wherefore 2. from 10. as before leaves 8. as at I. and lastly 1. borrowed, and 1. are 2. abated from 3. leaves 1. as at K. so the whole excess is found to be 18590.

E	F	G	H	I	K
30971	30971	30971	30971	30971	30971
12381	12381	12381	12381	12381	12381
	0	90	590	8590	18590

When one Number is to be subtracted from many, or many from one, first add all the Plural Numbers into one, and therewith proceed to *Subtraction* as before.

Examples of both. In the first 3496001 taken from 3424501. 1042601. and 200000. whose total is 4667102. and the Remainder 1171101. as at A. In the second 5423401. 800100. and 10081. are subducted from 6245002. and the Remain is 11420. as at M.

L		M	
3424501	Numbers from which Subtraction is made.	6245002	Greater Number.
1042601		5423401	Subtrahends.
200000		800100	
4667102	Total of the Addends.	10081	
3496001	Subtrahend	6233582	Total of the Subtrah.
1171101	Remain.	11420	Remain.

Probation

Proof of Sub-
traction.

Common.

This Proof by
9. uncertain.

Proof of Addi-
tion by Subtra-
ction 2. waies.

Probation serveth to demonstrate the Truth or Error of the Work, and is vulgarly performed by casting away nines from the Number from which *Subtraction* is made, and keeping the remain thereof equal to the remain left after rejection of nines from the Subtrahend, and Remain; as in the first Example above in this Chapter. 9. cast from 343. leaves 1. So also from 241. and 102. to be set thus $\frac{1}{9}$, or thus 1—1.

But for the Reason shewed before in Addition this way of Proof is uncertain. As the best proof therefore of *Addition* is by *Subtraction*; so the best proof of *Subtraction* is by *Addition*.

Addition may be proved by *Subtraction* two wayes. First by beginning at the left hand, and deducting in order from the particular places of the total, the Sum of the particular Files added, for if the work be right, 0. will remain at last.

1. As in the former instance where the total of 5423401. 800100. and 10081. was found to be 6233582. here 5. taken from 0. in the total leaves 1. then 8. and 4. in the next File make 12. abated from the 1. left, and 2. in the total, which are 12, leave 0. then 2. and 1. make 3. taken from 3. there rest 0. and so in all the rest; as at N.

2. Secondly, By cutting off from the Addends any Rank, and adding the Residue into one total; then subtracting this total from the first total, the remain will be the numbers first cut off; if the work be right.

As in the former instance, if 5423401. be cut off, and the total of 800100. and 10081. which is 810181. as at O. abated from 6233582. the remain will be 5423401. as at P.

N	O	P
5423401	5423401	6233582 First Total.
800100	800100	
10081	10081	810181 Second Total.
6233582	810181	5423401 Numbers cut off.
1000000		

Proof of Sub-
traction by
Addition.

Subtraction is proved by *Addition*. For if the Remainder, and Subtrahend be added together, the Number from which *Subtraction* is made will be returned.

As in the last instance, if 5423401. be added to 810181. the Total will be 6233582. as at Q.

	6233582 Greater Number.
	810181 Subtrahend
Q	5423401 Residue
	6233582 Total and Proof.

Both bottomed
on 2. Theorems.

The Proof of *Addition*, and *Subtraction* is bottomed on the two fundamental Theorems following, which give life mutually to *Addition*, and *Subtraction*.

1. 1. In *Addition* the Aggregate is equal to all the Addends, and *contra*, as $3+2=5$. and $5=2+3$.

2. 2. In *Subtraction* the Number to be subducted, and the difference are together equal to the Number from which *Subtraction* is made, and *contra*, because $3+2=5$. therefore $5-2=3$. and $5-3=2$.

Conseſſaries
from thence.

From hence spring these two Conseſſaries.

1. To know the one Addend, if the Total, and other Addend be given, subtract the given Addend from the Total.

2. To know the Subtrahend, if the Number from which *Subtraction* is made, and the Remain be given, subtract the Remain from the Number from which *Subtraction* is made.

Examples in Questions of Addition and Subtraction.

Questions in
Addition.

1.
2.

1. What Number is that to which if 52. be added it makes 397?

Answer. Subtract 52. from 397. and there remaineth 345. the Number sought.

2. What Numbers are those to which if there be added severally 32. 42. and 52. their Totals will be alike, and the Sum of each will be 130?

Answer. Subtract the given Numbers severally from 130. and the remains are the Numbers quesired, which are 98. 88. 78.

3. What

3. What Number is that from which if 40. be taken, the remain will be 375? *Questions in Subtraction.*
Answer. Add 50. to 375. and the total is 415. the desired Number.
4 What Numbers are they from which if 14. 15. or 16. be deducted, yet the remains will be equally alike 226?
Answer. Add severally to 226. the same Numbers, and the Totals will be 240. 241. 242. the Numbers required.

	1	2	3	4.
	397	130. 130. 130.	375	226. 226. 226.
Operation	52	32. 42. 52.	40	14. 15. 16.
	345	98. 88. 78.	415	240. 241. 242.

C H A P. VII.

Multiplication of Integers.

THE Prime, and Uncompound parts of *Numeration*, *Addition*, and *Subtraction* preceeding, next succeed the Compound, which are *Multiplication* and *Division*.

Multiplication is the Compound Genesis of Numbers, and a Conjunct part of *Numeration*, whereby one Number is led into, or increased by another; for the Number given to be multiplyed is so often added to it self as there be Units in the multiplying number, wherefore the third number, or number produced thereby, shall so often contain the first number, as there be Units in the second.

Multiplication serveth instead of many *Additions*, with more speed to augment a lesser number than *Addition* can. For *Multiplication* effecteth at once, what *Addition* could but do at many times.

Multiplication observeth *Collocation*, proceedeth to *Operation*, and concludeth with *Probation*.

Collocation, respecteth the *Nomenclator* or Artificial termes, and duely placeth every number thus. First set down the greater of the two given numbers for the number to be increased, or multiplyed, which is called the *Multiplicand*; and under him beginning at the right hand (observing due place, and denomination of Units, Tens, &c.) place the lesser given number for the multiplying number, called the *Multiplier*, yet as in *Addition* it is not material which standeth uppermost, for either may be exchanged for other, but it is most orderly to place the greater number at top; then draw a right line under them to seperate them from the number found out by their *Multiplication*, which is called the *Multiplee*, and ordinarily the *Product* because produced by the other two, sometime the *Offcome*, as coming off them, sometime the *Factus*, or *Fact*, being made by them, in reference to which the two given Numbers, *Viz.* the *Multiplicand* and *Multiplier* are called *Factores*, or *Factors*, and sometime also the *Product* is called the *Rectangle* or *Plain*, which implies that one of the propounded numbers is taken for the length and the other for the breadth of a *Rectangle*, or plain Figure geometrical, in respect to which the given numbers are called sides, and so shall the *Product* be understood for the *Content* or *Area* of that *Rectangle* figure.

Operation hath *Induction*, and *Perfection*; That sometime called *Simple*, and this *Compound Multiplication*.

Induction helpeth to know the product of any two digits multiplyed into themselves; without which knowledge great Numbers cannot be multiplyed; which is commonly thus done at the two ends of a Cross; on the left side place the two digits to be multiplyed, the one over the other; then subtract each digit from 10. and let the several remains respectively collateral to the digit from whence it came; again subtract either of these differences, it matters not which from the other digit, cross-wise, and this remain place under the digits: Lastly, Multiply the two first differences, and the number amounting place under them to the right hand of the number before set down, and this shall be the product of the two digits. As to know how much 7 times 8 is, they are placed as at A. then 7 abated from 10 rest 3, and 8 from 10 rest 2,

as at B. Lastly 2 from 7, or 3 from 8 the remain is 5, and 2 times or twice 3 is 6, as at C. So is 7 times 8 found to be 56.

A

7

8

X

B

7

8

X

3

2

C

Sub.

Dat.

7

Diff.

3

X

Mult.

2

56

Used by
Alsted.

Example.

Best way of
Induction.

This way is used both by *Alsted*, our Countrey-man *Record*, and Common School-masters, yet *Alsted* seems not well-pleased with it, calls it the Sluggards Rule, and prefers another before it, which is to break one of the digits into several parts, and multiply those small parts by the other digit, and add all these little Products together, as if in the former Example, 8 should be dissected into 2. 2. 2. 2. and each of them multiplied by 7 make 14. which set down 4 times, and added make 56 as before.

The Rule is undoubtedly true this Authour grounds this Operation on, (which is, If two Numbers one be cut into several parts, the product of the two Numbers shall be equal to the product of the one by the several parts of the other) but the purpose for which both this, and the former are brought, is not without exception; for to produce Rules to teach any thing, which require the knowledge of the thing pretended to be taught by the Rule, is not proper, and surely in *Logick* such reasoning would be accounted begging the Question, as to bring a thing doubtful to prove a thing in dispute: Yet such is the Mishap in these Rules which instead of teaching to know the Sum of any two digits, in order to learn to multiply; set the ignorant to multiply to find them out.

The better way for the Learner, is to set down one digit so many times as there be Units in the other digit, and add these into one total, as to set down 8 seven times, and the added total will be 56, as before.

But to avoid the prolixity of this, the best way to have *ad Unguem*, the value of any two digits multiplied together, is to learn by heart the Table commonly called *Multiplication Table*, where they are all expressed thus.

Multiplication

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

Table.

Table how far
enlarged by
Pythagoras.
Use of the
Table.

This Table *Pythagoras* had enlarged to 12, others to 10, but 9 being sufficient, it is contracted here, and because 8 times 7, and 7 times 8, or any such like, is all one, some convert it into a Triangular form, using only the part above the black Scale.

The Table is thus to be read, 2 times, or twice 2 is 4, twice 3 is 6, twice 4 is 8, &c. So 3 times 2 is 6, and 3 times 3 is 9, and 3 times 4 is 12. and so of the rest. So that to find the Product of any two digits multiplied together, is to find one of the digits in the left side, and the other in the head of the Table, and in the Angle common to both, is the product required: As to know how many 5 times 6 is, against 5 in the side, and 6 in the head, (or alternately contrary, 5 in the head, and 6 in the side) is found 30, and so much is 5 times 6, or 6 times 5.

Compleat Operation, proceedeth first, to multiply by Digits, and Articles, and afterward by mixt Numbers, and in either, the common, and select way of proceeding is observable.

The Compound
Multiplication
how wrought
commonly.

The

The Common way to multiply by digits, is after the numbers are placed as before, *By Digits.* to begin at the right hand, and multiply every figure of the *Multiplicand* by the multiplying digit, and what doth amount thereof place under the same if it be a digit, if an Article subscribe the Cypher, and reserve the figure of the Article (as in *Addition* was taught) to be added unto the Sum of the next multiplied figure; and if it be a mixt number, set down the digit, and reserve the Article as before.

Example. 5291 is to be multiplied by 2, after the Numbers set, as at D. say 2 times *Example.* or twice 1 is 2, which is subscribed, as at E. then twice 9 is 18, whereof 8 is subscribed, as at F. and 1. the Article borne in mind, again twice 2 is 4, and 1 carried is 5, to be set in the next place, as at G. and lastly twice 5 is 10. set as at H. So is the product of 5291 doubled or multiplied by 2, the Sum of 10582.

D	E	F	G	H	
5291	5291	5291	5291	5291	Multiplicand.
2	2	2	2	2	Multiplier.
	2	82	582	10582	Product.

In the same manner proceed to multiply by Articles, increasing the *Multiplicand* by *By Articles.* the signifying figure of the Article, the Cyphers whereof are best placed as at the 10. 11. and 12. Sections of this Chapter, but with inartificial Artists are set down as in the following instance of multiplying 5291 by 20, as at I.

	5291	Multiplicand.	
I	20	Multiplier.	<i>Example.</i>
	0000		
	10582		
	105820	Product.	

The Common way to multiply by mixt Numbers is, after all the figures of the *Multiplicand* be gone through as before by the first multiplying figure of the *Multiplier* *By mixt numbers.* then take the second figure of the *Multiplier*, and proceed therewith as before, and so with the third, fourth, fifth, &c. multiplying figures, increase each figure of the one given number by each figure of the other, till all be gone over, observing to set the product of the first multiplied digit of each, directly under the multiplying figure; and the rest gotten thereby orderly in a straight line to the left hand thereof; then under all these lines of production sometime called *Multiplees*, which will ever be as many, as there be figures or places in the *Multiplier*, draw another right line, and add all the particular products into one total product, and this shall be the desired number.

Example. To increase 402 by 349, the numbers placed as at K. after the *Multiplicand* *Example.* is gone over with 9 the first multiplying digit, the work stands as at L. then begin with 4, and say 4 times 2 is 8, which subscribe under 4, and 4 times 0 is 0, which set down to keep place, also 4 times 4 is 16, which makes the work as at M, and so proceeding do the like with the multiplying 3. Lastly, Adding those three lines of production together the total product of 402 multiplied by 349 is found as at N. to be 140298.

K	L	M	N	
402	402	402	402	Multiplicand.
349	349	349	349	Multiplier.
	3618	3618	3618	} Multiplees added.
		1608	1608	
			1206	
			140298	Total Product.

This Select way of *Multiplication* consisteth either in certain brief or compendious *Select ways of* Rules, or other choice Methods of proceeding, whereby the work of the Common *Multiplication-* way

way is abbreviated or meliorated, and these are both comprehended in these 18 following Sections.

1. If an Unite be in the Multiplier.

Because an Unite neither multiplieth nor divideth, wheresoever 1 is found in the Multiplier, it is but only to subscribe the *Multiplicand* in its due place among the *Multiples*, and add them as before; as to multiply 342 by 12, the product is 4104. Set as at O or P.

Example.

$\begin{array}{r} 342 \text{ Multiplicand.} \\ \hline \text{O } 684 \text{ Double.} \\ 342 \\ \hline 4104 \text{ Product.} \\ \hline \end{array}$	<p>P</p> $\begin{array}{r} 342 \\ 684 \\ \hline 4104 \\ \hline \end{array}$	$\begin{array}{r} 342 \\ 12 \text{ Common way.} \\ \hline 684 \\ 342 \\ \hline 4104 \\ \hline \end{array}$
---	---	--

2. If 2 be the Multiplier.

To multiply by 2 called *Duplication*, is nothing else but to double every figure of the *Multiplicand*, and subscribe the number amounting, if the double be not above a digit, but if an Article or mixt number subscribe the Cypher, or remaining figure, and for the Article account the next figure to the left hand one more than the double; as 4372 duplicated or multiplied by 2, for 2 is subscribed 4, for 7 the next amounting to an Article 4, and go 1. wherefore 3 the next is made 7, that is 1 more than the double, and the double of 4 is 8. Thus,

Example.

$\begin{array}{r} 4372 \text{ Multiplicand.} \\ \hline 8744 \text{ Product.} \\ \hline \end{array}$	$\begin{array}{r} 4372 \\ 2 \text{ Common way.} \\ \hline 8744 \\ \hline \end{array}$
---	---

3. If 3 be the Multiplier.

Example.

Triplication, or to multiply by 3, is to add the given number to the double of the same, as to multiply 4372 by 3 produce 13116, for 4 the double of 2 added to 2 make 6, and 14 the double of 7 added to 7 make 21, of which 1 subscribed, and 2 carried away, which 2, 3 and 6 the double of 3 is 11, whereof 1 set down in the next place, and the other 1 reserved, which at last added to 4 and 8 makes 13, and for more ease, some first put down the double: Thus,

$\begin{array}{r} 4372 \text{ Multiplicand.} \\ 8744 \text{ Double.} \\ \hline 13116 \text{ Product.} \\ \hline \end{array}$	$\begin{array}{r} 4372 \\ 3 \text{ Common way.} \\ \hline 13116 \\ \hline \end{array}$
--	--

4. If 4 be the Multiplier.

Example.

Reduplication, *Quadruplication*, or *Multiplication* by 4 is to double the *Duplication*, as to multiply 15 by 4 is to double 30 the double of 15, and the 60 amounting is the Product; so in the former Example 17488 is the double of 8744, which was the double of 4372, therefore 17488 is the Product of 4372 multiplied by 4.

$\begin{array}{r} 8744 \text{ Multiplicand doubled.} \\ \hline 17488 \text{ Product.} \\ \hline \end{array}$	$\begin{array}{r} 4372 \\ 4 \text{ Common way.} \\ \hline 17488 \\ \hline \end{array}$
--	--

5. Multiplication by 5.

Example.

To multiply by 5, called *Quintuplication*, adjoyn a Cypher to the right hand of the *Multiplicand*, and then take the half thereof as to multiply 468 by 5, the Product is 2340.

$\begin{array}{r} 468.0 \text{ Cypher adjoyned.} \\ \hline 2340 \text{ Product.} \\ \hline \end{array}$	$\begin{array}{r} 468 \\ 5 \text{ Common way.} \\ \hline 2340 \\ \hline \end{array}$
---	--

6. By 6.

Example.

Sextuplication, or to multiply by 6, adjoyn a Cypher to the given number as before, take half thereof, beginning at the right hand, and to the half add the figure standing next before; as to multiply 468 by 6, the Cypher adjoyned makes it 4680, then

then the half of 0 is 0, but 8 next before is 8, so the half of 8 is 4, and 6 next make 10, also the half of 6 is 3, to which the next 4 and the 1 reserved of 10 make 8 for the next place, and at last the half of 4 is 2. So the Product is found to be 2808, or the half may be set down and added.

468.0 Cypher adjoyned.	468	468
	2340	6 Common way.
2808 Product	2808	2808

Septuplication, or to multiply by 7, add half each figure to the double of the figure next before, a Cypher being first adjoyned, as to multiply 468 by 7, the half of 0 is 0. but the double of 8 is 16, of which 6 is subscribed, and 1 reserved, then the half of 8 is 4, being added to 1 before reserved, and the double of 6 make together 17, whereof 7 is set down, and 1 reserved, which with the half of 6 and double of 4 make 12, the reserved 1 of which with the half of 4 make 3 to be set at last, and so the Product is 3276

468.0 Cypher adjoyned.	468
3276 Product.	7 Common way.
	3276

Octuplication, or to multiply by 8. subtract the double of the given number from the same increased by a Cypher adjoyned to the right hand thereof, as to multiply 468 by 8. the double is 936, which taken from 468.0 leaves 3744 the Product desired.

468.0 Cypher adjoyned.	468
936 Double subtracted.	8 Common way.
3744 Product.	3744

Noncuplication, or to multiply by 9, adjoyn a Cypher to the Right hand of the given Number, as before, and from thence subtract the same *Multiplicand*, as to multiply 468 by 9, the Product will be

468 0 Cypher adjoyned.	468
468 Multiplicand subtracted.	9 Common way.
4212 Product.	4212

To multiply by 10, 100, 1000, &c. adjoyn so many Cyphers to the Right hand of the *Multiplicand*, as there be Cyphers in the *Multiplier*, as to multiply 468 by 100 two Cyphers being adjoyned, the Product is 46800.

468 Multiplicand.	468
100 Multiplier.	100 Common way.
46800 Product.	000
	000
	468
	46800

To multiply by 20, 30, 40, &c. 200, 300, 400, &c. multiply the given number by the signifying Figure of the *Multiplier*, and to the right hand of the *Product* place so many Cyphers as shall be in the *Multiplier*, as to multiply 468 by 200, the Product is 93600. for the 468 doubled or multiplied by 2, is 936, to which the 2 Cyphers in 200 are adjoyned.

468 Multiplicand.	468
200 Multiplier.	200 Common way.
93600 Product.	000
	000
	936
	93600

12. When Cyphers are at the right hand of the right hand.

Example.

To multiply when Cyphers are at the right hand of the *Multiplier* or both the given Numbers, first work with the signifying figures, and to the right hand of the Product place so many Cyphers as the *Multiplier* and *Multiplicand* had at the right hand places. As to multiply 3400 by 40, first 34 multiplied by 4 produce 136, to which 3 Cyphers adjoyned make the true Product 136000.

3400 <i>Multiplicand.</i>	3400
40 <i>Multiplier.</i>	40 <i>Common way</i>
136000 <i>Product.</i>	0000
	13600
	136000

13. When Cyphers come between.

Example.

To multiply when one Cypher, or more, fall between the signifying figures of the *Multiplier*; instead of making a line of Cyphers, place only one Cypher to keep place and then proceed with the next figure of the *Multiplier*. As to multiply 1432 by 204, the *Product* will be 292128.

1432	1432
204	204 <i>Common way.</i>
5728	5728
28640	0000
292128 <i>Product.</i>	2864
	292128

14. To multiply by nines.

Example.

To square any number of Nines, that is to multiply them into themselves, as 4 nines by 4 nines, &c. to the right hand of the number of nines to be multiplied place so many Cyphers, as there be nines, then add 1 to the first Cypher, and subtract 1 from the first nine, subscribe this 1 and 8 in their places, together with the Cyphers between 1 and 8, and the nines beyond 8, so is the *Product* obtained. *Example*, 9999 by 9999, produceth 99980001. Thus,

9999.0000	9999
abated 1 1 added	9999 <i>Common way.</i>
99980001 <i>Product</i>	89991
	89991
	89991
	89991
	99980001

15. To multiply without charging the memory.

Example.

To multiply without charging the memory by carrying the Articles. Multiply as before figure by figure, and what amounteth thereof set down if an article or mixt number, but if a digit subscribe the digit and before it a cypher in the place of an article, then multiply the next figure of the *Multiplicand*, and what amounteth subscribe the digit under the Article or Cypher before set down, and if there be an article, place it in the 100 place of the Sum, if no article, place a cypher as before instead thereof, and so going forward for every figure in the *Multiplier* make two lines of production. As to multiply 5142 by 43, because 3 times 2 is but 6 before it is a cypher placed, then 3 times 4 is 12, which is set down according to the former directions, the other like process of the work may be plainer discerned by the little lines drawn between the Numbers arising upon *Multiplication*.

5 1 4 2	5142
4 3	43 <i>Common way.</i>
0, 1, 0-6	15426
1-5 3 2	20568
0, 1, 0-8	221106
2-0 4 6	
2 2 1 1 0 6 <i>Product.</i>	

To multiply and bring the total production in the last line of the work, without the ordinary manner of *Addition*, or charging the memory, when the numbers are placed as usual, multiply figure by figure, as before, and what ariseth thereof set down the digit under the line, and the article above, or a Cypher instead thereof, then multiply the next figure, and there add the article before set down, cancelling him with a dash of the Pen, and this amounting Sum place as before, the digit under the line one place nearer to the left hand, and the article if any above, and so do till all the figures of the *Multiplicand* be run through, and the product of the last figure of the *Multiplicand*, set down the article beneath the line as well as the digit; and when the first line is done, draw a rectangle line toward the left hand, cutting off the right hand figure, then take the second *Multiplier*, and begin to multiply therewith, and add in their several places the uncanceled figures within the rectangle line, as well as the articles newly set down, and thus renew the rectangle lines, and proceed according to the number of the multiplying figures: So at last will all the figures be cancelled except those below the last rectangle line, and such right hand figures as were cut off by them, which being pulled down as they stand yield the desired product. As to multiply 8546 by 423 when 6 is increased by 3 the amounting 18 is placed 8 beneath the line, and 1 above, then 3 times 4 which is 12, and 1 above the line is 13, of which 3 is set beneath the line, and 1 above, and the first 1 cancelled, then 3 times 5 is 15, and 1 above the line uncanceled is 16, whereof the 6 subscribed and the 1 is set at top; again 3 times 8 is 24, and 1 at top make 25, which is subscribed, so is the first multiplying figure done with, and the work standeth as at Q. Then 8 cut off by the rectangle line, and 2 the second multiplying digit proceedeth with, beginneth with twice 6, which is 12 and 3 above the rectangle make 15, of which 5 subscribed in his place leaveth 1. to be set above the Rectangle, so twice 4 is 8, with 1 last set down, and 6 over him in all make 15, which is placed as before, and so proceeding to the end of the *Multiplicand* by this digit 2, the work appeareth as at R, and in like manner proceeding with the multiplying 4, the finishing Paragraph is represented at S, where are found under the line 36149, and cut off by the Rectangle, 5 and 8, which together make the product 3614958.

16. To multiply and bring the Total into the last line of Production.

Example.

Q	R	S	
8546	8546	8546	8546
423	423	423	423
xxx	xxx	xxx	Common way.
25638	25638	25638	25638
	19655	19655	17092
		322	34184
			3614958
			36149

To multiply by *Nepair's Bones*: Always Tabulate or Place the *Multiplicand* on the Bones, and by the help of the *Index* taking the Sum of every *Diagonal Square*, or *Rhomboid*, answering to the several multiplying figures in the *Multiplier* you have the several lines of production which dispose as before taught, viz. each line one place nearer to the left hand, then the other according to the multiplying figures, and add them into one total, and you have the product desired. As to multiply 16750 by 258, the *Multiplicand* 16750 being tabulated on the Bones, and the *Multiples* answering to 2. 5. and 8 in the *Index*, taken out are 33500. 83750. and 134000. which added make the product 4321500. See the *Bones* and *Index* at T. the Productions added at V. and as some set them at W.

17. To multiply by Nepaires Bones.

Example.

Bones.			
Index	I	16750	16750
	2	258	258
	3	33500	33500
	4	83750	83750
	5	33500	134000
	6	4321500	4321500
	7		
	8		
	9		

To

18. To multiply by the Back-side of the Bones.

Example.

To multiply by the back-side of the Bones, which some use when they would conceal the numbers they multiply from the present Spectators. First tabulate the *Multiplicand* on the Bones as before, then turn up the Bones just upside down, laying that Face which was underneath upward, and work with the number which here appeareth tabulated, as if it were the true number, to the product found thereby add the *Multiplier*, then subtract this total from the *Multiplier*, adding a sufficient number of Cyphers thereto, and the Remain shall be the Product desired. As in the former instance 16750 tabulated on the back-sides, will be tabulated 83249, which multiplied into 258 produceth 21478242, whereto 258 added, the product and total together are 21478500, this number subtracted from 258, and so many Cyphers adjoined as the *Multiplicand* had Figures, viz. 5, leaves the former Product 4321500.

$$\begin{array}{r} 25800000 \\ 21478500 \\ \hline 4321500 \end{array}$$

Bones were invented by Nepair.

They are Multiplication Table cut in pieces.

The use of these Bones or Rods have been sufficiently explained by others, it were but *actum agere* to insist upon it here: Only in brief, They are called *Nepair's Bones* from the Inventer the Honourable Lord of *Marchiston*, they are best made of *Ivory*, or *Wood* 4 square, of an equal thickness, about one fifth part of an Inch square. their length about 9 times their breadth, they are in number 10, but to have two or three sets of them will be convenient, because with one set can be tabulated but only, figures of one and the same *Species*; as 4 Cyphers, 4 Units, &c. but 20 or 30 Rods can tabulate 8 or 12. They have all the digits on them, and their *Multipliers* to 9, being only *Pythagoras's Table* cut in pieces. There is also an *Index* belonging to the Rods, of the same length and breadth. See more in the following Table.

The Description of all the four faces of every one of the Ten Rods, and the Index.

Figures of the Bones.

<div>0 1 9 8</div>	<div>0 2 9 7</div>	<div>0 3 9 6</div>	<div>0 4 9 5</div>	<div>1 2 8 7</div>
<div>0 2 8 1</div>	<div>0 4 8 1</div>	<div>0 6 8 1</div>	<div>0 8 8 1</div>	<div>2 4 6 1</div>
<div>0 3 7 2</div>	<div>0 6 7 1</div>	<div>0 9 7 8</div>	<div>0 2 7 1</div>	<div>3 6 4 1</div>
<div>0 4 6 3</div>	<div>0 8 6 2</div>	<div>0 1 6 2</div>	<div>0 6 6 2</div>	<div>4 8 3 2</div>
<div>0 5 5 4</div>	<div>0 1 5 3</div>	<div>0 5 5 3</div>	<div>0 2 5 2</div>	<div>5 1 4 3</div>
<div>0 6 4 5</div>	<div>0 2 5 4</div>	<div>0 1 5 4</div>	<div>0 2 5 4</div>	<div>6 2 4 4</div>
<div>0 7 3 6</div>	<div>0 4 6 3</div>	<div>0 2 6 3</div>	<div>0 2 6 3</div>	<div>7 4 5 4</div>
<div>0 8 2 7</div>	<div>0 6 7 2</div>	<div>0 2 7 2</div>	<div>0 3 7 2</div>	<div>8 6 6 5</div>
<div>0 9 1 8</div>	<div>0 8 8 1</div>	<div>0 2 8 1</div>	<div>0 3 8 1</div>	<div>9 8 7 6</div>
<div>1 3 8 6</div>	<div>1 4 8 5</div>	<div>2 3 7 6</div>	<div>2 4 7 5</div>	<div>3 4 6 5</div>
<div>2 6 1 2</div>	<div>2 8 1 0</div>	<div>4 6 1 2</div>	<div>4 8 1 0</div>	<div>6 8 1 0</div>
<div>3 9 2 1</div>	<div>3 1 2 1</div>	<div>6 9 2 1</div>	<div>6 1 2 1</div>	<div>9 1 2 1</div>
<div>4 1 3 2</div>	<div>4 1 6 2</div>	<div>8 1 2 2</div>	<div>8 1 2 2</div>	<div>1 2 1 6</div>
<div>5 1 5 3</div>	<div>5 2 1 0</div>	<div>1 1 5 3</div>	<div>1 2 1 5</div>	<div>1 5 2 3</div>
<div>6 1 8 3</div>	<div>6 2 4 3</div>	<div>1 2 1 8</div>	<div>1 2 4 3</div>	<div>1 8 2 4</div>
<div>7 2 1 4</div>	<div>7 2 8 5</div>	<div>1 4 2 1</div>	<div>1 4 2 8</div>	<div>2 1 2 8</div>
<div>8 2 4 4</div>	<div>8 3 2 4</div>	<div>1 6 2 1</div>	<div>1 6 2 5</div>	<div>2 4 3 6</div>
<div>9 2 7 5</div>	<div>9 3 6 5</div>	<div>1 8 2 7</div>	<div>1 8 3 6</div>	<div>2 7 3 6</div>

The INDEX.

0	1	2	3	4	5	6	7	8	9
---	---	---	---	---	---	---	---	---	---

Probation is here as before, to try the truth or falshood of the work. Particularly *Multiplication* performed by any of the Select Rules aforegoing, may be proved by the Common way. But generally the proof of all sorts of *Multiplication* may be had, either by *Addition*, Casting away Nines, or *Division*.

Addition, (especially in great *Multipliers*;) is a most tiresome and tedious way; yet it is thus set down the *Multiplicand* so many times as there be Units in the *Multiplier*, and add all together, the total will be the same with the *Product* if the work be right; As 1343 multiplyed by 3, the *Product* is 4029, and so is the total of that number set down 3 times.

1343	1343
1343	3
1343	—
—	4029 Product
4029 Total.	—

The truth of this dependeth on that *Theoreme*, which is the very Basis of *Multiplication*, viz. as an Unit to the *Multiplier*, so is the *Multiplicand* to the *Product*, and *contra*, for if 6 times 4 be 24, then 1 to 4 shall be as 6 to 24, and 1 to 6, as 4 to 24. But this way of Proof makes the remedy worse than the Disease, and therefore is rejected.

The Proof by casting away 9, is to cast 9 as oft as may be from the *Multiplicand* and *Multiplier* severally, and set the two remains at the opposite Angles of a Cross; then multiply these remains one into another, and if the *Product* thereof be under 9, set it down at another of the Angles of the Cross, if above 9, cast away 9, and set down the residue; Lastly cast away 9 from the *Product*, and the Remain thereof shall alwayes be equal to the Remain left before put down, if the work be right. As in the former Example proved by *Addition*, the nines cast from 1343 leave 2, which set opposite to 3 the *Multiplier*, and multiplyed thereby produce 6, and so much will be left when 9 is cast from the *Product* 4029 standing on the Cross thus,

$$\begin{array}{ccc} & 6 & \\ 2 & \times & 3 \\ & 6 & \end{array}$$

This kind of Proof upon the same account it was found faulty before in *Addition*, is fallible also here. But the most certain and infallible proof of *Multiplication*, is by *Division*; of which more in the next Chapter, wherefore to conclude this; take the two following Observations.

1. If any Number be multiplyed, it is so often added to its self as there be Units in the *Multiplier*.
2. If a Number be compounded of two Numbers, and that Number multiply another Number, the *Product* shall be equal to the *Product* of that Number multiplyed by those two Numbers. As 6 is compounded of 2 and 3, let 6 then multiply 9, the *Product* is 54, which is equal to 9, multiplyed into 2, that is 18, and 18 multiplyed into 3, which is 54.

C H A P. VIII.

Division of Integers.

Division, called by some *Partition*, is the Compound Analysis of Numbers, and that part of *Conjunct Numeration*, whereby one Number is subtracted from another so often as it is contained in it, and by that means is found how many of the one Number are contained in the other: So that the third Number, or Number artificially obtained by the two propounded Numbers shall so often contain an Unit, as the greater of the two given Numbers contains the lesser, and it serveth instead of many *Subtractions*, of which and *Multiplication*, it consists, and is composed.

How to place
the Numbers.

Dividend
what.

Divisor what.

Quotient what
and whence it
comes.

Parabola
what.

Remain if any
to be left un-
cancelled.

To be less than
the Divisor.

How to be set.

Synonimical
termes.

Induction called
Simple Divi-
sion.

Use of Multi-
plication Table
for Induction.

Complete Divi-
sion what
and how to be
performed.

§ Divis.

Division observeth *Collocation*, proceedeth to *Operation*, and concludeth with *Probation*. *Collocation* respecteth the *Nomenclature*, and duely placeth every number in manner following. First set down the Number to be divided, which is sometime called the *Dividuum*, but commonly the *Dividend*; and under it at the left hand (contrary to all the former Operations of *Arithmetick*) place the Number by which it is to be parted or divided, called the *Divisor*, observing to set the foremost figure to the left hand of the *Divisor*, under the foremost left hand figure of the *Dividend*, if the *Divisor* may be substracted from those figures of the *Dividend*, which stand above over them; but if not, then let the first left hand figure of the *Divisor* stand under the second left hand figure of the *Dividend*, and so in order proceeding to the right hand place all the other figures of the *Divisor*, then draw the Decrescent Lunular, or Seperatrix to distinguish these given Numbers from the Number to be found by *Division*, which is called the *Quotient*, and sometime *Parabola*; *Quotient* is derived from *Quoties*, because the number is gotten by enquiring how often the *Divisor* is contained in the *Dividend*. *Parabola* ariseth, from the application of a plain number, to a given Longitude, that a congruous Latitude may be found out; as will be more demonstrable in Figural numbers. If after *Division* be ended, any figures be left uncanceled, these are called the *Remain*, and denote that the *Dividend* could not be exactly parted by that *Divisor*; but there will be a number remaining, unless the *Dividend* be reduced to another Denomination: As in denominate numbers: And this Remain is alwayes to be less than the *Divisor*; and distinguished from the other figures by little Lunular or Rectangle Lines, like the Quotient Line; or set over the *Divisor* with a little Line between; as a *Fraction*. Lastly may be noted, That to part, measure or compare a number, are termes used synonimically, and signifie but to divide that number.

Operation hath both *Induction*, and *Perfection*; The one sometime called *Simple Division*; and the other *Compound*; and both presuppose the *Dividend* to be greater than the *Divisor*.

Induction by the help of *Multiplication Table*, sheweth how often any digit is contained in the *Multiplee* of any two digits: And consequently by suffrage of the same Table findeth how often the same digit is contained in all the intermediate Numbers that happen between such *Multiplees*.

To find how oft any digit is contained in the *Multiplee*, enter the Table with the *Multiplee*, among the *Areal* Numbers just against the given digit at the left side, and the number desired shall be the digit just over the *Areal* Number at top; but if the given digit be found in the head of the Table, the desired number shall be the digit at the left side just against the *Areal* number. As to find how often 2 is contained in 10, entring with 10, in the collateral Columne against 2 in the side, 5 is found at top over 10, as also against 2 in the head, and 10 in the perpendicular Collumne, which sheweth that 2 is contained 5 times in 10, and in like manner is found how often any of the other *Multiplees* below or above 10 do contain 2, as 4 containeth 2 twice, 6 three times, &c.

By consequence also may be found how often 2 is contained in all numbers intermediate, between those *Multiplees*, for 3 intermediate between 2 and 4 shall contain 2, but once and a half part more, and 5 intermediate between 4 and 6 shall contain 2. but twice and a part more, all intermediate numbers never exceeding the precedent *Multiplees* by an Unit, but only by a part or parts: for where there is but one number intermediate there will an Unit remain after the integral content is substracted; where two intermediate Numbers are, 1. will remain over the first intermediate number, and 2 over the second, and so accordingly in the rest.

Complete Operation proceedeth first to divide by digits and Articles, and afterwards by mixt numbers; and in either the common and select way of proceeding is considerable.

The Common way to divide by digits is, after the given numbers are placed as before, begin at the left hand, and see how many times the Dividing digit can be taken out of the figure of the *Dividend* standing over him, or to the left hand thereof, if the Dividing digit stand not under the first left hand figure of the *Dividend*, and having found how often this *Divisor* is contained in such figure or figures of the *Dividend*, set beyond the *Quotient* line a figure signifying how often; as 1 for once, 2 for twice, &c. then substract the content of the *Divisor* so many times as is set down in the *Quotient* from the *Dividend*, and let the *Remain* if any be over the figure of the *Dividend*, where it was left remaining, alwayes cancelling with a dash of the Pen the *Divisor* when done with, and those figures of the *Dividend* whence any thing was taken, then set down the *Divisor* one place nearer to the right hand, and thereby get another *Quotient* figure, and so proceed

proceed as before to the end of the *Dividend*; and so many times as the *Divisor* may be set down in the *Division*; so many figures or cyphers will be in the *Quotient*.

Example. To divide 3762 into 3 equal parts, the numbers being set as at A. inquire how many times the *Divisor* may be had out of 3. the *Dividend* figure over him, and finding it may be taken once, place 1 in the *Quotient*, and subtracting once 3 out of 3 there is 0 remaining; then cancelling the 3 at top, and the *Divisor*, the work stands as at B. Then placing the *Divisor* under 7, and inquiring as before, 2 is gotten and set in the *Quotient*, and twice 3 which is 6 subtracted from 7 leaves 1 remaining, as at C. Again placing 3 the *Divisor* under 6, by like inquiry is found 5 times 3, which is 15 to be contained in 16, therefore 5 is placed in the *Quotient*, and abating 15 from 16. leaves 1 to be set over 6, and the work appears as at D. Lastly, 3 placed under 2 by inquiry as before, 4 is obtained for the *Quotient*, which multiplying the *Divisor* 3 produceth 12 to be taken from the 12 before left of the *Dividend*. So there remains 0. And the *Quotient* is found to be 1254, which is the third part of the given *Dividend* 3762, and the Compleat work stands as at E.

	E	D	C	B	A
	21(0 Remainer	21	1		
Dividend	3762(1254 Quotient.	3762(125	3762(12	3762(1	3762(
Divisor	3333	333	33	3	3

In like manner procede to divide by Articles, dividing the *Dividend* by the signifying figure of the Article, for the Cyphers do but only keep place, and are best set as in the third *Section* of this *Chapter*; though with young Beginners they are set down as in the *Example* following to divide 1762 by 30 where the *Quotient* is 58, and 22 remain, as at F.

	F
	2(2
Dividend	1762(58 Quotient or 58 ²² / ₃₀
Divisor	300
	3

Example.

The Common way to divide by a mixt number is somewhat more difficult, yet is the former Method still observed, in placing the numbers and finding out the first *Quotient* figure only with this difference, that when the *Divisor* was a single digit or article, the same *Quotient* figure is to be as big, as could be subtracted from the figures of the *Dividend*, to the left hand thereof; now the first left hand dividing figure is to be taken no oftner from the figure or figures standing over him than that also every following figure of the *Divisor*, may be taken so often out of the figure or figures that stand over them, or are to the left hand of them, by borrowing as in *Substraction*, to supply the deficiency thereof; and though sometimes the first figures of the *Divisor* will seem to leave too much behind, yet the 2^d or 3^d figure may want: So as it will be convenient to reckon in mind a little before Operation whether the just figure be taken, which must never be above 9 at most, nor under so many times as the *Divisor* is contained, for then will the *Remain* be greater than the *Divisor*, which is not to be suffered.

Example. If 34633 were to be divided by 12, the numbers placed as before, and seeking how often 1 is contained in 3 there is found 3, if 1 were single, because 1 divideth not, but being joyned with 2, if 3 should be cancelled there would be but 4 left, and then thrice 2 which is 6 could not be taken from 4, therefore 1 is to be taken but twice out of 3, and the remaining 1 set over 3, and 2 is set in the *Quotient* as at G. Then seeing 1 is taken but twice out of 3 the other figure of the *Divisor*, 2 must be taken but twice, which is 4, from 4, at top resteth 0, to be set down because of 1 to the left hand as at H. Then setting the *Divisor* down again, and enquiring how many times 1 may be had out of 10 is found but 8, because there will not else be enough left to take the other figure of the *Divisor* so often out of the remaining figures to the left hand; therefore taking 8 for the *Quotient* out of 10 at top, resteth 2 to be set over 0, as at I. And 8 times 2 is 16 in like manner taken out of the figure over him, that is out of 6 cannot, but out of 16, borrowing 10 there remains 6, and the borrowed 1 out of 2 rests 1 as at K, moreover removing the *Divisor*, and by inquiry as before, 8 is found again for the *Quotient*, and twice 8 taken from 23 leaves 7, as at L. Lastly, Removing the *Divisor*, and seeking thereby as before, 6 is found for the *Quotient*, taken from 7 leaves 1, and twice 6 is 12 from 13 is left 1 remaining, when the *Division* is ended at M. where the work is compleat.

G	H	I	K	L	M
		2	2	22	222
1	10	10	100	1007	10071
34633(2	34633(2	34633(28	34633(28	34633(288	34633(2886.
12	12	122	122	1222	12222
		1	1	11	111

If the Dividend be less than the Divisor.

When the *Dividend* is less than the *Divisor* set the numbers in manner of a *Fraction*, with a right line between them; For as no *Subtraction* in Abstract Integral Numbers can be made of a greater Number from a lesser; so can no lesser *Integer* be divided by a greater in Abstract Numbers, but the further practise therewith must be referred to Fractionary or Contract Operations; As 15 cannot be divided by 16 in whole Numbers, but may be set thus $\frac{15}{16}$.

Select ways of Division.

The Select way of *Division* consisteth either in certain brief Rules, or other choice Methods of proceeding, bettering, or abbreviating the Common way; comprehend in the 18 Sections following.

1. If the Divisor be 1. with Cyphers.

First, To divide by an Unit with Cyphers, as 10. 100. 1000. &c. Cut off from the *Dividend* to the right hand, so many figures as there be Cyphers in the *Divisor*, and place the figures so cut off over the *Divisor* in form of a *Fraction*; as 3423 divided by 10, the *Quotient* is 342 $\frac{3}{10}$, and by 100. 34 $\frac{23}{100}$.

Example.

The Ground thereof.

The Reason of this Rule, and several others, is grounded upon that *Theorem*. That an Unit neither multiplyeth, nor divideth, for once 2 is but 2 still; and if application be made of 1 to 2, it may be had out thereof twice; and the like it is with any other Number besides 2.

2. If alike Number of Cyphers be in the Data.

If there be a like Number of Cyphers in the *Dividend*, and *Divisor*, and an Unit the only figure of the *Divisor* cut off the Cyphers by a perpendicular stroke of the Pen: and the residue of the *Dividend* to the left hand shall be the *Quotient*. But if the *Divisor* be more than 1, divide the remaining figures of the one by the Remains of the other given number, as 3400 divided by 100 gives 34 in the *Quotient* thus $\frac{34}{100}$. and 34800 divided by 12 gives 29 thus.

Example.

$$\begin{array}{r|l} 12 & 34800 \\ 122 & 00 \\ \hline & 00 \end{array}$$

3. If Cyphers be at the right hand of the Divisor.

If the *Divisor* only have one Cypher to the right hand or more, as it happeneth in Articles. Place the Cypher or Cyphers under the right hand figure or figures of the *Dividend*, and divide all the way by the signifying figure or figures of the Article unto the Cypher or Cyphers before set down, and the *Division* is done striking away the Remainder, as in the following Examples, 20 being *Divisor* to 8401, the Cypher of 20 is set under 1, and 840 is divided by 2, and the *Quotient* is 420 $\frac{1}{20}$, as at N. 800 dividing the same *Dividend*, *Quotient* is 10 $\frac{4}{5}$ as at O, and 150 dividing the same *Dividend*, the *Quotient* 56 $\frac{4}{15}$ as at P.

Example.

$$\begin{array}{lll} \text{N} & 840 \underline{1} (420 & \text{O} & 8 \underline{40} 1 (10 & \text{P} & 840 \underline{1} (56 \\ & 2220 & & 8800 & & 1550 \\ & & & & & 1 \end{array}$$

4. If a Cypher be taken in the Quotient.

In *Division*, If a Cypher at any time be taken in the *Quotient* (which happeneth when the *Divisor* being set down cannot be taken once from the numbers standing over him) then forthwith cancel the *Divisor*, and remove him one place nearer to the right hand, and meddle with none of the *Dividend* figures, and if this be foreseen the setting down the *Divisor* may also be spared by placing the *Divisor* two places nearer to the right hand: Nevertheless 0 must be set in the *Quotient*. As to divide 1340 by 13 at the second setting down the *Divisor*, 13 cannot be taken out of 4, wherefore 0 being set in the *Quotient* the *Divisor* is cancelled, and set down again, without altering the *Dividend*, because nought was taken therefrom.

Example.

$$\begin{array}{lll} 1340(1 & 1340(10 & 1340(103 \text{ Quotient} \\ 13 & 133 & 1333 \\ & 1 & 11 \end{array}$$

When

When it happeneth in *Division* that by multiplying two digits together, there ariseth an Article, meddle not with the digit of the *Dividend*, standing directly over head, but go to the place of the Article. As in dividing 853 by 24, in the second enquiry twice 5 will be 10, wherefore cancelling the 1 of the 13, the 3 is left, likewise 4 times 5 is 20, which taken from the left hand 3 of the 33, leaveth 13 remaining.

$$\begin{array}{r} 2 \overline{) 853} \\ 24 \overline{) 853} \\ 24 \overline{) 853} \\ 24 \overline{) 853} \\ 24 \overline{) 853} \end{array}$$

When in dividing Numbers at the right hand, it happeneth that the *Divisor* hath cut off all the figures of the *Dividend*, so that there remaineth nothing on the *Dividend* but Cyphers; then to the right hand of the *Quotient* adjoyn so many Cyphers as are yet remaining to the *Dividend*, without any part of the *Divisor* standing under them, and the work is finished. As 3654000 divided by 180 or 18, will declare by the following Operations.

$$\begin{array}{r} 3654000(20300 \\ 1888 \quad 0 \\ \hline \end{array}$$

$$\begin{array}{r} 3654000(203000 \\ 1888 \\ \hline \end{array}$$

To take the half of any Number called *Mediation*, *Bipartition*, or *Division* by 2, if the given Number consist of even figures begin at the left, or right hand, it matters not which, and take of every figure respectively the half, and subscribe under the same figures. But if any figure be odd, the best way is to begin at the left hand, and take the least part next the half of the odd figure, and augment the succeeding figure by 10. As to subtract the half of 42983, thus the half of 4 is 2, of 2 is 1, of 9 is 4, of 8 is 9, and of 3 being the last 1. So is the full half thereof 21491.

In like sort it may be accustomed to take the third, fourth, fifth, sixth, seventh, eighth, and ninth parts, of any number, and some of these also may be otherwise obtained sooner than to set down the *Divisor* so often as in the Common way of *Division*, for *Tripartition* is but *Division* by 3. *Quadripartition* to divide by 4, &c. Example, to take the third part of 3687, the fourth part of 5460.

$$\begin{array}{r} 3 \overline{) 3687} \\ 1229 \text{ Quotient.} \end{array}$$

$$\begin{array}{r} 2 \\ 3687(1229 \\ 3333 \end{array}$$

$$\begin{array}{r} 4 \overline{) 5460} \\ 1365 \text{ Quotient} \end{array}$$

$$\begin{array}{r} 122 \\ 5460(1365 \text{ Com way.} \\ 4444 \end{array}$$

Quadripartition, or to divide by 4, may also be thus performed by half the half of the given number, as in the former instance 5460, half is 2730, from whence half abated, leaves 1365 the quarter part as before:

$$\begin{array}{r} 4 \overline{) 5460} \\ 2 \overline{) 2730} \text{ half.} \\ 2 \overline{) 1365} \text{ Quotient.} \end{array}$$

$$\begin{array}{r} 122 \\ 5460(1365 \text{ Com. way.} \\ 4444 \end{array}$$

Quintipartition, or to divide by 5 may likewise be effected thus, double the number given, and cut off the right hand figure or cypher; if any figure be cut off, take half that figure. So 2340 doubled is 4680, from whence 0 cut off, leaves 468 the fifth part thereof. So also 4569 parted by 5 gives 913.

$$\begin{array}{r} 5 \overline{) 2340} \\ \text{Quotient. } 468 \end{array}$$

$$\begin{array}{r} 2340(468 \text{ Com. way.} \\ 555 \end{array}$$

$$\begin{array}{r} 5 \overline{) 4569} \\ 913 \overline{) 8} \\ \text{Quotient. } 913 \end{array}$$

$$\begin{array}{r} 1(4 \\ 4569(913 \text{ Com. way.} \\ 555 \end{array}$$

10. To divide
by 6.
Example.

Sextipartition, or to divide by 6, also may be thus done; take half the third part of the given number. As to take the sixth part of 3684, the third part is 1228, half which is 614 the number sought.

$$6) 3684$$

$$3) 1228 \text{ Third part.}$$

$$2) 614 \text{ Half. Quotient.}$$

$$\begin{array}{r} 2 \\ 3684(614 \text{ Com. way.} \\ 666 \end{array}$$

11. To divide
by 8.
Example.

Octipartition, Or to divide by 8, is but to take half the quarter part of the given Number, as to get the eighth part of 5460, the quarter part is 1365, whereof the half is 682, and one half the quesired number.

$$8) 5460$$

$$4) 1365 \text{ Quarter part.}$$

$$2) 682 \frac{1}{2} \text{ Half. Quotient.}$$

$$\begin{array}{r} 624 \\ 5460(682 \text{ Com. way.} \\ 888 \end{array}$$

12. To divide
by 9.
Example.

Nonupartition, or to take the ninth part of a Number, is to take the third part of the third part of the given Number. As to divide 5463 by 9, the *Quotient* will be 607. So will the remain, if the third part of 1821 be taken, which is the third part of 5463.

$$9) 5463$$

$$3) 1821 \text{ Third part.}$$

$$3) 607 \text{ Third Part. Quotient.}$$

$$\begin{array}{r} 5463(607 \text{ Com. way.} \\ 999 \end{array}$$

13. To divide
by 20, &c.
Example.

From hence it follows, that to divide by 20. 30. 40. &c. 200, 300, 400, &c. after the Cyphers are placed, as *Section 3.* above, the half, third, fourth part, &c. of the remaining figures of the *Dividend* may be taken. As to divide 45156 by 50, when 0 is set under 6, the fifth part of 4515 is taken, which is 903, and six fifties are left remaining.

$$50) 4515 \overline{) 6}$$

$$903 \frac{6}{50} \text{ Quotient.}$$

$$\begin{array}{r} 4515(6(903 \text{ Com. way.} \\ 5550 \end{array}$$

14. To divide
by Nines.

Example.

When any squared number of nines is given to be divided by the number of nines whereof it was produced, place the *Divisor* under the right hand places of the *Dividend*, add them together, and from the total cut off all the Cyphers; the Remain shall be the *Quotient*. As to return 99980001 being the square of 9999 into the Root, I add 9999 to 99980001, and the total is 99990000, which 4 Cyphers cut off, the Remain is the Root.

$$\begin{array}{r} 99980001 \\ 9999 \\ \hline \text{Radix } 99990000 \end{array}$$

$$\begin{array}{r} 8 \\ 198 \\ 070 \\ 080 \\ 18988 \\ 99799 \\ 008888 \\ 1887999 \\ 99980001(9999 \text{ Common way.} \\ 9999999 \\ 99999 \\ 999 \\ 9 \end{array}$$

To cut short the borrowing work in the top figures in *Divisors* consisting of many figures (a most commendable practise) do thus, after the numbers are placed, and the first Quotient figure found out as before; then begin with the right hand figure of the *Divisor*, and take him so many times as the quotient figure denotes from the figure that standeth over him if it may be, if not, borrow in imagination One, or more Tens, as occasion is, that *Subtraction* may be made, and place the remain at top as before, then take the next figure of the *Divisor* and do the like with him, adding in for every 10 borrowed before 1, and so proceed toward the left hand with all the dividing figures, then set down the *Divisor* again, and reiterate this manner of work till the *Division* be ended.

is. To cut short the borrowing, a good and useful way.

As to divide 10816010 by 1234 when the numbers are placed, and 8 found for the first quotient figure, then beginning at 4, and taking him 8 times, which is 32, because but 6 standeth over 4 cannot wholly be taken without borrowing 3 tens, but 2 may be taken out of 6, and leave 4 at top, or reckoning entire 32 may be taken out of 36 and leave 4, either way 3 is reserved to be added in the next place. Then coming to 3 in the *Divisor*, and taking 3 times 8 which is 24, with the former reserved 3, make 27, which are to be taken out of 1, but cannot without borrowing 3 tens again, and then 27 out of 31 leaves 4 to be set at top. Again 2 in the *Divisor* multiplied into 8 in the quotient, produce 16, and 3 before borrowed is 19, where but 2 tens need be borrowed, and out of them and 8 in the *Dividend* which are 28 if 19 be taken there will remain 9. Lastly 1 taken 8 times, and 2 last borrowed added make 10, which abated from 10 at top leaves 0, not set down because no figure standeth to the left hand thereof. So is the work with the first setting down the *Divisor* ended, Then the *Divisor* removed one place nearer to the right hand, and the like work reiterated till the whole *Division* be ended, the *Quotient* is 8765, and this work thus performed hath 13 figures less at top than that wrought the Common way, and much more will it shorten great *Divisions*, and after a little practise be as ready and facil as the other. The several Paragraphs of the work, at Q. R. S. T. are further Exemplary.

Example.

Q	R	S	T
944 10816010 8 1234	80 9442 10816010 87 12344 123	6 801 94427 10816010 876 123444 1233 12	6 801 94427 10816010 8765 1234444 12333 122 1

Some to favour the memory practise the *Italian* way, to multiply the *Divisor* by the several quotient figures as they are found, and accordingly subtract the several *Multiplies* from the *Dividend* one after another: And they place the numbers most conveniently in this form. viz. The *Dividend* between two Parallel Lines, to the right hand whereof the *Quotient* beyond the Decrescent Lunular, as before; and to the left hand beyond an Increscent Lunular the *Divisor*; and to represent the place of the first figure of the *Divisor*, under the *Dividend* a Cypher, and when a quotient figure is gotten, and the *Divisor* multiplied thereby, this *Multiple* is first set down under the *Dividend*, and subtracted, and the remainder set at top, then another Cypher is placed under the *Dividend* to denote the remove of the *Divisor*, and so another quotient figure obtained, the process is as before, only in setting down the several *Multiplies*, there are two formes; the one as at V. and the other as at W. The first of which is the foregoing Example, wherein the several Paragraphs of the work are distinct. The other in dividing 144980099 by 1798 containeth all the difficulties this kind of *Division* can have, and is set entire.

16. The way called the Italian way not chargeable to the memory.

Numbers ready for Work.	First Paragraph.	Second Paragraph.	Examples.
Divisor. Dividend. Quotient.	944	80 9442	
1234) 10816010 (1234) 10816010 (8	1234) 10816010 (87	
0	0	00	
	8 9-8-72	78 9-8-7-28	
	9872	8-6-3'	
		9872	
		8638	

Third

Third	6	and Fourth	6
8 0 1	8 0 1	8 0 1	8 0 1
8 4 4 2 7	8 4 4 2 7	8 4 4 2 7	8 4 4 2 7
1234) 10818010 (876	1234) 10818010 (876	1234) 10818010 (876	1234) 10818010 (876
000	000	000	000
678	9-8-7-284	5678	9-8-7-2840
9872	8-6-3'0'7'	9872	8-6-3'0'7'
8638	7-4'	8638	7-4'1'
7404		7404	6'
		6170	10816010 Proof.

In this Example the figures of each *Multiplee*, are marked with a little line for the plainer evidence, though not needful, the following Example is plain without.

	(1	673(6 Remain	
	11412 5(7		
Divisor.	1798) 144980099 (80634 Quotient.		
	00000		
	43608	14384	W
	14384	0000	
	0000	10788	
Multiplees	10788	5394	
	5394	7192	
	7192	144979932	Total of the Multiplees.
		167	Remain added.
Proof.	144980099	Dividend Returned.	

17. To divide and set the Remains below.

Some not only when they have enquired, and obtained a quotient figure, multiply the *Divisor* thereby, and place the *Multiplee* underneath for *Subtraction*, as before, but also set what remaineth after *Subtraction* beneath the same *Multiplee* without cancelling any figures, and then having the *Divisor* in a moveable piece of paper to apply to the *Dividend* at pleasure, they enquire from those remains with the residue of the *Dividend*, for another quotient figure, and so continue the work till the *Division* be perfected.

Example.

As to divide 34636 by 12, the first quotient figure is found to be 2, whereby 12 multiplied, the *Multiplee* is 24, which subtracted from 34, leaves 10 to be subscribed, then apply the *Divisor* in the moveable paper to 10 the remain, and 6 in the *Dividend* next to it, and 8 may be found for the quotient, and so the like is done with the several remains after the subtraction of 96, 96 and 72 the Products of the *Divisor* multiplied by the Quotient figures, and the last remaining 4 is left at the bottom alone thus,

Divisor.	Dividend.	Quotient.	Proof.
12)	34636	(2886	
6882	24		24
			96
24	10		96
96	96		72
96			
72	10		34632 Total of the Multiplees.
	96		4 Remain added.
			34636 Dividend returned.
	7		
	72		
Remain	4		

Example.

Remain. 8

Another Variety.

Examples.

13. To divide
by Napier's
Bones.

Example.

such subtracted numbers, are to be set in the *Quotient*. As to divide 1396788 by 5678, the *Divisor* 5678 is tabulated on the Bones, and the several *Multiplees* taken out, and 7 in the *Dividend* pricked, where the right hand place of the *Divisor* should stand; the next lesser number to 13967 in the *Dividend*, among the *Multiplees* is 11356, which hath 2 for the *Index*, therefore 2 placed in the *Quotient*, and subtraction made, 8 is pricked in the *Dividend*, and among the *Multiplees* 22712 is found to be the next lesser to 26118, which remain to the last prick of the *Dividend*, and the *Index* of 22712 is 4 to be set in the *Quotient*. Lastly the right hand 8 in the *Dividend* pricked, and finding the next lesser number to 34068 to be the *Multiplee* of 6, this is therefore set in the *Quotient*; and after Subtraction nothing remaineth. As here appeareth.

1	5	6	7	8
2	10	12	14	16
3	15	18	21	24
4	20	24	28	32
5	25	30	35	40
6	30	36	42	48
7	35	42	49	56
8	40	48	56	64
9	45	54	63	72

5678
11356
17034
22712
28390
34068
39746
45424
51102

5678)	1396788	(246
	22712	
	2611	
	22712	
	3406	
	34068	

General Proof
of Division.
By Subtraction
most tedious.

Probation is the Proof whether the *Operation* be true or false, and is either general or particular.

General Proof is for any kind of *Division*, and this may be effected three wayes.

First by *Subtraction*. For if the *Quotient* be subtracted from the *Dividend*, so many times as there be Units in the *Divisor*, there will remain at last no more than the remainder of the *Division*, if the work be well wrought. As in the former instance, pag. 31. where 3762 is divided by 3, the *Quotient* is 1254. If therefore 1254 be taken 3 times from 3762, there will 0 remain, as on the *Division*.

3762	Dividend.
1254	Quotient abated
2508	Remain.
1254	Second Subtraction.
1254	Second Remain.
1254	Third Subtraction.
0	Last Remain.

11(0
3762(1254
3

Grounded on a
Theorem that is
the basis of
Division.

This is a most laborious, and therefore an useless way of trial, yet the truth of this is grounded on the *Theorem*, which is the foundation of *Division*, viz. The *Dividend* to the *Divisor* is as the *Quotient* to an *Unit*, and *contra*. For if 20 divided by 5 give 4 in the *Quotient*; then 20 to 5 is as 4 to 1, and 1 to 4 is as 5 to 20.

Proof by 9 un-
certain.

Secondly, *Division* is commonly proved by casting away nines thus. Cast away 9 from the *Divisor* as oft as may be, and place the remain at one Angle of a Cross, and the remain, after 9 in like sort is cast from the *Quotient*, place at the opposite Angle, then multiply both these remains together, and to the amounting number add the remain of the *Division* if any, and if the total be under 9, place it at another Angle of the Cross, but if above 9, cast away 9 as before, and set down the residue. Lastly, cast away all the nines from the *Dividend*, and set this last remain opposite to the remain last before set down; which if the work be right will be equal thereto, otherwise not. As in the second Example before, the *Divisor* 30 leaving 3, and the *Quotient* 58 leaving 4 when the nines are rejected; these 3 and 4 multiplyed produce 12, which added to 2 and 2 the remain of the *Division* maketh 16, from whence 9 cast resteth 7, and so much will be left when the nines are cast from 1762 the *Dividend*.

7
3 X 4
7

2(2
176(2(58
33 0

This kind of Proof for the uncertainty before noted in *Addition* is laid by.

Thirdly, as *Addition* and *Subtraction*; So are *Multiplication* and *Division*, mutually the most excellent and infallible proofs of each other.

Multiplication is proved by *Division* thus. Divide the *Product* by the *Multiplier*, and the *Quotient* will return the *Multiplicand*; or contrarily the *Product* divided by the *Multiplicand*, the *Quotient* shall return the *Multiplier* when the work is duely wrought. As for instance in the Example of the 16 Section of the precedent Chapter where the *Product* is 3614958, the *Multiplier* 423, the *Multiplicand* 8546.

2

195

24042

3614958

423333

4222

44

Quotient.
Multiplier.

8546
Multiplicand.

256

19653

3614958

854666

6344

85

Quotient.
Multiplier.

423
Multiplier.

Division on the contrary is proved by *Multiplication* thus. Multiply the *Quotient* by the *Divisor*, and to the *Multiplies* add the Remain of the *Division*, if any be, the total *Product* shall be equal to the *Dividend*, if the *Division* be right. As for instance in the last Example of the 16 Section of this Chapter where the *Dividend* is 144980099, the *Divisor* 1798, the *Quotient* 80634, and the *Remain* 167.

Multiplicand.
Multiplier.

80634
1798

Quotient.
Divisor.

645072

7257067

5644386

806341

} Remain added

Product.

144980099

Dividend.

From hence flow these two Consecutaries. First the *Multiplicand* may be known if the *Multiplier*, and *Product* be given; or the *Multiplier* if the *Multiplicand* and *Product*: By dividing the *Product* by the other given Number. As above in the Proof of *Multiplication*.

Secondly, If the *Quotient*, and *Divisor* be given, the *Dividend* may be known by *Multiplication* of the one given number into the other; as above in the proof of *Division*.

And if such numbers were quesited, the questions may in like manner be resolved.

Examples in Questions of Multiplication and Division.

1. What number was that which multiplied by 15, produced 1380?
Answer, 92. for 1380 divided by 15 giveth 92 in the Quotient.
2. What number being divided by 15 will give in the Quotient 92?
Answer, 1380. for 92 multiplied by 15 will produce 1380.

Questions in
Multiplication
and Division.

Operations.

3

1380

155

1

92
15
460
92
1380

Quotient.
Product.

Particular Proof of *Division* is peculiar to some particular methods of Operation; and will not serve for all sorts of *Division*. And so the *Divisions* wrought according to the 16, 17, and 18. Sections of this Chapter may be proved by *Addition*; for the several *Multiplies* added together with the Remain of the *Division* if any be, will infallibly return the *Dividend*; and all the select wayes of *Division* may be tryed by the Common way; as in the Sections before may be seen, and therefore needles here to be repeated, only for a Close to this Chapter, and also to this first part of this Book, concerning *Integers*, take these few Observations and Theorems following.

1. If

Observations
and Theorems.

1.

1. If a Quantity, either Magnitude or Number, be made of other two Quantities by their Multiplication, the one of them being the *Factor*, will measure (that is, evenly divide) the same Quantity made by the other, (being the other *Factor*.) As 20 made of 4 into 5, the one of them will measure 20 by the other 4, and so will 4 measure the same 20 by 5. For the measure of 5 is 4 times in 20, and the measure of 4 is 5 times in 20.

2.

2. If a Quantity, either Magnitude or Number, be made of two other Quantities by their Multiplication, it is all one whether any other Number be divided by that one Quantity, or by those other two Quantities. As if 20 made of 4 and 5 as before, divide 60, the Quotient is 3, so is it all one if 60 be divided first by 4, and the Quotient thereof 15 by 5, for this last Quotient will be 3.

3.

3. *Fractions* arise from *Division*, when the *Dividend* is lesser than the *Divisor*, or cannot be evenly measured thereby, a *Fraction* will remain. As several Examples of *Division*, pag. 31. and elsewhere demonstrate.

4.

4. When the *Quotient* and *Divisor* are equal, and nothing remain on the *Division*, the *Dividend* is a square number, and the *Divisor* the Root thereof. As in the instance, Sect. 14. of this Chapter 99980001 is a Square Number, and 9999 the Root, *De quibus in suo loco*.

Partis primæ Libri primi

F I N I S.

T H E

THE
SECOND PART
OF THE
FIRST BOOK.

CHAP. I.
Of FRACTIONS.

In *Integers* dispatched in the former part of this Book. The next sort of *Homogeneous Fractions* need to *Integers* to be examined are *Fractions*, sometime called *Fragments*, sometime *Parts*, sometime *Broken Numbers*. This shall be done generally and particularly. *dealt with.*

In general, it is requisite to know what they are, whence they come, and how to express them. *Fractions what they are.*

By what passed before Chap. 2. pag. 5, and 6, *Fractions* may be understood to be *Broken Numbers*, or broken parts of a *Whole Number*, extending infinitely under an Unit as *Integers* do above.

They arise from the Division of their *Integers*, as was observed in the last Chapter before, and in sum are expressed with the same Notes or Characters, as the *Integers* from whence they come. Those arising from *Abstract Integers* are first to be treated of; and the proper subject of this Part, where *Fractions* without any other note of distinction are to be taken for such. To proceed then, *Whence they arise.*

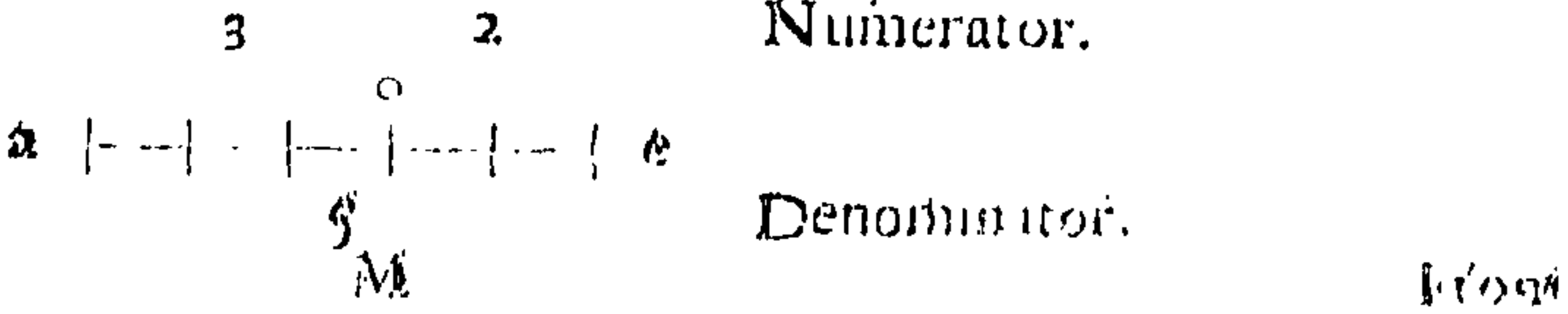
Fractions as they are broken numbers, so are they brokenly expressed, verbally, or in writing, by two termes or numbers, dissected as it were the one from the other, but *Integers* are uniforme. *Abstract Fractions first treated of.*

The two termes or numbers expressing a *Fraction*, are by some set one before or over the other, with a *Colon* between them. As Two thirds thus, 2 : 3, or thus $\frac{2}{3}$, but the most usual form is to set them one over another, with an interjacent line. As Two Thirds thus, $\frac{2}{3}$.

Of these two termes, the neathermost under the line, or to the right hand of the *Colon*, denoteth the Unit to be divided into so many equal divisible parts, and is called the *Denominator*. As if it be 2, it noteth the Unit is parted into halves, if 3 into thirds, if 4 into fourths, &c. *Broken Numbers brokenly expressed by 2. Terms.*

The uppermost, or foremost to the left hand, sheweth how many of those parts into which the Unit is broken, are signified to be contained in, or taken from the content or value of the *Fraction*, and this is called the *Numerator*, and sometime *Nominator*. So that the *Numerator* alway containeth the number of the parts signified by the *Fraction*; and the *Denominator* the greatness of those parts. *Which of them Numerator, and which Denominator.*

As in $\frac{3}{5}$, the *Numerator* is 3, the *Denominator* 5, and the *Fraction* signifieth three parts of any one *Integer* divided or broke into five equal parts, which content may be demonstrated by supposing the Unit or *Integer* to be the line a. e. equally divided into 5 parts, the Division made by o upon the line toward a. shall be equal to the *Fraction* $\frac{3}{5}$ of that line, and the residue of the line marked with o. e. shall be $\frac{2}{5}$, and make up the whole line, &c. *Example*



Inlarging the Denominator or Numerator what effect.

The Principle on which the effect depends.

Which Terms are Heterologal or Heterogeneal and the contrary.

Numerator is pronounced before the Denominator.

That by Cardinal, and this by Ordinal Numbers.

Fractions diversly divided.

From hence it appears. The more the *Denominator* is increased, the farther is the value of the *Fraction* from the whole Number or *Integer*, and the more the *Numerator* is augmented, the nearer is the value to the whole. Again the *Numerator* may be enlarged till the value be more than the whole, and contrarywise the *Denominator* may be diminished till the *Fraction* become an Unite or more; but the *Denominator* though never so great, yet will ever import a part of the *Integer*, though never so small. For every *Fraction* if the *Denominator* be increased is less, and if decreased is greater in quantity. All this dependeth on that fundamental principle. Such proportion as the *Numerator* beareth to the *Denominator*, the same beareth the parts signified by the *Fraction* to an Unite as in $\frac{3}{4}$ of a Shilling, it is evident that 3 to 4 is as $\frac{3}{4}$ to 1 shilling, or 12 pence, three quarters of which being 9 pence 9 to 12, shall in proportion be as 3 to 4.

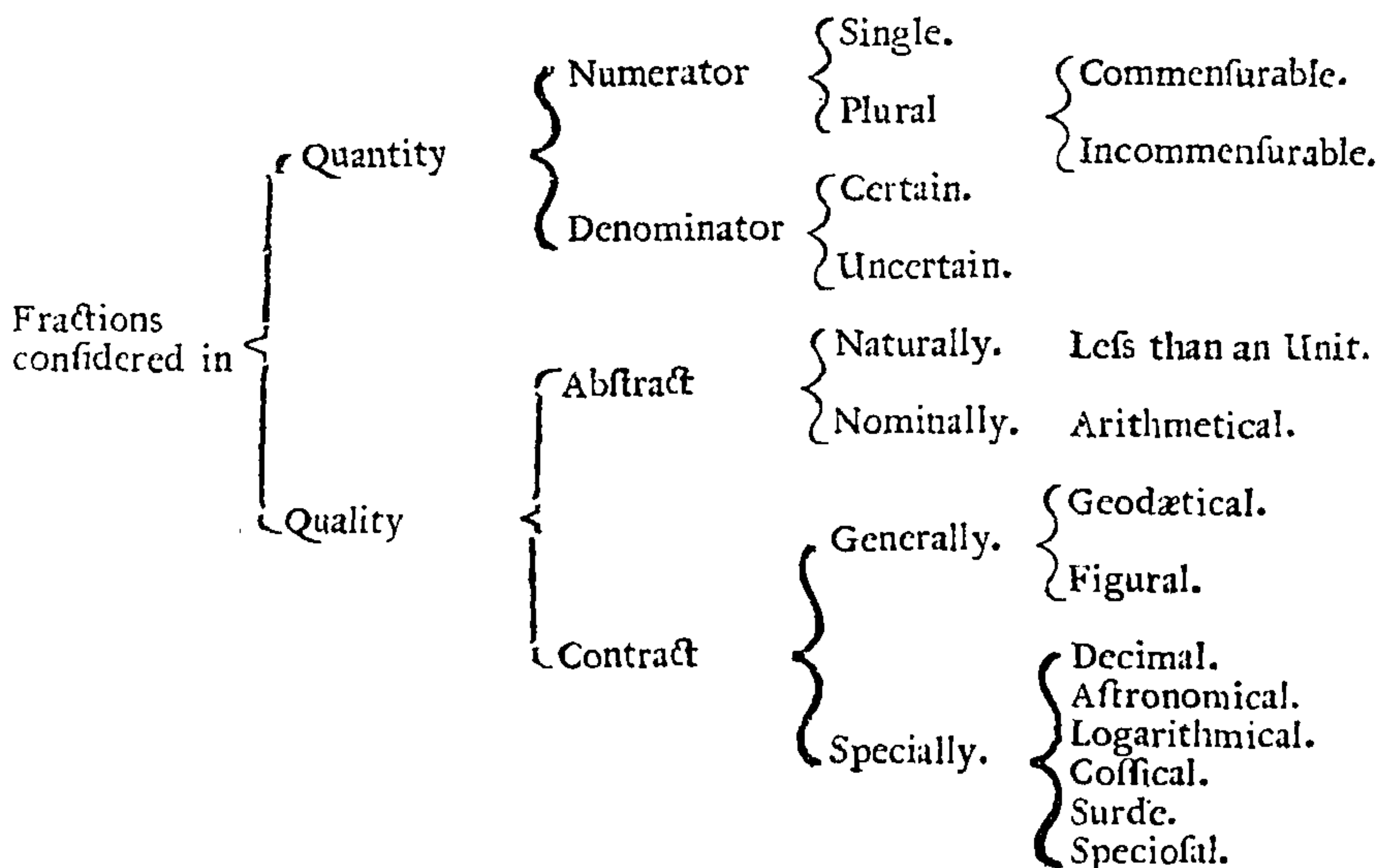
Numerators, and *Denominators*, compared one to another, are called *Heterologal*, and sometime *Heterogeneal* Terms; but either of them by themselves, are called *Homologal*, and sometime *Homogeneal*.

As in *Integers* the Numeration of their Quantities precedeth their *Denomination*, so in *Fractions*, the *Numerator* is alwayes pronounced before the *Denominator*.

The *Numerators* are verbally expressed by *Cardinal Numbers*, as One, Two, Three, Four, &c. The *Denominators* are best pronounced by the *Ordinals*, as halves, thirds, fourths or quarters, fifths, &c. As $\frac{1}{2}$, $\frac{3}{4}$, $\frac{6}{7}$, $\frac{1}{17}$, thus, one half, three quarters, six sevenths, seventeen twentieths, &c.

In particular, *Fractions* (as well as *Integers*) are capable of being diversly considered, in reference both to their quantity and quality; as they are single or plural, abstract or contract; generally or specially. A short narrative whereof followeth in the ensuing Table.

A Table of the Nature of Fractions.



Quantity of a Fraction how signified.

Numerator notes the content, if Single or Plural.

Plural are Commensurable or Incommensurable.

Common and Greatest Common Divisor what.

The Quantity of every *Fraction*, as before noted, being signified by his *Numerator*, and *Denominator*, both Terms are to be considered distinctly for the particular knowledge of the *Fraction*.

The *Numerator* denoting the content of the *Fraction*, shews whether it be a single, or a plural *Fraction*; that is, whether it contain only a part of the *Integer*. As $\frac{1}{2}$, $\frac{1}{4}$, &c. or many parts. As $\frac{3}{4}$, $\frac{1}{4}$, &c.

These Plural *Fractions* are again distinguished into Commensurable and Incommensurable.

Commensurable, called *Compositi inter se*, or Numbers compound among themselves, are such as besides an Unit have some Number that will divide exactly both the Terms without leaving any Remain. As $\frac{1}{5}$ is Commensurable, for both the Terms may be evenly divided by 5.

The Number so exactly dividing the Terms, is called the Common Divisor, or Measurer, and because some Terms have more Numbers than one that will thus divide them, the greatest of these Numbers is called the greatest Common Measure, or Divisor, and in Latine *Communis mensura maxima*, or *Communis Divisor maximus*. As $\frac{6}{12}$ may be exactly divided by 2, but the Greatest Common Measure is 6.

Incommensurable,

Incommensurable, called *Primi inter se*, or prime among themselves, are they that cannot be measured by any Number but an Unit, which not dividing properly, they have no number that will serve for a Common Divisor evenly to divide both the Terms without leaving some remaining number upon the Division of one of them. As $\frac{2}{3}$, where 2 may be exactly divided by no number but 3, and 3 cannot exactly divide 20, but 2 will remain. Such *Fractions* are said to be in their least Terms.

Incommensurable in their last Terms.

The Product of any Incommensurable Terms multiplied one into another, is called in Latine *Dividuus communis minimus*, and in English the least common Dividend, of which little use is made, but the Common Divisor is very useful, as may be further seen in the next Chapter of *Reduction*. Here by the way may be seen the Difference between Prime and Compound Integers and Prime and Compound Fractions. Uncompound Integers can be measured by no number of multitude exactly. As 3. 5. 7. 11. 13, &c. But *Fractions* may have either of the termes taken singly, or apart, a Compound Integer; yet may be *Primi inter se*, considered joynly. As $\frac{2}{3}$ is Incommensurable, though both 8 and 9 are Compound Integers.

Common Divisor is useful.

Difference between Integers and Fractions.

Fractions are again considered in respect to their Denominators, which are either Certain or Uncertain.

Denominator certain or uncertain.

The Denominators are certain in *Decimals*, *Astronomicals*, and *Logarithmes*, the first always increasing or decreasing by 10. The second by 60. The third according to the *Radii*, and are omitted; because certainly known.

Where certain, and omitted.

Geodaticals certain in regard their Denominators are known by their Denominations, but various according to such Denominations, and the Denominators of *Fractions* sometime used with them are uncertain.

Geodaticals how certain and uncertain.

All other Denominations besides these are uncertain, that is may be any absolute Number or Quantity whatsoever; and therefore of necessity must be expressed.

Where uncertain and expressed.

As *Fractions* are considerable in their quantity, so in their quality. As whether they are Abstract or Contract. Abstract are free from any restraint, but may be a part or parts of any quantity or magnitude. Contract on the contrary have some annexed Denomination, or special Denominator.

Quality of a Fraction to be Abstract or Contract.

Abstract absolute *Fractions*, have community in their nature with others of what sort soever that is to be inferior to an Unit be they single or plural in their Numerators; but they have a name proper to them for distinction sake to difference them from other sorts, that is *Arithmetical Fractions* some time called *Abstract*, *Vulgar*, or *Common Fractions*. These are such as before in this Chapter are instanced, and hereafter in this part at large handled.

Abstract what. The names of Abstract Fractions.

Contract Fractions, like *Integers*, are more generally contract in *Geodaticals* and *Figural Numbers*, and more especially in *Decimals*, *Astronomicals*, *Logarithmes*, *Cosicks*, *Surdes*, and *Species*. In the two first, and three last principally by reason of their Denominations: In the other three by their special Denominators, as well as in respect of their Denominations. The Nature and Operations of which were briefly before touched, Chap. 2. but are particularly to be unfolded in the several parts of the two next Books.

Contract Fractions General or Special.

Denominator and *Denomination*, though sometime used promiscuously, differ in respect of Quantity and Quality, the Denominator sheweth as before the greatness or smallness of the parts the Integer is understood to be divided into, the *Denomination* declareth the Nature of the Integer, of what kind or quality it was, whither Pounds, Ounces, Yards, Ells, Men, Moneths, &c.

How Denominator and Denomination differ.

The *Denomination* is sometime called the Surname, this annexed makes any abstract Fraction contract, and thereby makes the number seem to be doubly denominate, viz. As to the *Denominator* and *Denomination*, therefore are *Fractions* accounted before less absolute than *Integers*.

Surname of a Fraction what.

Arithmetical, or *Vulgar Fractions*, in this part dealt with as they arise from *Integers*; so in some things are like them, but in others different. As a greater Integer cannot be subtracted from, nor divide a lesser: But a lesser Fraction may be divided by a greater, though in *Substraction* they agree, that the *Subtrahend* shall be the lesser number. They agree to be composed, and dissolved, but their Operations therein are different, and the most easie in the one, are most difficult in the other, though in the prime and compound parts of Original Numeration they take part with *Integers*, to be added, subtracted, multiplied, and divided, yet because of their *Denominators*, they resemble the mode of Contract Numbers when their *Denominators* are unlike, and will first be reduced before they can be added, or subtracted, and so compared to *Integers* in their Simple Elements go in a Retrograde motion, placing Ortive Numeration before Original. They mix well enough with *Integers*, and are as hand-maids to them, whence arise the first sort of *Heterogeneous Numbers*, who in this part with them receive their Operations and Resolution.

How Abstract Fractions agree and differ with Integers.

Whence Improper Fractions come. Resolutions. In Fine from the mixture of *Integers* and *Fractions*; issue *Improper Fractions* in reference to which others are called *Proper Fractions*, between both which by increasing the Numerators of *Proper Fractions*, or the Denominators of *Improper Fractions*, *Equal Fractions* spring forth, and are all easily known thus.

Proper Fraction what. *Proper Fractions* always have the Numerator less than the Denominator, for then the parts signified are less than an *Unit* or *Integer*, though the Terms be never so great. As $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, &c.

Equal Fraction what. *Equal Fractions* have the Numerator, and Denominator always equal, and then the *Fraction* is equal to an *Unit*. As $\frac{1}{1}$, $\frac{2}{2}$, $\frac{3}{3}$, &c. and it were better to express such by 1 seeing they are but 1. These are improperly called *Fractions*, because they contain one *Integer*, but are thus represented for convenience in work.

Improper Fraction what. *Improper Fractions* have always the Numerator greater than the Denominator, for then is the *Fraction* greater than the whole. As $\frac{3}{2}$, $\frac{4}{3}$, $\frac{5}{4}$, &c.

Proper Fraction Conjunct. *Proper Fractions* are of two kinds, *Conjunct*, and *Divided*. *Conjunct*, sometimes called *Fractions of Integers*, are such as are conjoined together by the Copulative [*and*]. As $\frac{1}{2}$ and $\frac{3}{4}$, &c. implying more broken parts than one.

Divided. These are Fractions of Fractions. *Disjunct* or *divided*, are frequently called *Fraction of Fractions* differenced from the other by the intervening [*of*] instead of [*and*]. As $\frac{1}{2}$ of $\frac{3}{4}$, &c. denoting only a part of a *Fraction*.

Improper Fractions of 2 sorts. *Improper Fractions* also are of two sorts, *Viz.* either they include several *Integers*. As $2\frac{1}{2}$, $3\frac{2}{3}$, &c. or else one or more *Integers* with some part or parts of the *Integer*. As $5\frac{1}{2}$, $6\frac{3}{4}$, &c.

Elements of Fractions. The Simple Elements of *Fractions* in sum may be concluded under two heads; either to increase or decrease, their Terms or their Value, these as more essential are comprehended under *Original Numeration*. Those as accidental under that part of *Ortive Numeration*, called *Reduction*.

CHAP. II.

Reduction of Fractions.

Reduction what and how useful. **R**eduction in general, is that part of *Ortive Numeration*, whereby one Number, or Magnitude is reduced to another. Useful in *Fractions* and *Contract Numbers*, that they may receive a more apt form of Operation, in those to bring them from one Term to another; in these from one Denomination to another, yet in both retaining the same value or content.

What it consists in. *Reduction* consisteth principally in *Multiplication*, and *Division*, yet occasionally converseth with *Addition* and *Subtraction*, as necessity requireth.

The work thereof to be seen in the Sections following. *Reduction* so far as concerns *Arithmetical Fractions* now in hand will be understood in two things, *Operation* and *Probation*.
Operation may be thus methodized.

Reduction of	Proper Fractions	{	To their least Termes	{ by the great Common Measure.	§. 1.	
				{ by Bipartition, Tripartition, &c.	§. 2.	
		{	To like Denominators.	Conjunct by Multiplication.	§. 3.	
				Disjunct by Multiplication.	§. 4.	
	Improper Fractions	{	To Integers, or mixt Numbers, by Division.		§. 5.	
			From mixt Numbers, by Multiplication and Addition.		§. 6.	
	Integers.	{	To the form of an Improper Fraction, by an Unite.		§. 7.	
			To any desired Denominator, by Multiplication.		§. 8.	
	Proper or Improper Fractions to any desired Denominator by Multiplication and Division.					§. 9.
	Integers with Proper, and Improper Fractions.					§. 10.

§. Abbreviation by the Common Divisor. The first sort of *Reduction* belonging to *Proper Fractions* is to reduce them to their least termes. This is commonly called *Abbreviation*, because it doth abbreviate or cut short the terms of a *Fraction*, and so consequently the work therewith. And because by *Euclide*, lib. 7. Prop. 17. if one Number multiply two Numbers their Products will retain the same proportion

proportion either to other the Numbers did before *Multiplication*; of necessity if one Number divide two Numbers, the Quotients must be still proportional. And hence it is evident that the termes of the same *Fraction* may be infinite, and yet bear the same proportion one to another. As $\frac{4}{3}$ may be expressed by $\frac{8}{6}$, $\frac{12}{9}$, $\frac{16}{12}$, &c. multiplying both terms by 2, and by $\frac{2}{1}$, $\frac{3}{2}$, &c. multiplying by 3, every of which *Fractions* is still in value, but $\frac{4}{3}$. And as in denominate numbers it is not proper to speak of 100 pence, 500 shillings, or such like, when there are other Denominations to abbreviate the numbers. So neither in *Fractions* is it proper to mention any termes save the least that they may be reduced into. Wherefore then upon the whole matter, because Abbreviation is but brief Division, it appears that if the Greatest, or Commensurable termes of any *Fraction* be divided by any number that will equally measure or divide them without leaving any remain, then shall the *Fraction* be expressed in lesser terms by help of this Common Divisor; that is to say, by dividing both *Numerator* and *Denominator*, thereby observing to let the Numerator of the new Fraction, be the Quotient of the old Numerators Division: And the Denominator his Quotient likewise.

Most proper to set Fractions in their least Termes.

Commensurable terms sometimes admit several Common Divisors, among which some one is the greatest. As in the foregoing Chapter was observed; wherefore it is necessary to know how to find this great Common Divisor, because thereby the *Reduction* is perfected at once, when the lesser Common Divisors repeat their operations several times.

How to find the Greatest Common Divisor.

The Greatest Common Measure of two Numbers, by *Euclide*, lib. 7. prop. 2. is found by *Subtraction*, but the far better way is by continual *Division* of the greater by the lesser, and of the Divisor by the Remainder till either a Cypher or an Unite remain. For that Divisor which first divideth the Dividend without leaving any remain, is the Greatest Common Measure of both the numbers, and both the numbers thereby are found to be Commensurable or Compound among themselves. But if no such Divisor be found till an Unit remain, the numbers are Incommensurable, and will not be reduced to lesser terms.

Example. If $\frac{4899}{5888}$ were the Fraction given, and it were desired to know if the Numbers were Commensurable, and if so, what is the greatest Common measure, and consequently their least terms. Then dividing 5888 by 4899, and 4899 by 989 the remain of the first Division, and again 989 by 943 the second remain, and so continuing this manner of *Division* 23 is found to be the first Divisor, that leaveth 0 remaining on the *Division*, wherefore the Fraction is Commensurable, and the Great common Divisor is 23. by which both terms divided, the new Fraction is found to be $\frac{213}{256}$ which are the least terms of $\frac{4899}{5888}$ and numbers Incommensurable. For dividing $\frac{213}{256}$ as before, no Divisor will be found evenly to divide them till an Unit remain which as aforesaid neither multiplyeth nor divideth.

Example.

4899

5888

(989

5888(1

4899

(943

4899(4

989

(46

989(1

943

(2

943(20

466

4

(0

46(2

23

26

4899(213 Numerator.

2333

22

213

256

(43

256(1

213

(41

213(4

43

(2

43(1

41

4(1

2(20

2

1

2

2(2

1

x

x23

5888(256 Denom.

2333

22

By frequent practise some Common Divisor or other if not the greatest may quickly be espied. To spare therefore such multifarious *Division* in getting the Greatest Common Divisor. It is common to use *Bipartition*, *Tripartition*, or any such select *Division* by the Digits, as in the Chapter of *Division* before was set forth, if the given terms of the Fraction will exactly be divided by any of them; placing the Numerators along upon a line, and the Denominators beneath, and separating the several Quotients in the work, with a down right line or dash of the Pen. As if $\frac{288}{576}$ were to be reduced into its least termes; first mediating, or taking half the Numerator and Denominator, As below till $\frac{1}{2}$, be brought forth, and then the third part of both numbers accordingly. At last the Fraction is reduced to $\frac{1}{2}$ thus.

§. 2. How to abbreviate the Common way.

Example.

288

576

144

288

72

144

36

72

18

36

9

18

3

6

1

2

N

Also

Where Cyphers
are at the
right hand.

Also where Cyphers are in the terms of the Fraction to the right hand as before in *Division*, a like number of them from both Numerator and Denominator may be cut off by a dash of the Pen, and the residue left, or reduced lower if it may be. As $\frac{300}{400}$. thus $\frac{3}{4}$ will thereby appear to be but $\frac{3}{4}$. Also if from $\frac{200}{300}$ be cut off the two right hand Cyphers, the residue $\frac{2}{3}$ straight way is seen to be but $\frac{2}{3}$.

But if no such Common Divisor can be found among the Digits, then the great Common Measure, if any is to be sought, as before; and the Fraction reduced to its least terms thereby.

§. 3. To bring
Conjunct Fra-
ctions into one
Denominator.

The next sort of *Reduction* belonging to proper Fractions is to reduce them from Different to like Denominators, that they may be expressed in more Commodious numbers. For as in Integers, Units must be added to, or subtracted from Units, Tens to or from Tens, &c. And as in Denominate numbers Pounds must be added with, or taken from Pounds, Shillings, with or from Shillings, &c. So in Fractions, must halves be added to halves, thirds to thirds, &c. and subtracted accordingly, or else if the Denominators be unlike, both must be reduced to like Denominators before *Addition* or *Subtraction*.

Common Den-
ominator what.

The Reduction of different Denominators into like Denominators is sometime called Reduction to one Denomination, and the Denominators reduced being all one, the same need be set down but once to all the given Fractions, which Denominator so placed is called the Common Denominator.

This Reduction differs as the Fractions to be reduced are Conjunct, or Disjunct.

Proper Fractions conjunct are two, or more.

When only 2
Fractions are
given, and the
Denominator
Incommensura-
ble.

When two Fractions are given to be thus reduced, dispose them for form sake on either side of a Cross, and if the Denominators be Incommensurable multiply both the Denominators together, and that Product shall be the New or Common Denominator to both the given Fractions; Then for new Numerators multiply Crosswise the Numerator of the one Fraction by the Denominator of the other, and place the Product respectively overt that Fraction whose Numerator was in the Composition.

Example.

As to reduce $\frac{4}{5}$ and $\frac{2}{3}$ to one Denomination. They are set as at A. and then multiply 5 by 3, the Product 15 is the Common Denominator, set as at B. or C. also multiplying 4 by 3 the Product 12 is the Numerator to be set over 4, and multiplying 2 by 5 the Product 10 is the other Numerator to be set over 2, as at D. or E. So are the two reduced Fractions $\frac{12}{15}$ and $\frac{10}{15}$.

$$\begin{array}{ccccc} \text{A} & \frac{4}{5} & \frac{2}{3} & \text{B} & \frac{4}{5} & \frac{2}{3} & \text{C} & \frac{4}{5} & \frac{2}{3} & \text{D} & \frac{12}{5} & \frac{10}{3} & \text{E} & \frac{12}{5} & \frac{10}{3} \\ & \text{---} & \text{---} & & \text{---} & \text{---} & & \text{---} & \text{---} & & \text{---} & \text{---} & & \text{---} & \text{---} \\ & 5 & 3 & & 5 & 3 & & 5 & 3 & & 5 & 3 & & 5 & 3 \\ & & & & 15 & 15 & & 15 & 15 & & 15 & 15 & & 15 & 15 \end{array}$$

When the De-
nominators are
Commensurable.

If the Denominators of the given Fractions be Commensurable, then first reduce them to their least terms, and alternately place the least term of the one under the other Fraction, and thereby multiply the Numerators respectively for new Numerators, and either of the Denominators multiplied by the least terms under him shall be the Common Denominator,

Example.

As to reduce $\frac{16}{12}$ and $\frac{7}{12}$ to one Denominator 16 and 12, being Commensurable reduced by 4 their greatest Common Measure, make their least terms 4 and 3, which 3 coming of 12 placed under 16, and 4 coming of 16 under 12, the work appears as at F. then multiplying 13 by 3, and 7 by 4, the Numerators 39 and 28 are produced at G. and 16 by 3, or 12 by 4 the Common Denominator is found to be 48, and the work stands compleat as at H. where is found $\frac{13}{16} = \frac{39}{48}$ and $\frac{7}{12} = \frac{28}{48}$ being reduced.

$$\begin{array}{cccc} 4) \frac{16}{12} & \left(\frac{4}{3} \right. & \text{F} & 4) \frac{13}{16} \text{ and } \frac{7}{12} \\ & & & \begin{array}{cc} \dots & \dots \\ 3 & 4 \end{array} \\ & & & \text{---} \end{array} \quad \begin{array}{cccc} & 39 & & 28 \\ & \dots & & \dots \\ G & 4) \frac{13}{16} \text{ and } \frac{7}{12} & & \\ & \begin{array}{cc} \dots & \dots \\ 3 & 4 \end{array} & & \end{array} \quad \begin{array}{cccc} & 39 & & 28 \\ & \dots & & \dots \\ H & 4) \frac{13}{16} \text{ and } \frac{7}{12} & & \\ & \begin{array}{cc} \dots & \dots \\ 3 & 4 \end{array} & & \\ & & & 48 \end{array}$$

When

When more than two Conjunct Fractions are given to be reduced to like Denominators multiply all the Denominators together for the Common Denominator, and to find new Numerators, multiply each Fractions Numerator, into the Denominators of all the other Fractions except its own Denominator.

When more than 2 are given, and the Denominators Incommensurable.

As to reduce $\frac{2}{3}$ and $\frac{4}{5}$ and $\frac{6}{7}$ into like Denominators, multiplying 2 the Numerator of the first by 5 the Denominator of the second Fraction, and the Product 10 by 7 the Denominator of the third, the amounting Product 70 is the Numerator of the first Fraction. For the second Multiply 4 the Numerator of the second by 3 the Denominator of the first, and the Product 12 by 7 the Denominator of the third. So the Product 84 is the second Fractions Numerator. Then multiply 6 the Numerator of the third by 5 the Denominator of the second, and the Product 30 by 3 the Denominator of the first. So is 90 the Numerator of the third Fraction, unto whom the Common Denominator shall be 105. for $3 \times 5 = 15 \times 7 = 105$, and the three reduced Fractions set as at I. or K.

Example.

Denominators. Numerators.

$\frac{3}{5}$	$\frac{2}{5}$	$\frac{4}{3}$	$\frac{6}{5}$
$\frac{15}{7}$	$\frac{10}{7}$	$\frac{12}{7}$	$\frac{30}{3}$
$\frac{105}{70}$	$\frac{70}{84}$	$\frac{84}{90}$	

I

K

$$\frac{70}{105} \text{ and } \frac{84}{105} \text{ and } \frac{90}{105}$$

Some deliver the Rule thus. Multiply all the Denominators together for a Common Denominator, and divide that Common Denominator by each several Denominator, and multiply the several Quotients by their respect Numerators. As in the last Example.

Variety of work.

$\frac{3}{5}$	$\frac{105(35}{2}$	$\frac{105(21}{4}$	$\frac{105(15}{6}$
$\frac{15}{7}$	$\frac{70}{84}$	$\frac{90}{90}$	
Common Denominator.	New Numerators.		

Either of these wayes will serve if the Denominators be Incommensurable, but if the Denominators, or any of them be Commensurable, the Fractions thus reduced will not be in their least terms. Therefore to reduce Conjunct Fractions of unlike Commensurable Denominators to one Common Denominator, and yet keep the Fractions in the same value, and least terms, do thus. Reduce all the Denominators to their least termes, if they be all Commensurable, and by these least terms get a Common Denominator, and thereby new Numerators, as in the Operation last before.

When the Denominators are Commensurable.

Example.

reduce $\frac{3}{4}$ and $\frac{5}{6}$ and $\frac{7}{8}$. Thus the Denominators abbreviated to their least terms are 2. 3. 4. the Common Denominator gotten thereby is 24, and the several Numerators 18. 20. 21. As by the Operation appears.

$\frac{3}{4}$	$\frac{5}{6}$	$\frac{7}{8}$	$\frac{2}{3}$
$\frac{18}{24}$	$\frac{20}{24}$	$\frac{21}{24}$	$\frac{16}{24}$
2.	3.	4.	24

When all the Denominators are not Commensurable but only some of them, then either first reduce those Commensurable to one Denominator, and after work with this reduced, and the other remaining Fraction or Fractions, or else reject those lesser Denominators which are even parts of the Greater Compound Denominator, and work with the other Denominators, and the greater Compounds.

When not all the Denominators are Commensurable.

As to reduce $\frac{3}{4}$ and $\frac{5}{8}$ and $\frac{2}{5}$ to like Denominators; because 4 is a part of 8 Example. it may be 10, 20, and only 8 and 5 multiplyed for the Common Denominator. Or

Or else first reducing $\frac{3}{4}$ and $\frac{5}{8}$ the new Fractions are $\frac{6}{8}$ and $\frac{5}{8}$ which reduced with $\frac{2}{5}$ make $\frac{30 \text{ and } 25 \text{ and } 16}{40}$, as in the Operations following.

$$\begin{array}{r}
 \begin{array}{c} \dots 6 \dots \\ 4 \overline{) \frac{3}{4}} \\ \dots 2 \dots \end{array} \quad \text{and} \quad \begin{array}{c} \dots 5 \dots \\ 8 \overline{) \frac{5}{8}} \\ \dots 1 \dots \end{array} \\
 \hline
 8
 \end{array}
 \qquad
 \begin{array}{c}
 40 \overline{) 5} \\
 \underline{8} \\
 30
 \end{array}
 \qquad
 \begin{array}{c}
 40 \overline{) 5} \\
 \underline{8} \\
 25
 \end{array}
 \qquad
 \begin{array}{c}
 40 \overline{) 8} \\
 \underline{5} \\
 16
 \end{array}
 \qquad
 \begin{array}{c}
 30 \text{ and } 25 \text{ and } 16 \\
 \hline
 \frac{3}{4} \text{ and } \frac{5}{8} \text{ and } \frac{2}{5} \\
 \hline
 40
 \end{array}$$

§. 4. Fractions of Fractions reduced. Example.

To reduce Proper Disjunct Fractions; Multiply all the Numerators one into another for a new Numerator, and in like manner all the Denominators together, and the new Denominator is produced. So $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{8}$ reduced will be $\frac{15}{64}$.

Numerators.	1		2	Denominators.
3		15	4	
3		$\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{8}$	8	
5			8	
15		64	64	

When some of the terms be Commensurable.

If any of the Alternate Heterologal Terms in the Fractions given to be reduced be Commensurable, the new Fraction gotten as above, will not be in its least terms, unless such Heterologal termes be first abbreviated to their lowest, and operation made there.

Example.

with as before. So $\frac{1}{4}$ of $\frac{2}{5}$ reduced without abbreviation will be $\frac{2}{20}$, but if the Heterologal terms 2 and 4 be first reduced to their least terms, and then Multiplication made of these new Homologal terms as before, the new Fraction will be $\frac{1}{10}$ in its least terms.

$$\begin{array}{c}
 2 \\
 \hline
 \frac{1}{4} \text{ of } \frac{2}{5} \\
 \hline
 20
 \end{array}
 \qquad
 \begin{array}{c}
 1 \\
 \hline
 2 \overline{) \frac{1}{4} \text{ of } \frac{2}{5}} \\
 \underline{4} \\
 \dots 2 \dots \\
 \hline
 10
 \end{array}$$

§. 5. Improper Fractions reduced to Integers.

The next kind of Reduction belongeth to Improper Fractions, and is double. First, To reduce Improper Fractions into Integers or mixt numbers. Divide the Numerator by the Denominator, and if any thing remain after Division adjoyn it to the Quotient in

Example.

form of a Fraction by setting the Remain over the old Denominator. As $\frac{15}{3}$ shall give 5 Integers, and $\frac{15}{4}$ the mixt number $3 \frac{3}{4}$.

Improper Fraction $\frac{15}{3}$ (5 Integers $\frac{3}{4}$ Mixt Numbers.

§. 6. Integers and Fractions reduced to Improper Fractions

This Reduction is oft-times needful at the end of Operation, that the Resolution of the Question in hand might be more plain.

Secondly on the contrary, when any mixt number is to be reduced into an Improper Fraction. Multiply the Integer by the Denominator of the Fraction, and to the Product add the Numerator of the Fraction, and this Product shall be the Numerator of the Improper

Example.

Fraction, the old Denominator shall serve still. As $13 \frac{3}{4}$ reduced into an Improper Fraction shall be $\frac{55}{4}$. For 13 multiplied by 4 produceth 52, to which 3 added the Numerator is 55 to the Denominator 4.

This

Example.

Example:

Example:

$$\frac{2}{3} \text{ of } \frac{1}{1000}. \text{ Or by Reduction } \frac{1}{1500}.$$

$$\begin{array}{r} 60 \\ 3 \\ \hline 180 \end{array} \quad \begin{array}{r} 3 \\ 180 \\ \hline 5 \end{array} \left(\begin{array}{r} 36 \\ 60 \end{array} \right) \quad \begin{array}{r} 1000 \\ 2 \\ \hline 2000 \end{array} \quad \begin{array}{r} 22(2) \\ 2000 \\ 3 \end{array} \left(\begin{array}{r} 666 \\ 1000 \end{array} \right) \quad \text{and } \frac{2}{3} \text{ of } \frac{1}{1000}.$$

with $\frac{2}{3}$ of $\frac{1}{5}$ be given to be reduced with 2 Integers, and $\frac{7}{2}$ the Improper Fraction, to

one Denomination. After $\frac{2}{3}$ of $\frac{1}{5}$ are reduced to one single Fraction, and an Unit *Example.*

placed under the 2 Integers, the Common Denominator is found out as before to be 210, and the several Numerators 30. 28. 420. 735.

$$\frac{30}{7} \text{ with } \frac{28}{2} \text{ of } \frac{1}{5} \text{ and } \frac{2}{1} \text{ and } \frac{7}{2}$$

<u>30</u>	<u>28</u> 2	<u>420</u>	<u>735</u>	<u>7</u> 15	<u>210(30)</u> 7 1	<u>210(14)</u> 15 2	<u>210(210)</u> 1 2	<u>210(105)</u> 2 7
				105	—	—	—	—
				2	30	28	420	735
<hr/>				<u>210</u>				

210

This Proof of one kind of Reduction is alternately by another ; because the *Fractions* are to keep the same equality in value, how ever differently they are expressed in greater or lesser terms ; Whence it is that Fractions duely reduced to one Denominator, may be returned to their least terms again by Abbreviation, and Fractions abbreviated

viated to their least terms may be converted to their former greater terms; that by *Division* with the great Common Measure, and this by *Multiplication* therewith. As in the last Example $\frac{1}{7}$ was reduced with others to $\frac{30}{210}$ which as *termini convertibiles*, may by 30 the great common measure or any other common *Divisor* be reduced from the one to the other. For $\frac{1}{7} = \frac{30}{210}$.

$$30) \frac{30(1}{210(7}$$

$$\frac{30 | 1}{210 | 7}$$

Improper.

Likewise Reduction of Improper Fractions reciprocally prove one sort the other: As *Multiplication*, and *Division* in Integers mutually do, they being performed thereby, as above may be seen. For $3\frac{3}{4}$ is but the Quotient of $\frac{15}{4}$, and $\frac{15}{4}$ the Product of $3 \times 4 + 3$.

$$\text{Wherefore } 3\frac{3}{4} = \frac{15}{4}.$$

Integers set like Fractions by subjecting 1. As in the 7 *Seet.* above are soon reverted into their old form by taking away the subjected Unite. Integers also reduced to given Denominators are returned back by *Division* of the Numerators by the Denominators, like Improper Fractions: And so $\frac{40}{4}$ shall return 10.

In like manner, *Fractions* reduced to a given Denominator may be abbreviated by Common *Divisors* till the first terms be returned. In the Instance above $\frac{3}{5}$ made $\frac{36}{60}$ therefore by the great Common measure 12 shall $\frac{3}{5}$ be returned.

But if a *Fraction* remained; as in turning $\frac{2}{5}$ into thousandths, the Proof is somewhat more difficult than the work because of the divided *Fraction*. Yet neither such nor *Fractions* of *Fractions* are destitute of trial. For in those if the Fragment be added to the Quotient, and reduced to their least terms, the former given *Fraction* will be returned. And in *Fractions* of *Fractions* if the *Fraction* of one Denomination be divided by either of the parts multiplyed, the other will be returned in the Quotient, because *Multiplication* and *Division* prove each other. And besides, in *Contract Fractions* another kind of proof may be had; by finding the value denominate. As in the next Book of *Geodetics* may be seen. But forasmuch as *Addition* and *Division* of *Fractions* are not yet taught, and both these *Reductions* are performed by *Multiplication* or *Division*, or both, as if they were *Integers*: It may satisfy as to the truth of these *Reductions*, if the *Multiplications*, and *Divisions* be found right.

Proof of the Reduction of Fractions of Fractions, See pag. 53. Of Contract Fractions, See Geodetics.

C H A P. III.

Addition of Fractions.

Proper Fractions are increased by Addition, and Division diminished by Subtraction & Multiplication. Improper, how they increase, or decrease. Mixt, how they increase, or decrease.

HOW to increase or decrease the terms of a *Fraction* hath been seen in *Reduction*; it remains now to see how to increase or decrease their value.

Proper Fractions like to *Integers* have their value increased by *Addition* and diminished by *Subtraction*.

But contrary to *Integers* are lessened by *Multiplication*, and increased by *Division*.

Improper Fractions by *Addition* increase, and *Subtraction* decrease their value according to *Integers*, and *Proper Fractions*. In *Multiplication* they are redundant by their value, and increase as *Integers*; defective by their *Fractions*, and decrease as *Fractions*. But in *Division* are altogether like *Integers*.

Proper Fractions mixt with *Improper*, or *Integers*; are augmented or diminished in *Addition* or *Subtraction*, as before. In *Multiplication* they follow the manner of

Improper

Improper Fractions; in *Division* if the *Fraction* be the *Dividend* the *Quotient* is decreased, if *Divisor* the contrary.

Addition of Fractions, and mixt Numbers contains Operation and Probation.

In Operation are four Cases. The two first general, the two last special.

1. Case. When the Denominators are alike, add the Numerators together, as

Addition of Fractions included under 4 Cases.

Integers, and beneath the total subscribe the Common Denominator. As $\frac{2}{7}$ and $\frac{4}{7}$ ad-

1. *Denominators alike.*

ded together make $\frac{6}{7}$, And so $\frac{3}{20}$ and $\frac{13}{20}$ make $\frac{16}{20}$, and by Abbreviation $\frac{4}{5}$ by some

Examples.

set as at A. by others as at B.

$$A \quad \frac{2}{7} \quad \frac{4}{7} \quad \frac{6}{7}$$

$$B \quad \frac{2}{7} + \frac{4}{7} = \frac{6}{7}$$

$$A \quad \frac{3}{20} \quad \frac{13}{20} \quad \frac{16}{20}$$

$$B \quad \frac{3}{20} + \frac{13}{20} = \frac{16}{20} \text{ or } \frac{4}{5}$$

2. Case. When the Denominators are unlike, first reduce them to one Denomination, then as before add the Numerators, and under the total subscribe the Common De-

2. *Denominators unlike.*

nominator. As $\frac{3}{4}$ and $\frac{4}{5}$ reduced make $\frac{15 \text{ and } 16}{20}$ then added make $\frac{31}{20}$ the Improper

Examples.

Fraction, or $1 \frac{11}{20}$ set as at C. or D. So $\frac{2}{3} \& \frac{3}{4} \& \frac{2}{3}$ first reduced make $\frac{18 \& 9 \& 18}{12}$,

then added are $2 \frac{11}{12}$. As at E. or F.

$$C \quad \frac{31}{20} \quad \frac{31}{20} \quad \frac{31}{20} \quad D \quad \frac{3}{5} \quad \frac{4}{4} \quad \frac{31}{20} \text{ or } 1 \frac{11}{20}$$

$$E \quad \frac{8}{3} \quad \frac{9}{4} \quad \frac{18}{2} \quad \frac{35}{12} \text{ or } 2 \frac{11}{12}$$

3. Case. If Integers or mixt Numbers are to be added with Fractions, either add the Integers after the manner of Integers, and Fractions after the manner of Fractions se-

3. *Integers or mixt Numbers with Fractions.*

verally by themselves; or else reduce the mixt numbers into Improper Fractions, and then proceed as above. As if $3 \frac{1}{3}$ and $6 \frac{1}{4}$ were to be added with $\frac{2}{5}$ either 3 and 6

Examples.

the Integers added make 9, and the Fractions $\frac{1}{3} \frac{1}{4} \frac{2}{5}$ make $\frac{59}{60}$ or reducing the mixt

numbers $3 \frac{1}{3}$ and $6 \frac{1}{4}$ into Improper Fractions, and then proceeding the same total is

at last resulting to $9 \frac{59}{60}$ as at G. or H.

$$G \quad \frac{3}{6} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{2}{5} \quad \frac{59}{60}$$

$$H \quad \frac{200}{3} \quad \frac{375}{4} \quad \frac{24}{5} \quad \frac{599}{60} \text{ or } 9 \frac{59}{60}$$

4. Case. If Integers, mixt Numbers, Fractions, and Fractions of Fractions, are to be added together, either they may first be severally added, and afterwards their se-

4. *Integers and mixt Numbers with Fractions.*

veral Totals into one total; or else reduced to one Denomination, and then added.

Example. To add 2 Integers, $3 \frac{1}{5}$ a mixt Number, $\frac{1}{3}$ a Fraction, and $\frac{1}{2}$ of $\frac{1}{4}$ a Fra-

Examples.

tion of a Fraction. As at I. or K.

$$I \quad \frac{2}{3} \quad \frac{1}{5} \quad \frac{1}{3} \quad \frac{1}{2} \text{ of } \frac{1}{4} \quad \frac{79}{120}$$

$$K \quad \frac{2}{3} \quad 3 \frac{1}{5} \quad \frac{1}{3} \quad \frac{1}{2} \text{ of } \frac{1}{4} \quad \frac{679}{120}$$

Proof of Addition of Fractions.

Addition of Fractions like absolute Integers is proved by *Subtraction* and reserved to the next Chapter. But *Addition* being learned: The Proof of *Reduction* in the 9. Sect. of the last Chapter may be remembered, and exemplified, whereafter the first *Division* remained a Fraction of a Fraction, in reducing of $\frac{2}{3}$ to 1000^{ths} there was $\frac{666}{1000}$ and $\frac{2}{3000}$ both which added by the second case above will return $\frac{2}{3}$. As before noted, and prove the *Reduction* right.

$$\begin{array}{r} 2 \overline{) 1000} \\ 1998 \quad \dots 2 \\ \hline 1000 \overline{) \frac{666}{1000}} + \frac{2}{3000} = \frac{2}{3} \\ \hline 3 \quad \quad \quad 1 \\ \hline 3 \overline{) 1000} \end{array}$$

CHAP. IV.

Subtraction of Fractions.

To find which of 2 Fractions given is the greatest.

Examples.

BECAUSE a greater Fraction cannot be taken out of a lesser, it is convenient first to know how to find which of any two propounded Fractions is the Greater, that so the Propositions may not be impossible. Which to do, Multiply the Numerator of the one into the Denominator of the other, and the Product which is the greatest shall demonstrate that Fraction biggest, whose Numerator was one of the Factors. As to know which is the biggest Fraction of $\frac{3}{4}$ or $\frac{4}{5}$ multiplying 3 by 5, the Product is 15, and 4, by 4 yieldeth 16, which sheweth $\frac{4}{5}$ to be greater than $\frac{3}{4}$ and import a bigger part of the Integer. So likewise $\frac{2}{3}$ is more than $\frac{3}{5}$, because 10 exceeds 9, and in like manner Equal Fractions may be found, as $\frac{2}{5}$ and $\frac{4}{10}$.

$$\text{Minor } \frac{15}{4} \times \frac{16}{5} \text{ Major. } \frac{10}{2} \times \frac{9}{3} \text{ Minor both } \frac{20}{2} \times \frac{20}{10} \text{ Equal.}$$

Subtraction of Fractions included under 8. Cases.

1. Denominators alike. Examples.

Subtraction of Fractions, and mixt Numbers, includes Operation, and Probation. In Operation may happen Eight Cases the two first general, the six last particular.
1. Case. When the Denominators are alike, then subtract the lesser Numerator from the greater, as in Integers, and place the remain over the Common Denominator. As $\frac{2}{5}$ from $\frac{3}{5}$ leaves $\frac{1}{5}$. So $\frac{3}{8}$ out of $\frac{7}{8}$ there resteth $\frac{4}{8}$, and by Abbreviation $\frac{1}{2}$ set as at A by some, as at B by others.

$$\begin{array}{cc} \text{A} & \text{B} \\ \frac{2}{5} \times \frac{3}{5} & \frac{3}{5} - \frac{2}{5} = \frac{1}{5} \end{array} \quad \begin{array}{cc} \text{A} & \text{B} \\ \frac{3}{8} \times \frac{7}{8} & \frac{7}{8} - \frac{3}{8} = \frac{4}{8} \text{ or } \frac{1}{2} \end{array}$$

2. Denominators unlike. Examples.

2. Case. When the Denominators are unlike, first reduce them to one Denomination; then abate the lesser Numerator out of the greater, and under the Remain subscribe the Common Denominator. As in subtracting $\frac{5}{7}$ from $\frac{7}{9}$ being reduced they are $\frac{45}{63}$ then

then $\frac{45}{63}$ subtracted from $\frac{49}{63}$ leaves $\frac{4}{63}$ set as at C. or D. So $\frac{13}{20}$ out of $\frac{4}{5}$ leaveth remaining $\frac{3}{20}$. As at E. or F.

$$\begin{array}{l} \text{C} \quad \frac{4}{\frac{45}{7} \frac{49}{9}} = \frac{4}{63} \\ \text{D} \quad \frac{7}{9} - \frac{5}{7} = \frac{4}{63} \\ \text{E} \quad 5) \frac{4}{5} \frac{13}{20} = \frac{3}{20} \\ \text{F} \quad \frac{4}{5} - \frac{13}{20} = \frac{3}{20} \end{array}$$

3. Case. If mixt Numbers consisting of Integers and Fractions are given, or happen in the work, then first reduce them into Improper Fractions; and afterwards proceed as before. So $1 \frac{5}{12}$ taken from $1 \frac{13}{16}$ shall leave $\frac{19}{48}$ as at G. and $3 \frac{1}{4}$ from $7 \frac{3}{4}$ leaves $4 \frac{1}{2}$ as at H.

$$\begin{array}{l} \text{G} \quad 4) 1 \frac{13}{16} - 1 \frac{5}{12} = \frac{19}{48} \\ \text{H} \quad 7 \frac{3}{4} - 3 \frac{1}{4} = \frac{18}{4} \text{ or } 4 \frac{1}{2} \end{array}$$

4. Case. If many Fractions are to be subtracted from one, or one from many, then first add them that are to be subtracted together, if more than one, into one total, and likewise those Subtraction is to be made from, and afterwards subtract the total of the Subtrahend from the other total, as before. As to take $\frac{7}{8}$ and $\frac{9}{16}$ from $\frac{3}{4}$ and $\frac{5}{6}$, first $\frac{7}{8}$ and $\frac{9}{16}$ reduced and added, their Total is $\frac{23}{16}$, then $\frac{3}{4}$ and $\frac{5}{6}$ reduced and added, their Total is $\frac{19}{12}$. Lastly, $\frac{23}{16}$ subtracted from $\frac{19}{12}$, there remaineth $\frac{7}{48}$, the Operations appear at I. K. L.

$$\begin{array}{l} \text{I} \quad 8) \frac{7}{8} + \frac{9}{16} = \frac{23}{16} \\ \text{K} \quad 2) \frac{3}{4} + \frac{5}{6} = \frac{19}{12} \\ \text{L} \quad 4) \frac{19}{12} - \frac{23}{16} = \frac{7}{48} \end{array}$$

5. Case. If in two mixt given numbers the lesser Fraction belong to the Subtrahend, then to work with the Integers severally after the manner of Integers, and with the Fractions by themselves after their manner, is the best way for brevity, because it saves many times Reduction and great Multiplications. As to abate $19 \frac{1}{5}$ from $48 \frac{3}{5}$, the Integers 19 from 48 leave 29, and $\frac{1}{5}$ from $\frac{3}{5}$ leaves $\frac{2}{5}$. So is the whole Remain $29 \frac{2}{5}$ as at M. Also $40 \frac{1}{4}$ from $63 \frac{1}{2}$ leave $23 \frac{1}{4}$ for 40 withdrawn from 63 leaveth 23, and $\frac{1}{4}$ from $\frac{1}{2}$ leaveth $\frac{1}{4}$ as at N, which as many others sometime happen are so commonly known, that upon sight, without further work may be discerned, but if need be, Operation may be made for the Fraction as before; yet will the work be far shorter than if all the Numbers were turned into Improper Fractions.

$$\begin{array}{r}
 48 \frac{3}{5} \text{ Greater Homogeneous.} \\
 \hline
 19 \frac{1}{5} \text{ Subtrahend.} \\
 \hline
 M \quad 29 \frac{2}{5} \text{ Remain.}
 \end{array}$$

$$\begin{array}{r}
 63 \frac{1}{2} \\
 \hline
 40 \frac{3}{4} \\
 \hline
 N \quad 23 \frac{1}{4}
 \end{array}
 \quad
 \begin{array}{r}
 \frac{1}{2} \\
 \hline
 \frac{1}{4} \\
 \hline
 \frac{1}{4}
 \end{array}
 = \frac{1}{4}$$

5. *Next Numbers and the Greater Fraction be in the Subtrahend.*
Example.

6. Case. If the Fractions of the two given mixt Numbers be of one Denomination, and the Fraction belonging to the Subtrahend be greater than the other Fraction, then add the Denominator of the Number from which *Subtraction* is to be made to the Numerator, and abate an Unit from his Integer, and afterward make *Subtraction* as before. So if $3 \frac{5}{6}$ were to be subducted from $9 \frac{1}{6}$, because $\frac{5}{6}$ is the major Fraction,

6 is added to 1, and 1 taken from 9. So is the number thus altered $8 \frac{7}{6}$, from whence

$3 \frac{5}{6}$ substracted there is left $5 \frac{2}{6}$ or $5 \frac{1}{3}$. For $9 \frac{1}{6} = 8 \frac{7}{6} - 3 \frac{5}{6} = 5 \frac{2}{6}$, or $5 \frac{1}{3}$

7. *One Fraction, and that in the Subtrahend.*
Examples.

7. Case. If there be but one Proper Fraction in the two given Numbers, and that belong to the *Subtrahend*, then abate the Numerator from the Denominator, and subscribe under the remain the Denominator, and accompt the Integers in the Subtrahend

1 more, or those in the Greater number 1 less. As to take $\frac{2}{3}$ from 4 Integers; first 2 taken from 3 leaves 1 to be set over 3, then 1 from 4 leaveth the whole remain $3 \frac{1}{3}$

As at O. So if $6 \frac{3}{7}$ be abated from 9 Integers, 3 taken out of 7 leaves $\frac{4}{7}$, and 6 and 1 out of 9, or 6 out of 8, there remaineth $2 \frac{4}{7}$, as at P.

$$\begin{array}{r}
 4 \text{ Integers.} \\
 \hline
 0 \frac{2}{3} \text{ Subtrahend.} \\
 \hline
 O \quad 3 \frac{1}{3} \text{ Remain.}
 \end{array}$$

$$\begin{array}{r}
 9 \text{ Integers.} \\
 \hline
 6 \frac{3}{7} \text{ Subtrahend.} \\
 \hline
 P \quad 2 \frac{4}{7} \text{ Remain.}
 \end{array}$$

8. *Subtrahend an Integer.*
Example.

8. Case. If of the two given Numbers, the Subtrahend be Integral, then keep the Fraction intire to the Remain, and make Subtraction as in Integers. As to take 3 Integers from $5 \frac{1}{4}$, take 3 from 5, and the remain shall be $2 \frac{1}{4}$.

Proof of Subtraction of Fractions.

Probation of Subtraction and Addition, as in Integers, so in Fractions is reciprocal, the one by the other; wherefore if one of the Addends be substracted from the Total, the Additionary work will be proved by the Remain equal to the other Addends; so if the Subtrahend and Remain be added, the Subtractionary work will be proved, for the Sum shall amount to the greater Number, from which *Subtraction* was made, and as in *Subtraction of Integers*, if the Remain be subducted from the Greater Number, this Remain shall be the Subtrahend *vice-versa*. Examples in the Answer of these 2 Questions.

Questions in Addition and Subtraction of Fractions.

1. What Number is that from which if $\frac{1}{20}$ be substracted, the Remain will be $\frac{3}{4}$?

Answer, $\frac{4}{5}$, for so much is the Total of $\frac{1}{20}$ and $\frac{3}{4}$ added together.

2. What Number was that to which $\frac{1}{2}$ added the Total was $\frac{4}{5}$?

Answer, $\frac{3}{4}$, for such is the Remain after $\frac{1}{20}$ is substracted from $\frac{4}{5}$.

$$5) \frac{\frac{4}{5} - \frac{1}{20} = \frac{15}{20} \text{ or } \frac{3}{4}}$$

Multiplication of Fractions.

Examples.

$$\begin{array}{r} \text{G} \quad \begin{array}{r} 2 \\ \hline 2) \frac{4}{15} \times \frac{5}{6} = \frac{2}{9} \\ 5) \end{array} \end{array}$$

$$\text{H} \quad \begin{array}{r} 9 \\ \hline 2) \frac{3}{2} \times \frac{6}{5} = 1 \frac{4}{5} \\ 5) \end{array}$$

$$\text{I} \quad \begin{array}{r} 9 \\ \hline 5) \frac{5}{7} \times \frac{9}{5} = 1 \frac{2}{7} \\ 7) \end{array}$$

3.
Mixt Numbers
both, or one a
Fraction or
Integer.
Examples.

3. Case. If Integers or mixt Numbers with a Fraction, or both mixt Numbers be given to be multiplied, subject an Unit as in *Reduction* under the Integers, and reduce the mixt Numbers into Improper Fractions, and then proceed as above. So 3 by $2 \frac{2}{7}$ will produce $\frac{48}{7}$, as at K. and 4 by $\frac{2}{3}$ produce $\frac{8}{3}$, as at L. Also $4 \frac{1}{3}$ by $3 \frac{1}{2}$ produce $\frac{91}{6}$, as at M.

$$\text{K} \quad \begin{array}{r} 48 \\ \hline 16 \\ 3 \times 2 \frac{2}{7} = 6 \frac{6}{7} \\ 1 \end{array}$$

$$\text{L} \quad \begin{array}{r} 8 \\ \hline 4 \times \frac{2}{3} = 2 \frac{2}{3} \\ 1 \end{array}$$

$$\text{M} \quad \begin{array}{r} 91 \\ \hline 13 \times \frac{7}{6} = 15 \frac{1}{6} \\ 1 \end{array}$$

4.
Integer and
Fraction or
mixt Number.
Examples.

4. Case. When an Integer and a Single Fraction, or an Integer and a mixt Number whose Fraction is single, the Denominators of the Fractions being digits, be the two given Numbers; then the Integer may be made *Multiplicand*, and the other Number the *Multiplier*, which if a mixt Number, multiply the *Multiplicand* by the Integers of the *Multiplier*, and for the Fraction of the mixt Number take $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. as the Fraction is, of the *Multiplicand*, and add to the former Product, and the total product shall be the desired Number. But if the *Multiplier* be only a single Fraction, then either take the half, third part, fourth part, &c. of the Integer according to the given Fraction, or else duplicate, triplicate, quadruplicate, &c. the Fraction according to the given Integer, for the Product desired: For to multiply any Integer Fraction-wise by $\frac{1}{2}$ is Bipartition, by $\frac{1}{3}$ Tripartition, &c. And contrary-wise to multiply any Fraction by 2 is but to double the Numerator, or take half the Denominator; by 3 is to triple the Numerator, or take the third part of the Denominator, &c. And hence sometimes in like sort *Multiplication by Plural Fractions*, if their Denominators be Digits is made use of, as sooner accomplished then by *Reduction into Improper Fractions*.

Examples of all these varieties, at N. O. P. in the one 48 is multiplied by $9 \frac{1}{3}$ in the other by $\frac{1}{5}$ and in the third by $12 \frac{3}{4}$; so plainly they need no illustration.

$$\begin{array}{r} \text{N} \\ 48 \text{ Multiplicand.} \\ 9 \text{ Multiplier} \\ \hline 432 \text{ Product of the Integers.} \\ 16 \text{ Third Part added.} \\ \hline 448 \text{ Total Product.} \end{array}$$

$$\begin{array}{r} \text{O} \\ 48 \text{ Multiplicand.} \\ \frac{1}{5} \text{ Multiplier.} \\ \hline 9 \text{ Product.} \end{array}$$

$$\begin{array}{r} \text{P} \\ 48 \text{ Multiplicand.} \\ 12 \text{ Multiplier.} \\ \hline 96 \text{ Product of 12.} \\ 48 \text{ } \\ 36 \text{ } \text{ } \text{ of 48 added.} \\ \hline 612 \text{ Total Product} \end{array}$$

5.
Both mixt Numbers
with single Fractions,
&c.

5. Case. When two mixt Numbers whose Fractions are single, and their Denominators Digits, are given to be multiplied; after *Multiplication* by their Integers, and the the Fraction of the *Multiplier*, as last above-mentioned, then multiply the Numerator of the *Multiplicand Fraction* by the Integers in the *Multiplier*, and the Product divide by the Denominator of that Fraction; and add this Quotient to the numbers before set down with the Product of the two Fractions multiplied, the Sum of all these shall be the

the total Product. As to multiply $48 \frac{1}{2}$ by $12 \frac{3}{4}$, first 48 by 12 produce 576, then $\frac{3}{4}$ of 48 is 36, then 12 halves is 6 whole ones, or 12 multiplying 1, and dividing by 2 is all alike. Wherefore 6 added to the other, and $\frac{3}{8}$ the Product of $\frac{1}{2}$ into $\frac{3}{4}$ make together 618 $\frac{3}{8}$ for the Total Product; as at Q. examined by the Common way at R. and found alike.

Q

48
12

Multiplicand.
Multiplier.

96
36
6
3

Product of 12
 $\frac{3}{4}$ of 48.
12 halves.
 $\frac{1}{2}$ of $\frac{3}{4}$.

added

618 $\frac{3}{8}$

Total Product.

R

4947

97
51

48 $\frac{1}{2}$ x 12 $\frac{3}{4}$

8

97
485

4947

$48 \frac{1}{2} \times 12 \frac{3}{4} = 618 \frac{3}{8}$

16(3
4947(618
8

6. Case. When the Heterologal Terms either way are equal, cancel them, and let the other Terms stand as they are for the Product. But when they are equal both ways take 1, for the Product shall always be an Unit or equal Fraction. As in multiplying $\frac{1}{3}$ by $\frac{3}{4}$, the Product shall be $\frac{1}{4}$ cancelling 3, and 3. And in multiplying $\frac{3}{4}$ by $\frac{4}{3}$, the Product shall be 1, or 1.

S

1

3) 1 x 3 = 1

4

T

1

3) 3 x 4 = 1

4

Besides the particular Proof of the different *Multiplications* one by another in General; The Proof of *Multiplication* as in *Integers* so in *Fractions*; is by *Division*, and to be sought in the next Chapter.

C H A P. VI.
Division of Fractions.

Division of Fractions may be called their Composition as increasing both their terms and value if the Fractions be proper, and Homologal terms Incommensurable; but Improper and mixt Numbers partly increase and partly decrease their Quotients as before noted in *Addition of Fractions*. Chap. 3.
Fractions and mixt Numbers, are divided with much facility, both Operation and Proportion being performed by *Multiplication*; that after the manner of *Integers*; this of *Fractions*.
Operation concludeth with Six Cases, the three first Essential, and three last Accidental.
1. Case. If the two given Numbers be Proper or Improper Fractions, or the one Proper, and the other Improper, and the Homologal terms Incommensurable; then multiply

multiply as in *Integers* the Heterologal Terms, that is to say the Numerator of the Dividend by the Denominator of the Divisor, for the Numerator of the Quotient, and the Denominator of the Dividend by the Numerator of the

Example.

Divisor, for the Denominator of the Quotient. As to divide $\frac{1}{2}$ by $\frac{2}{3}$ and $\frac{3}{2}$ by $\frac{5}{3}$, and $\frac{1}{3}$ by $\frac{3}{2}$, the several Quotients are $\frac{3}{4}$, $\frac{9}{10}$, $\frac{2}{9}$ variously set as at A. or B. C. or D. E. or F.

$$\begin{array}{l} \text{A } \frac{2}{3} \overset{3}{\times} \frac{1}{2} \quad \text{B } \frac{2}{3} \overset{1}{\times} \frac{3}{2} \left(\frac{3}{4} \right) \quad \text{C } \frac{5}{3} \overset{9}{\times} \frac{3}{2} \quad \text{D } \frac{5}{3} \overset{3}{\times} \frac{2}{2} \left(\frac{9}{10} \right) \quad \text{E } \frac{3}{2} \overset{2}{\times} \frac{1}{3} \quad \text{F } \frac{3}{2} \overset{1}{\times} \frac{2}{3} \left(\frac{2}{9} \right) \end{array}$$

2.
Homologal
Terms Com-
mensurable.
Examples.

2. Case. If the Homologal Terms or either of them be Commensurable; first reduce them to their least Terms, and then multiply the new Heterologal Terms as before, and so the Quotient shall be also kept in its least Terms. As $\frac{2}{3}$ divided by $\frac{4}{5}$, because 2 and 4 may be abbreviated to 1 and 2 the Product of 2 into 3 shall be the Denominator of the Quotient; and the Product of 1 into 5 the Numerator, as at G. So the Quotient of $\frac{2}{3}$ divided by $\frac{5}{6}$ shall be $\frac{4}{5}$, as at H. and the Quotient of $\frac{5}{8}$ by $\frac{15}{16}$ shall be $\frac{2}{3}$, As at I.

$$\begin{array}{l} \text{G } 2) \overset{2}{\dots} \frac{4}{5} \overset{1}{\dots} \left(\frac{5}{6} \right) \quad \text{H } 3) \overset{5}{\dots} \frac{5}{6} \overset{2}{\dots} \left(\frac{4}{5} \right) \quad \text{I } 5) \overset{3}{\dots} \frac{15}{16} \overset{1}{\dots} \left(\frac{2}{3} \right) \end{array}$$

3.
Mixt Numbers
both or one a
Fraction or
Integer.
Examples.

3. Case. If *Integers*, or *mixt Numbers*; with *Fractions*, or two mixt Numbers, are given to be divided; Subscribe an Unit as in *Reduction* under the Integers, and reduce the mixt Numbers into *Improper Fractions*, and then proceed as above. So 4 divided by $\frac{3}{4}$ shall give in the Quotient $\frac{16}{3}$ or 5 $\frac{1}{3}$ as at K. and 4 divided by $\frac{2}{2}$ as at L. shall give $\frac{8}{9}$, also 4 $\frac{1}{3}$ dividing 3 $\frac{1}{2}$ the Quotient shall be $\frac{21}{26}$, as at M.

$$\begin{array}{l} \text{K } \frac{3}{4} \overset{4}{\times} \left(\frac{16}{3} \text{ or } 5 \frac{1}{3} \right) \quad \text{L } \frac{9}{2} \overset{4}{\times} \left(\frac{8}{9} \right) \quad \text{M } 4 \frac{1}{3} \overset{13}{\times} \frac{7}{2} \left(\frac{21}{26} \right) \end{array}$$

4.
Integer and
Fraction or
mixt Number.

4. Case. When an *Integer* is given to divide an *Improper Fraction*, or a *Single Fraction*, whose Denominators are digits, or such a *single Fraction* given to divide an *Integer*, then if the *Integer* be Dividend, double, triple, quadruple, &c. the Dividend according to the Denominator of the *Fraction*. But if the *Integer* be Divisor, and the *Improper Fraction* or other *Fraction* be Dividend, then either double, triple, quadruple, &c. the Denominator of the Dividend according to the given *Integer*, or else accordingly take the half, third part, quarter, &c. of the Numerator, which may

Examples.

best be done. As to divide 3 by $\frac{1}{2}$ the 3 doubled shall make the Quotient $\frac{6}{1}$. But $\frac{1}{2}$ divided by 3, because the third part of 1 cannot be had, 2 shall be tripled, and make the Quotient $\frac{1}{6}$. And in dividing $\frac{3}{2}$ by 3 either 2 may be tripled, or the third part of the Numerator which is 1 taken, and this is best, because the Quotient $\frac{1}{2}$ will be in its least Terms, otherwise it would be $\frac{3}{6}$, and need Abbreviation. See the Common Operations at N. O. P.

$$\begin{array}{l} \text{N } \frac{1}{2} \overset{3}{\times} \left(\frac{6}{1} \right) \quad \text{O } 3 \overset{1}{\times} \left(\frac{1}{6} \right) \quad \text{P } 3) \overset{1}{\dots} \frac{3}{1} \overset{1}{\dots} \left(\frac{1}{2} \right) \end{array}$$

5. Case.

5. Case. When an Improper Fraction is given to divide an Integer greater in quantity than the Numerator of the Fraction, Division may be made after the manner of Integers thus ; Divide the Dividend by the Numerator of the Fraction, and subtract this Quotient from the Dividend, and lastly divide the Remainder by the Integers contained in the Fraction, and this last Quotient shall be the quesited Number. As to divide 1480

by $3\frac{1}{12}$, or $\frac{37}{12}$, after 1480 is divided by 37, and the Quotient 40 subtracted, the Remainder 1440 is to be divided by 3, and this Quotient 480 is the Number sought. As at Q. agreeing with the Common way at R.

$$\begin{array}{r} \text{Q } 3\frac{1}{12} \overline{) 1480} \\ \underline{1440} \\ 40 \end{array} \quad \begin{array}{r} \text{R } 37 \overline{) 1480} \\ \underline{1480} \\ 0 \end{array}$$

6. Case. When the Numerators are equal, cancel them, and place the Denominator of the Divisor for the Numerator of the Quotient, over the Denominator of the Dividend. But when the Denominators are equal reject them, and *vice versa* place the Numerator of the Divisor under the Numerator of the Dividend for the Denominator of the Quotient. And if both the given Fractions be equal, for the Quotient take 1, for the new Fraction in such case shall alwayes be equal ; and generally may be observed, if the Dividend be the greater of the two propounded Fractions, the Numerator of the Quotient will be greater than the Denominator, but if the Divisor be the major Fraction the contrary. As to divide $\frac{2}{5}$ by $\frac{2}{3}$, the Quotient will be $\frac{3}{5}$. So $\frac{1}{3}$ divided by

$\frac{2}{3}$ gives $\frac{1}{2}$ in the Quotient, and $\frac{1}{3}$ by $\frac{1}{3}$ makes the Quotient an Unit. As at S. T. V.

Numerators Equal.

Denominators equal.

Fractions equal.

$$\text{S } \frac{2}{3} \overline{) \frac{2}{5}} = \frac{3}{5}$$

$$\text{T } \frac{2}{3} \overline{) \frac{1}{2}} = \frac{1}{2}$$

$$\text{V } \frac{1}{3} \overline{) \frac{1}{3}} = 1$$

As in Integers, so in Fractions, Division and Multiplication alternately prove each other, and though particularly one sort of work may be tried by another sort, yet regular and general Probation is by dissolving the Numbers compounded, and compounding the Numbers dissolved. Wherefore if the Product of any Fraction be divided by either of the Factors, the other Factor shall be found in the Quotient, and if the Quotient of any Fraction be multiplied by the Divisor, the Dividend shall be returned. Example in the Answer of these two following Questions.

1. What Number is that which being multiplied by $\frac{1}{5}$ shall produce $\frac{3}{10}$?

Answer. $1\frac{1}{2}$, for dividing $\frac{3}{10}$ by $\frac{1}{5}$, the Quotient is $\frac{3}{2}$ or $1\frac{1}{2}$.

2. What Number being divided by $\frac{1}{5}$ will give in the Quotient $\frac{3}{2}$?

Answer. $\frac{3}{10}$, for such is the Product of $\frac{1}{5}$ multiplied by $\frac{3}{2}$.

Proof of Division.

Proof of Multiplication.

$$\begin{array}{r} 3 \\ \frac{1}{5} \times \frac{3}{2} = \frac{3}{10} \end{array}$$

$$\begin{array}{r} \frac{1}{5} \overline{) \frac{3}{10}} \\ \underline{\frac{2}{10}} \\ \frac{1}{10} \end{array}$$

Hence it is evident that Reduction of Fractions of Fractions may be proved ; if the reduced Fraction, being but the Product of their Multiplication, be divided by any one of the Fragments, for then will the Quotient be the other Fragment, or the Sum of the other Fragments, if more than two were in the Composition. As in the former Instances,

Proof of the Reduction of Fractions of Fractions in p. 48.

Instances, 2. Chap. 4. Sect. of Reduction, $\frac{1}{4}$ of $\frac{2}{5}$ reduced became $\frac{1}{10}$, if therefore $\frac{1}{10}$ be divided by $\frac{1}{4}$, the Quotient will be $\frac{2}{5}$, or by $\frac{2}{5}$ it will be $\frac{1}{4}$, as at U. and W.

And the Reduction of $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{8}$ unto $\frac{15}{64}$ will be found true; for if $\frac{15}{64}$ be divided by any of the three Fragments, the Sum of the other two will appear in the Quotient, as at X. Y. Z.

$$\begin{array}{c}
 \text{U} \qquad \qquad \text{W} \qquad \qquad \text{X} \qquad \frac{15}{32} \\
 2) \frac{1}{4} \left) \frac{1}{10} \left(\frac{2}{5} \right. \quad 5) \frac{2}{5} \left) \frac{1}{10} \left(\frac{1}{4} \right. \quad 2) \frac{1}{2} \left) \frac{15}{64} \left(\frac{15}{32} = \frac{3}{4} \text{ of } \frac{5}{8} \right. \\
 \frac{2}{5} \qquad \frac{1}{2} \qquad \frac{1}{32} \qquad \frac{3}{32}
 \end{array}$$

$$\begin{array}{c}
 \text{Y} \qquad \qquad \text{Z} \\
 3) \frac{1}{3} \left) \frac{5}{16} \left(\frac{5}{16} = \frac{5}{16} \text{ of } \frac{5}{8} \right. \quad 5) \frac{1}{8} \left) \frac{3}{8} \left(\frac{3}{8} = \frac{3}{8} \text{ of } \frac{3}{4} \right. \\
 4) \frac{3}{4} \left) \frac{15}{64} \left(\frac{15}{64} = \frac{15}{64} \text{ of } \frac{5}{8} \right. \quad 8) \frac{5}{8} \left) \frac{15}{64} \left(\frac{15}{64} = \frac{15}{64} \text{ of } \frac{3}{4} \right. \\
 \frac{1}{16} \qquad \frac{1}{16} \qquad \frac{1}{8} \qquad \frac{1}{8}
 \end{array}$$

This kind of Proof was mentioned before in *Reduction*, but reserved till after *Division* was taught; as not probable before to be understood.

Partis secundæ, & Libri primi

F I N I S.

T H E

ARITHMETICK.

The Second BOOK,

CONCERNING
Numbers generally contract;

In Two PARTS.

WHEREIN
GEODÆTICALS } are { Declared.
FIGURALS } { Demonstrated.

AND THEIR
SIMPLE ELEMENTS.

CHAP. I.

Of GEODÆTICALS.

Sufficient hath been said of *Absolute Integers* and *Abstract Fractions* in the former Book, it is necessary now to proceed to *Contract Numbers*.

Contract Numbers were before declared to be Numbers restrained by some annexed Denomination, or special Denominator, and Book 1. Part 1. Chap. 2. divided into two sorts, *General* and *Special*.

General are such whose Denominations are generally known in most Nations, and usual not only in *Mathematical Sciences*, but also in *Mechanical Arts*, and of Common and Vulgar Frequentment in Traffique, Merchandizing, Buying, Selling, &c.

Numbers generally Contract are *Geodetical* or *Figural*.

Geodeticals include all Numbers contracted by Vulgar Names or Denominations according to the common and usual distinctions, divisions, dimensions, or legal institutions, customs, or usages of Nature or Nations, As Men, Women, Horses, Sheep, Weights, Measures, &c.

After Abstract Numbers the Contract treated of. Contract of 2 sorts. General are

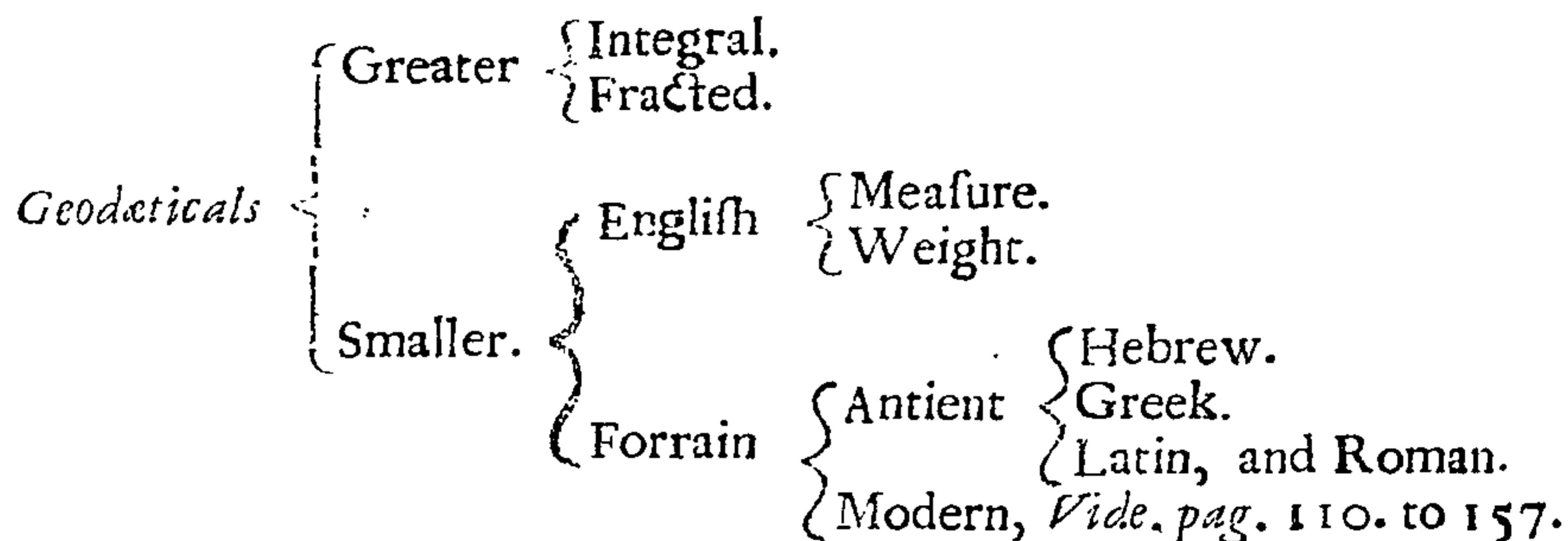
Geodeticals and Figurals. Geodeticals what.

Whence the Word.

The word *Geodetical* comes from *Geodasia*, and this from the two *Greek* words, *γῆ* and *μέτρον*, signifying a Division, Measure or Dimension of the Earth, and may seem too short to denote what is intended thereby; yet because all Denominations are truly but measures of quantities or gravities, accompted according to the Standart of earthly dimensions instituted by the Law of Nature or Nations upon the Terrestrial Globe; the Term *Geodetical* may fitly stand for the purpose here used till another more fit be found out.

Geodeticals distinguished.

Numbers thus concluded may be distinguished as in the Order following.



Greater Geodeticals.

The greater *Geodeticals* are such Denominate Numbers whose Names or Denominations admit not smaller artificial divisions, or are the highest denominations of that kind or quality, and if they may be divided into smaller parts yet cannot be heightened into greater. Of the first sort are Angels, Men, Women, Cities, Towns, Horses, Sheep, Ships, and a multitude of such other Nounes. Of the other sort are Leagues, Years, Circles, Bales, Butts, Tons and such like, which are capable of being divided into smaller parts or denominations.

Integral or Fraçted.

These greater *Geodeticals* are *Integral* or *Fraçted*.

Integral what.

Integral, When such denomination is annexed to any *Abstract Integer*, as 10000 Angels, 36 Men, 800 Ships, &c.

Fraçted what.

Fraçted, When such denomination is annexed to any *Arithmetical Fraçtion*, as $\frac{1}{2}$ of a Ship, $\frac{3}{4}$ of a Ton, &c.

Geodeticals how they come to be reckoned among Integers denominate.

And because several of these greater denominations may be parted into lower and subtiler parts, every of which parts having Proper Names, and being thereby known, those parts so named become reckoned for *Integers* denominate, and the greater denominations omitted as before, *Chap. 2. Of the Nature of Numbers* was declared. For $\frac{3}{4}$ of a Ton, because a Ton is divided into 20 Hundreds shall be called 15 Hundreds, which is $\frac{3}{4}$ of 20, and pass for a denominate *Integer*, and the great denomination [Ton] omitted: So because a Pound of Money is divided into Shillings, and Shillings into Pence, parts of a Pound shall be accompted by Shillings, and parts of a Shilling by pence, &c.

Smaller Geodeticals.

The smaller *Geodeticals* arise from such of the greater as admit of subdivisions and lesser parts under particular names and denominations which being abbreviated with the greater denomination may be reduced back again from a smaller denominate *Integer* to a fraçted *Geodetical* of the greater Contraçtion; As 10 Shillings abbreviated with 20, the Shillings in one pound shall be reduced to $\frac{1}{2}$ l.

English or Foreign.

Geodeticals of the smaller sort may be comprehended under two heads, *English* and *Foreign*.

English take in Measure and Weight.

English, Include both *Measure* and *Weight*.

Measure what.

Measure is the accompt of any Quantity or Magnitude taken either in length or breadth, or according to its solid thickness, or hollow capacity, by some Standart approved as a known Measure.

Weight what.

Weight is the accompt taken of the gravity of any Quantity or Magnitude according to some approved Standart or known Measure.

Both doubly considered.

Both these, *Viz.* *Weight* and *Measure* referring to some denominations are pure as considered *per se*, to others impure as considered *inter se*, but in Authours found somewhat confusedly, as Money and Bread passing only as numbred or accompted, and yet notwithstanding to have their due proportion of Weight, and Woollen Cloth generally passing by Measure in length, ought also to have due breadth and weight.

English Measures.

Under *English* Denominations that refer to measures of length may be contained not only things once measured by the Inch, Foot, Yard, Ell, Rod, &c. but also several things accompted by Number, or as commonly called Tale, as Fish, Skins, Paper,

per, &c. for it is sufficiently evident that every Number in some sense is but a lineary Measure or Multitude of Units supposed to stand in length one by another.

Long Measures therefore comprise the Denominations used in *Astronomy of Time and Long. Motion*. In *Cosmography and Geography* from the measure of Leagues down to the length of Early-Corns. In Merchandise the denominations of many sorts of Silks, Stuffs, Linnen Cloth, Fish, Skins, Paper, Parchment, &c.

Broad Measures imply things twice measured by a line, once in respect to length and *Broad.* again to breadth, and contain the Denominations used in *Geometry* from *Acres* downward, and in the square measures of Glass, Plank, Pavements, &c. In Merchandize Woollen Cloth, which also is to have its proportion of Weight.

Measures of Thickness or Bodily Capacity are either Solid, as the Sizes of Fuel, *Thick.* Measure of Timber, Stone, &c. or Concave which have hollow Capacity. Concave Measure is divided into dry and liquid; Dry take in the Measures of Corn, Salt, Coals, Lime, &c. Liquid, Ale, Beer, Honey, Butter, Fish, Oyl, Sope, Wine, &c.

Among *English Weights* but two are considerable, *Viz.* *Troy* and *Avoirdupois*.

English Weights
2. considerable.

The Book intituled *The Path-way to Knowledge* Translated out of *Dutch* into *English* by *W. P.* 1596. makes mention of three other Weights. *Dutch Book mentions.*

1. The *Pound Tower* usual for Mintage, in the names of its divisions like the *Pound Troy*, but in quantity less, for 16 *Pounds Tower* make but 15 *Pounds Troy*. *Pound Tower.*

2. The *Pound Subtil* or *Suttle* so termed, for that in small quantity it may be made *Pound Subtil.* ratable to represent any other greater Weight whatsoever, as *Four Penny weight Troy*, or less, to answer in due proportion unto the whole *Pound Troy* with all his parts, every part sensible and severally to be handled. This Weight is private to Assay-Masters and such as can make trial of *Minerals*, and not known to many other, neither in ordinary Accompts is there any use thereof.

3. The *Pound Foile*, is lesser than the *Pound Troy* by $\frac{1}{12}$ part of the *Pound Troy*, and *Pound Foile.* hath small use, save only amongst those that make *Gold Foil* and *Wyre*.

But the *Troy* and *Avoirdupois* Weights are of common use, the first of force by Statute, the other by Custom, yet confirmed by Statute. Why so called I could never satisfy my self, but crave leave to conjecture that *Troy Weight* was the Weight anciently used in Towns and Cities, and *Verstigan* tells us that *Troy Novant*, is as much as to say *New Town*; and *Avoirdupois*, *Averdupois*, *Averdepois*, or *Avoirdupoys* however written seems to sound like an Over-Weight, from the old *Norman Language*, and was either allowed upon drossy and coarse Goods, or probably sold in the Countrey and the Over-Weight allowed in lieu of Carriage to a Market. *Troy and Avoirdupois Weights of common use in England. Why so called probably. Troy weight quasi Town weight. Avoirdupois quasi to have Over weight of 12. on 100. What weighed by Troy weight. What weighed by Avoirdupois weight.*

By the *Troy-Weight* is accompted the Assize of Bread and some Liquid Measures constituted, also Gold and Silver Plate and Bullion, Rings, Jewels, Precious Stones, Musk, Civit, &c. are weighed thereby.

By the *Avoirdupois-Weight* are bought and sold all base Mettals, as Copper, Tinn, Steel, Iron, Lead, and several other Merchandizes, as Allom, Rozin, Pitch, Tar, Wax, Tallow, Hemp, Flax, Beef, Cheese, Meal, Corants, Raisins, Prunes, Figgs, Almonds, Spices, Druggs, Sugar, Tobacco, Wooll, &c. So Sope and Butter when sold by Retail, and many other Retailed Commodities whereof any Garble, Refuse or Waste comes; But Butter, Sope, and some other things when sold in gross, pass by the Barrel or Firkin where the Content or Measure of the Vessel is also considerable.

Coyne respecting both Weight and Number or Tale which is but long Measure as before noted may be placed under either of them, but though vulgarly passing by Tale, yet properly belongs to *Troy-weight* according to the divisions whereof, for the most part, *English Money* is divided, and ought to be valued in Weight and Fineness. *Coyne respects Weight and Number.*

Of all these in order, it is convenient something be said, and their Dimensions and Divisions seen, before Foreign Measures and Weights be spoken of.

English Measures.

The Measures of Time and Motion being properly *Astronomicals*, having certain Denominators and special Operations belonging to them are referred to the second part of the next Book. *Measures of Time and Motion. See Book 3. Par. 2. Barley-Corn the beginning of Long Measures.*

Cosmographical and *Geographical* Measures of length begin at a Barley-Corn, and increase upward to a League, in Latin called *Leuca*, which is the greatest denomination principally in use with Mariners, for with Geographers commonly the greatest denomination

League from
the Latin
Leuca,
Mile from
Mille.
A Table of En-
glish Long Mea-
sures for Land.

mination is a Mile, generally thought to be derived from the Latin *Mille*, signifying 1000, because a Mile with the Latins contained just so many Paces, but an English Mile containeth 1056 Paces.

In 1 League are 3 Miles. In 1 Mile 8 Furlongs, &c. As in the following Table.

League.	1	Mile.					
Miles. *. 1.	3	1	Furlong.				
Furlongs. *. 2.	24	8	1	Perch.			
Perches. *. 3.	960	320	40	1	Foot.		
Feet. *. 4.	15840	5280	660	161	1	Inch.	
Inches.	190080	63360	7920	198	12	1	
Barley-Corns *. 5.	570240	190080	23760	594	36	3	Barley-Corns.

Length of an
English Mile.

*. 1. That an *English* Mile shall contain so many Furlongs, Perches and Feet may be seen in the Statute. *Anno 35. Eliz. Cap. 6.* Intituled, *An Act for the Restraint of New Building, &c.* and long before the making of that Statute was a Mile of the same content.

Pulton misprint-
ed in the num-
ber of Furlongs.

*. 2 The Collection of the Statutes at large by *Ferdinando Pulton*, Printed 1640. pag. 1191. in the Statute of 35. *Eliz.* last mentioned deserves Correction, who there makes 1 Mile but 5 Furlongs, whereas it should be 8, as by another Printed Copy by me, and other good Authours may appear.

Perch, several
names thereof.
Less than a
Rood.
The Sorts.

*. 3. A Perch or Pearch hath other names, as Pole, Rod, and Lugge, but they are grossly mistaken who call it a Rood, for a Rood is a quarter of an Acre, a far greater quantity than a Rod.

Perch for the
Measure of
Woodland.
Church-land.
Forests.

The Pole, Rod or Perch, for they are the most usual names, is by the Statute afore- said to be but 161 Feet, yet by the usage of some Countreys the Pole doth vary, for in some places it is 18 Feet, in some 21 Feet, and in others 24. And Mr. *Osborne* (as witnesseth *Dalton* in his *Countrey-Justice*, Chap. 65.) writeth that the measure of 18 Feet to the Perch is commonly called Woodland Measure, 21 Feet to the Pole is called Church Measure (*Scilt.* of Land which doth or did belong to some Church), and 24 Feet to the Pole is called, (and that rightly) Forest Measure.

Foot, the length
thereof.

*. 4. That a Foot shall contain 12 Inches, and one Inch 3 Barley-Corns laid end to end, (or as some say 4 in thickness being dry and round and taken out of the midst of the Ear) is evident by the Statute made *Anno 33. Edw. 1. An. Dom. 1306. De terris Mensurandis, & De compositione Ulnarum & Perticarum.* Foot is often used in the Plural as well as Singular; as 2 Foot, 3 Foot, &c. for 2 Feet, 3 Feet.

Often used Plu-
rally.

Barley-Corn no
measure in it
itself.

*. 5. A Barley-Corn is in it self no Measure, but the least thing in a Measure, where- of as it were Measure is made, and whereby it is rectified by the Ordinance, Intituled, *Compositio Ulnarum & Perticarum.* 3 Barley-Corns laid end to end make an Inch, 12 Inches a Foot, &c.

Measures for
acres, &c.
Yard the length
Ell, the length.

Besides these, some measure lengths whether depths, heights, or distances by Scores, Goads, Fathoms, Paces, Ells, and Yards, which are thus divided.

In 1 Yard are 3 Feet, or 36 Inches.

In 1 Ell are 3 Feet 9 Inches, or 45 Inches.

Both these ordained by Statute, and of general use in *England*, for the measure of Silks, Stuffs, Cloth, Lace, Ribband, and several other Wares and Merchandize; com- monly Woollen Cloth by the Yard, and Linnen Cloth, Silks, &c. some by the one, and some by the other, as is well known to Tradesmen, but seldom used for Land. Both Yard and Ell divided equally into halves, quarters, half quarters and Nails, or Neils. So 1 Yard or Ell containing 16 Nails.

Measures of
Cloth how much
in the

To some Cloth and Silks belong greater denominations than Ells and Yards, as Chefs, Bolts, Pieces, Hundreds, Roules, &c.

Chef.

In 1 Chef of { Fine Linnen, Silks—————10 } Ells.
 { Fustian—————14 }

Bolt.

In 1 Bolt of { Pole Davies————— } 28 Ells.
 { Spruce Elbing————— }

In 1 Piece of	Barbers Aprons, or Checks	} 10 Yards	Piece.
	Curle Sipers		
	Borratoes or Bombasines, Buffins	} 15 Yards.	
	Moccadoes and Lile Grograins, Buftians		
	Carrels, Dornix, Strip't or Tufted Canvas		
	Rafhes, Flanders Serges, &c.		
Beaupurs, Frizado, Hounscot Say	24 Yards.	Hundred.	
English Tufted Canvas	30 Yards.		
Ribband	36 Yards.		
In 1 Piece of	Cambrick	} 13 Ells.	Rowle.
	Lawn		
In 1 Hundred of	Lockram called	{ Treagers Broad Dowlafs	Hundred.
	Canvas	} 120 Ells.	
	Sackcloth		
Soulthwitch			
In 1 Roul of	Tiking and Twill of Scotland	{ 1500 Ells.	Rowle.
	Minfters		
	Ozenbrigs		

In 1 Pace or rather Pafs, from the Latin *Paffus*, are 5 Feet, called a Pace *Geometrical*, to difference it from other Paces of greater or leffer content, and is properly a Foreign Measure for Land.

In 1 Fathom are 6 Feet, used in meafuring Depths, and Sounding at Sea.

In 1 Goad 1 1/2 Yard, or 4 1/2 Feet, a Measure in some places for Land and Cloth received by Custom.

In 1 Score 20 Yards or 60 Feet, a Measure also not ordained by Statute.

Some Denominations frequent in *English* Books, or Digits, Palms, Spans and Cubits great and small, are no usual *English* Measures, but came from other Nations hither, and retaining their Foreign Dimensions and Divisions, are to be sought among the Foreign Accompts.

The Measure called a Handful used in meafuring the height of Horfes, by 27. *Hen. 8. Chap. 6.* is ordained to be 4 Inches.

In Merchandize under Long Measures as before noted, fall sundry Lengths, commonly called Number or Tale

Many small Wares called Habberdashery, and some other Commodities are sold by Dozens, Scores, Shocks, Hundreds, Thousands, Lasts, Grosses, &c.

Every dozen contains 12, and of some things in some places 13.

Every Score 20, and in some places 21 for 20.

Every Shock 60.

The Hundred is more or less according as the Commodity is, and the Thousand and Last greater or lesser as the Hundred.

The Gros is small or great, the small Gros is 12 dozen, the great Gros is 12 small Gros.

A Breviat of some Merchandizes accompted by these and such like Denominations.

Alphabets.	In 1 Set—24.	Set.
Balkes	} In 1 Hundred—120.	Hundred.
Barrel board		
Bome spars.		
Bookes.	In 1. Maund. 2. Fats.	Maund, Fat &c.
Bowstaves.	In 1 Handred 120.	Hundred.
Boxes called Sope-Boxes.	In 1 Shock 60.	Shock.
Bracelers or Necklaces of Glas.	In 1 small Gros 12 Bundles or Dickers.	Gross, Bundle.
Bread, in several places.	In 1 dozen 13 Penny Loaves.	Dozen.
Buttons.	In 1 great Gros 12 small Gros. In 1 small 12 ordinary dozen of Buttons.	Grosses, &c.
Canes.	In 1 Shock 60.	Shock.
Cantspars	} In 1 Hundred 120.	Hundred.
Capravens		
Clapholt or Clapboard.	In 1 Great Hundred 12 Rings. In 1 Ring 2 small Hundred.	Hundred, &c.
	In 1 small Hundred 120 Boards. So that 1 great Hundred contains 24 small Hundred, or 2880 Boards.	
Deales.	In 1 Hundred, 120.	

Length of Fish
fold by Tale.

Fresh Fish how
fold.
Maund thereof.
Mary.

Cod, &c. the
100.
Eeles the Bind,
Strike.
Herrings.

A Table of the
Last of Her-
rings.

Fish, Of the greater sort barrell'd called Countable or Tale-Fish, ought to contain in length from the Bone in the Finn to the third joynt of the Tale, 26 Inches at the least, by the 22. *Edw. 4. Cap. 2.*
Most fresh Fish is fold by the Common Dozen or Score, Sometimes by the Maund, if the Fish be small; the Maund or Moane, holdeth about a Gallon; and Six Maunds full set together, and an heap on them all at top is called a Mary.
Besides these some quantities of Fish fresh and Salt fold out of Cask have Common Denominations.
Cod, Also Haberdine Ling, and Newland Fish. In 1 Hundred 124.
Yet the Book of Rates reckons but 120 to the Hundred.
Eeles, In 1 Bind, 10 Strikes, In 1 Strike 25 Eeles. So is 1 Bind 250 Eeles.
Herrings Fresh or Salted at Sea, called carn'd or corbed by the Last in some places thus divided.

Last.	1	Thousand		
Thousands.	10	1	Hundred.	
Hundreds.	100	10	1	Warpe.
Warpes.	3200	320	32	1
Herrings.	12800	1280	128	4

White Herrings.
Red Herrings.

Hundred.
Turn.
Hundreds.

But by the Statute of Herrings made 31. of *Edw. 3. An. Dom. 1357. Cap. 2.* there is appointed but Six Score to the Hundred, so one Last shall contain but 12000.
Yet at *Yarmouth* they sell 33. Warpe to the Hundred.
Accordingly White Herrings that is Salted in Barrels is fold by Retail, and Red Herrings that is dryed in the Smoak, in some places are accompted by 120 to the Hundred.
A Cade or Carde of Red Herrings ought to contain in 1 Cade 5 Hundred, that is 600 Herrings, and one Last 20 Cades.
Shrimpes. In 1 Hundred, 120.
Soles. In 1 Turn 4.
Sprats. The Hundred as Herrings. The Cade of Red Sprats 5000. Yet by the Book of Rates Outwards but 1000.
Stock-fish. In 1 Hundred, 120.
Raftal. 8 Title, Weights and Measures, saith the Hundred of hard Fish must be 8 Score.
Garlick. In 1 Hundred 15 Ropes. In 1 Rope 15 Heads. So is 1 Hundred, 225 Heads.
Gloves. In 1 Dicker, 10 Pair.
Horsshoes. In 1 Dicker, 10 Shooes.
Iron. In 1 Dozen 6 pieces. Raftal 8 Weights and Measures.
Knives. In 1 Bundle 6 Dickers. In 1 Dicker, 10 Knives.
Leather. In 1 Last 20 Dickers. In 1 Dicker 10 Hides. So the Last is 200 Hides.
Oars. In 1 Hundred, 120.

Ropes.
Dickers.
Dozen.
Bundle, Dicker.
Last, Dicker.
Hundred.

Paper.

Parchment.

Paper and
Parchment how
reckoned.

Bale.	1	Ream.	
Reams.	10	1	Quire.
Quires.	200	20	1
Sheets.	5000	500	25

Rowle.	1	Dozen.	
Dozens.	5	1	
Skins.	60	12	Skins.

Bind of Skins. Skins. In 1 Bind, 33 Skins.

Timber of Skins, or Furrs of { Ermines. Letwis.
Fitches. Martrons.
Grayes. Minkes.
Jenners. Sables. } In 1 Timber 40 Skins.
Budge. }
Cat. } In 1 Hundred 5 Score Skins.
Coney. }

Skins

Skins of

Calves.

Goats.

Kidds.

Lambs.

Sheep.

In 1 Dozen 12.

In 1 Kippe. 50.

In 1 Hundred, 5 Score.

Yet by the Book of Rates Outward 6 Score.

Skins.

Dozen

Kippe

Hundred

of Skins.

Trayes of Wood. In 1 Shock, 60.

Shock.

Measures long and broad reach the Denomination used about Land Measuring, called Acres. It is sufficiently manifest by the Statute *An. 33. Edw. 1.* aforesaid, that an Acre of Land is to contain in length 10 Perches, and in breadth 16. So if the breadth be 1 Perch, the length shall be 160; if the breadth be 2, the length shall be 80; and so proportionably, that is to say alwaies 160 Square Perches. The other proportions of the length and breadth of one Acre of Land mentioned in that Statute were observed by Record in his Book of *Arithmetick* long since, not to be exact. The Acre is thus divided.

Acre.	1	Rood.	
Roods. *.	4	1	Daies work.
Daies Works.	40	10	1
Square Perches.	160	40	4

Square Perches.

A Table of English Square Measures.

*. A Rood is sometime called a Farthendele, and sometime a Yardland, but as to the latter very corruptly, for a Yardland containeth much more than an Acre.

Several Denominations about Land Measure besides a Yardland are found in the Law Books, as Hides, Plowlands, Carves, Carucates, and Oxeganges, but are grown so obsolete, That the Lawyers themselves can hardly agree about the Content thereof; the first 4 seem to be all one, and are reckoned to contain by some 85, by most, 100 Acres, yet *Norden* in his *Surveyors Dialogue*, and others, make a difference between a Hide of Land and the other three, and say that a Hide of Land containeth 4 Plowlands, and every Plowland, Carve or Carucate, which are all one, 4 Yardlands, and every Yardland 30 Acres. So shall one Plowland contain 120 Acres, and one Hide of Land 480 Acres, *Cambden* and *Hollingshead* will have one Hide of Land to contain 100 Acres, and others say, 8 Oxeganges make a Hide or Plowland, and every Oxegange containeth 15 Acres. *Dalton* in his *Counrrey-Justice* saith, that the Common Account in the East part of *Cambridge-shire* of a Yardland is but 24 Acres. And *Sir Edward Coke* in the first part of his *Institutes* under the Title *Escuage* (perhaps prudently foreseeing these differences irreconcilable) is of Opinion that a Plowland is of no certain Content, but is rather to be reckoned by the Value than Content, and that more in one place and lesser in another shall be a Plowland according to the quantity that one Plow may till in a Year. But all agree that a Yardland called in Latin *Quatrona terra* is much more than an Acre, and therefore ought not to be used for a Rood, which is but a quarter of an Acre.

Rood how called Yardland, more than an Acre.

Difference in the Accompts of Hides, Plowlands. &c.

Sir Edward Coke's Opinion of a Plowland.

Yardland how called.

Among long and broad Measures fall in next, Glafs, Plank, Pavements, of which Glafs-Windows are commonly measured by the Foot Rule of 12 Inches to the Foot. So one Square Foot shall contain 12 Inches in length, and as many in breadth, that is 144 Square Inches. Unwrought-Glafs sometime sold by Weight. See among *Weights*.

Glafs, &c. how measured.

Pavements are sometimes measured by the Yard, commonly by the Foot-Rule, and a parcel of Pavement or Tiling of 10 Foot long and 10 Foot broad is ordinarily in these parts called One Square. And equivalent to such a Square shall be the laying of 100 Gutter Tiles, or Redge-Tiles, though of the latter some count but 50.

Pavements, Roofs, how measured.

The Square thereof.

Plank or Board is commonly accounted among Square Measures, yet more properly belongeth to Timber Measure, for that besides the length and breadth respect is had to the thickness of the Plank, whether it be Inch, Inch and half, Two Inch, Three Inch Plank or more, all commonly measured by the Foot of 12 Inches. The Accompt of the Load of each is referred to Timber Measure.

Plank or Board how measured.

All Square Measures, whether of Acres, Glafs, Pavements, Plank, &c. do properly belong to Figural Numbers treated of in the next part of this Book.

Square Measures belong to Figurals.

But Woollen Cloth though as before noted, must have length, breadth and weight, yet being commonly accompted only by length, and accordingly by Retail sold by the Yard keeps place among *Geodæticals*, and as to the making, and Wholesale thereof duely placed here.

Woollen Cloth.

Statute for the
due make
thereof.

The due making of Woollen Cloths is declared in a Statute made *An. 4. Jac. Chap. 2.* being an Epitome of all former Acts to that purpose, in which may be seen their Weight, Breadth and Length, Workmens Orders, with the Viewing, Searching and Forfeitures or Abatements of and for the same.

One Sack of Wooll (the Content whereof is found among Weights) is accompted to make 4 Standard Clothes of clean Wooll called sorting Clothes, weighing 60 lb the Cloth, and being 24 yards long and 6 quarters $\frac{1}{2}$ broad or thereabouts within the remedy or allowance of 2 lb weight upon a Cloth.

Retailed by the
Yard and Inch.

The length is prescribed by the Statute to be measured wet within the list of the Cloth, by the Yard and Inch, instead of which Inch is the allowance of the Thumb in Retailing usual.

Cloth to be
weighed.

In Weighing, which is to be by the *Avoirdupois* Weight is to be observed that the Clothes be well scoured, thickned, milled, and fully dryed, the Weight of a Cloth seems more to be regarded than the measure, because the Weight containeth substance which may be abused by stretching into Measure.

Affize of Cloth
by Statute.

The Affizes of Woollen Cloths by the Statute An. 4. Jacobi. Cap. 2.

	Length. Yards.	Breadth. Quarters.	Weight. Pounds.
Long Broad Cloth, and Clothes of Died Woolls and mingled Colours of <i>Kent, Yorkshire</i> and <i>Reading</i> , between—	30 & 34	6 $\frac{1}{2}$	86
Whites of <i>Worcester, Coventry</i> and <i>Hereford</i> —	30	33—7	78
Plunkets, Azures, Blews, and long Whites of <i>Suffolk, Norfolk</i> , and <i>Essex</i> .—	29	32—6 $\frac{1}{2}$	80
Sorting Clothes, <i>Suffolk, Norfolk</i> , and <i>Essex</i> —	23	26—6	64
Fine short <i>Suffolks</i> —	23	26—6 $\frac{1}{2}$	64
Handiwarpes—	29	32—7	76
Plunkets, Azures, &c. of <i>Wilts, Somerset, &c.</i> —	26	28—6 $\frac{1}{2}$	68
<i>Yorkshire</i> short Clothes—	23	25—6 $\frac{1}{2}$	66
Broad listd Whites and Reds of <i>Wilts, Gloucester, Oxford</i> and East part of <i>Somerset</i> —	26	28—6 $\frac{1}{2}$	64
Narrow listd White and Red—	26	28—6 $\frac{1}{2}$	61 White 60 Red
Fine plain listd Clothes of the <i>Shires</i> last mentioned—	29	32—6 $\frac{1}{2}$	72
<i>Tauntons, Bridgwaters</i> , and <i>Dunsters</i> —	12	13—7	30
Short Cloths of Died Wooll, &c.—	23	25—6 $\frac{1}{2}$	66
Narrow of <i>Somerset, &c.</i> —	24	25—4	30
<i>Devon</i> Kerfies called Dozens—	12	13—4	13
Check Kerfies, Straights and Plain Graies—	17	18—4	24
Ordinary Pennistone or Forest Whites—	12	13—5 $\frac{1}{2}$	28
Sorting Pennistones—	13	14—6 $\frac{1}{2}$	35
Whashers of <i>Lancashire</i> and others—	17	18—	17
Cogware, Kendal, and Karpmeales at pleasure—	20	at least.	

Cottons by the
Goad.

Some accompt *Manchester, Cheshire*, and *Welch* Cottons by the Goad, allotting the *Lancashire* Cotton to be in length between 20 and 21 Goads, in breadth $\frac{3}{4}$ within the list, and in weight 21 pound. The *Manchester* and *Cheshire* 22 Goads in length, in breadth as the *Lancashire*, and in weight 30 Pound.

Measures Bodily
for Bodies
thick, solid, or
concave.

The next sort of Measures are bodily, whither Solid or Concave, and properly belong to Figural Numbers handled in the next part, yet because many of them are not Rooted Numbers, and in Common Commerce reckoned by Number or Tale rather than in respect to their Capacity, they may as to their Denominations stand among Geodæticals.

Among Solid Measures are the Affizes of Fuel, Plank, as before noted, Timber, Stone, Laths, Tiles, &c.

Fuel the Measure.

Fuel contains Billets, Cordwood, Faggots, Talwood, and Coals. But Coals are sold by Bushel, and therefore placed among Concave Measures

Billets the
Affize.

Every Billet by the 7. *Edw. 6. Chap. 7.* must be 3 Feet 4 Inches in length, and is accompted for 1 or more, according to the bigness thereof. For if it be but 7 $\frac{1}{2}$ Inches about, it shall be but a single Billet. If it were 10 Inches about it was called a Cast, and was marked with 1 notch within 4 Inches of the end, and to pass for 2 Billets. If the two Billets were 14 Inches about it was called a Cast of two, and marked with 2 notches, within 6 Inches of the middle, and to pass in Tale for 4 Billets

But by the 43. *Eliz. Chap. 14.* the Affize was altered as to the Casts, Cleft Billets, affized, and provision made that no single Billet should be cleft : Thus,

	Round.	Half Round.	Quarter cleft.	Length.
A Single Billet.	7 $\frac{1}{2}$			
A Cast marked 1.	11	13	12 $\frac{1}{2}$	} 40 Inches.
A Cast of 2 marked 2.	16	19	18 $\frac{1}{2}$	

Billets marked with 3, 5, or 7 notches are to pass for so many single Billets, and to be proportional. Billets are commonly sold by the Hundred, 5 Score to the Hundred.

Cordwood is Wood of the bigger sort of Firewood, measured by a Cord or Line, of which there are two Measures. That called the Fourteen Foot Cord is to be 14 Feet in length, 3 Feet in breadth, and 3 Feet in height. *Cordwood the Load or Cord.*

The other Cord is to be 8 Feet long, 4 Feet high, and in breadth 4 Feet, yet in some places 3 of the 4 Feet high is 4 Foot Wood, and the other Foot but 3 Foot Wood.

Faggots called Two bands by the last mentioned Statutes are to be 3 Feet in length, and the band 24 Inches about, besides the knot. Of such Faggots 50 go to one Load. *Faggots the Load.*

Faggots of smaller Wood called Bavin and Spray are sold by the Hundred, and 100 accounted for a Load.

The Affize of Round Talshide Ordained by 7. *Edw. 6. Chap. 7.* is confirmed by 43. *Eliz. Chap. 14.* and Talshide half round and quarter cleft affized thus. *Talshide the Affize.*

Number of Notches or Marks. Round. Half Round. Quarter cleft.

Every Talshide named	{ 1 2 3 4 5	} must be	{ 16 23 28 33 38	—	{ 19 27 33 39 44	—	{ 18 $\frac{1}{2}$ 26 32 38 43	} Inches about within a Foot of the middle.

Plank or Board customarily is accounted by the Load according to the thickness of the Plank, and to be measured by the Foot Rule ; the Load thus reckoned. *Board or Plank the Load.*

	Feet long.	Feet broad.	Inches thick.	
Plank or Board	600	—	1	} make 1 Load.
	400	—	1 $\frac{1}{2}$	
	300	—	2	
	240	—	2 $\frac{1}{2}$	
	200	—	3	
	171 $\frac{3}{4}$	—	3 $\frac{1}{2}$	
	150	—	4	

The Load how much.

If the breadth be more than 1 Foot, the length must be less proportionably.

Timber well hewn and perfectly squared, viz. 1 Foot broad, and 1 Foot thick 40 Feet long make 1 Ton or Tun. And 50 such Feet 1 Load. If the breadth or thickness be more, the length must be less, if less, the length must be more. *Timber the Load the Ton.*

Stone sometimes is measured by the Foot after the manner of Timber, and sometimes reckoned by the Ton Weight. *Stone how measured.*

Lath, Tann, and Tile, because in them respect is had to their length, breadth and thickness may fitly be placed here.

Lathes are sold in gross by the Load bound up in Bundles, every Load 30 Bundles. In Retail by the Bundle, every Bundle 100 Laths. Every Lath ought to be 5 Feet long, 2 Inches broad, and $\frac{1}{4}$ Inch thick, if the Lath be but 4 Feet long, then there must be 6 Score to the Hundred. *Laths the Load and Bundle, how much.*

Tann, 1 Load must be 60 yards long, 1 yard high, 3 Rinds thick set up on each side of a Pole laid along to rest against, and 2 Rinds at top. Yet 45 yards thus set is a good Waggon Load. *Tann, the Load how much.*

Tiles are of 3 sorts, 1st Plain Tile or Thack Tile. 2nd Gutter Tile, or Corner-Tile. 3rd Roof Tile, Crestle or Ridge Tile, commonly sold by the Hundred in some places of 6 Score, and some 5 Score to the Hundred, all affized by the Statute *An. 17. Ed. 4.* *Tiles the Affize.* *Chap. 4.* thus.

	Inches long.	Thick.	Broad.
Plain Tile	10 $\frac{1}{2}$	and $\frac{1}{4}$ of $\frac{1}{4}$ with convenient Thickness and Breadth.	6 $\frac{1}{4}$
Gutter Tile	10 $\frac{1}{2}$		
Ridge Tile	13		

T

Concave

Concave Measures.

Concave Measures whether Dry or Liquid had their Original from Weight, and therefore some place them among Weights, but being formally Bodies, and common use respecting rather their Content or Quantity than their Weight, they may fitly stand here.

Dry.

Dry Measures are those by which Dry Goods are measured, as Corn, Salt, Coals, Lime, &c. And may be distinguished into *Winchester Measure* and *Water Measure*. *Winchester Measure* is the Standart Measure. *Water Measure* greater.

Corn Measure.

Corn is measured by Troy Weight, and weighed by *Averdupois*, that is the Measures for Corn are according to Troy Weight, but Corn sold by Weight shall be weighed by *Averdupois* Weights.

*The Affize
thereof.
Winchester
Measure.
The Weight.*

By 51 Hen. 3. An. 1266. 31 Edw. 1. & 12. Hen. 7. Chap. 5. The Content of a Gallon of Wheat is to be 8 Pounds or Pints Troy, 8 Gallons 1 Bushel London Measure, and 8 Bushel 1 Quarter. This is called *Winchester Measure*.

By the Book of Affize of Bread fet forth by *John Povel*, the Bushel is to contain 56 Pounds or Pints of *Averdupois* Weight, and so proportionably for half Bushels, Pecks, &c;

*Bushel above
8 Gallon in
some places,
and by heap.*

Custom hath begotten in some places greater Bushels, then 8 Gallons, as 9, 10, &c. Also greater quantities than the Quarter have their denominations, yet all are to be reckoned according to the measure of the Bushel and Gallon ordained by the Statutes.

Uſage in ſome places hath continued Meaſure by heap, although ſome Statutes order it by Strike, and allowance in ſome places is 21 for 20.

The Divisions and Subdivisions of Corn Measure called *Winchester* Measure, may be inspected in the Table following.

<i>A Table of the</i>	Laſt.	1	Load.										
<i>Laſt of Corn</i>	Loads.	2	1	Quarter.									
<i>Meaſure.</i>	Quarters or Seams.	10	5	1	Coomb.								
	Corncocks or Coombs.	20	10	2	1	Strike.							
	Striks or half Coombs.	40	20	4	2	1	Buſhel.						
	Buſhels.	80	40	8	4	2	1	Tovit.					
	Tovits or half Buſhels.	160	80	16	8	4	2	1	Peck.				
	Pecks.	320	160	32	16	8	4	2	1	Gallon.			
	Gallons.	640	320	64	32	16	8	4	2	1	Pottle.		
	Pottles.	1280	640	128	64	32	16	8	4	2	1	Quart.	
	Quarts.	2560	1280	256	128	64	32	16	8	4	2	1	
	Pints or Pounds Troy.	5120	2560	512	256	128	64	32	16	8	4	2	Pints.

Apples, &c.
the Last Barrel.
Charcoal the
Load, Sack.
Meal sold by
Measure.

Apples, Nuts, Oatmeal, In 1 Last 12 Barrels. 1 Barrel 3 Bushels:

Charcoal is sold sometimes by the Load. In 1 Load 80 Bushels. Sometime by the Sack. In 1 Sack 4 Bushels, 7. *Edm. 6. Cap. 7.* and sometime they reckon 88 Bushels, or 22 Sacks *Winchester* Measure to a Load.

Meal in some places sold by Measure. In 1 Bushel 12 Gallons striked.

*Lime by Water
Measure, how
much the Bushel
Salt, the Hun-
dred Wey.*

Lime, Salt, Seacoal are measured by Water Measure, the Bushel whereof by the 11. Hen. 7. Cap. 4. is to contain 10 Gallons of Winchester Measure, nevertheless in some places is 12. 14. &c. Gallons.

Salt is reckoned by the Hundred and Wey. In 1 Hundred of Salt 10½ Weyes, in 1 Wey 40 Bushels. So 1 Hundred contains 420 Bushels, Water-Measure.

*Seacoale the
Last Chaulder.*

Seacoale is accompted by the Laft and Chaulder. In 1 Laft of Seacoal *Newcastle* Measure 7½ Chaulders. The Chaulder generally 32 Bushells, but differs at several places according to the quantity of the Bushel. In 1 Chaulder Rye-Measure 32 Bushels, in 1 Bushel 12 *Wincheſter* Gallons by heap, and the Cop on the Bushel was equal to 4 Gallons more: Whereupon of late, upon new making the Bushel, order was given to make a Bushel that should hold the old Bushel with the Cop, and this new Bushel is still in use filled up to the brim, but not by heap.

Chandler at
Rye.

*Concave Mea-
sures Liquid.*

Liquid Measures are Cask that contain Moist or Liquid Commodities; As Ale, Beer, Butter, Fish, Honey, Oyl, Soap, Wine, &c.

In Measures of this Nature two things are to be heeded, viz. the Content of the Vessel which is alwaies by the *Winchester* Measure, and the weight of the empty Cask when the Commodities are sold by weight.

Ale 23. Hen. 8 Cap. 4.

A Table of Ale Measures.

Laft.	1	Barrel.				
Barrels.	12	1	Kilderkin.			
Kilderkins.	24	2	1	Firkin.		
Firkins.	48	4	2	1	Gallon.	
Gallons.	384	32	16	8	1	Pottle.
Pottles.	768	64	32	16	2	1 Quart.
Quarts.	1536	128	64	32	4	2 1
Pints.	3072	256	128	64	8	4 2 Pints.

Beer 23. Hen. 8. Cap. 4.

A Table of Beer Measures.

Laft.	1	Ton									
Tons.	2	1	Pipe.								
Pipes.	4	2	1	Hoghead.							
Hogheads.	8	4	2	1	Bunn.						
Barrels or Bunn.	12	6	3	1½	1	Barrel.					
Herring Barrels.	16	8	4	2	1½	1	Kilderkin.				
Kilderkins.	24	12	6	3	2	1½	1	Firkin.			
Firkins.	48	24	12	6	4	3	2	1	Gallon.		
Gallons.	432	216	108	54	36	27	18	9	1	Pottle.	
Pottles.	864	432	216	108	72	54	36	18	2	1	Quart.
Quarts.	1728	864	432	216	144	108	72	36	4	2	1
Pints.	3456	1728	864	432	288	216	144	72	8	4	2 Pints.

Butter and Sope are oflike measure, and the Content of the Barrel, Kilderkin, and Firkin of Sope is ordained by 23. Hen. 8. Cap. 4. to be as the Ale Measure, that is the Barrel 32 Gallons, &c. And the Weight of the empty Cask as followeth.

Barrel	26	} Averdupois.
Kilderkin or half Barrel	13	
Firkin	6½	

Weight of the Cask.

So that of Butter and Sope that is besides the Weight of the Cask should be { 56 Butter. 60 Soap.

Fish, as Eeles and Salmon by the 22. Edw. 4. Cap. 2. and other Statutes have one and the same Measure, Viz.

Laft.	1	Butt.				
Butts.	6	1	Barrel.			
Barrels.	12	2	1	Half Barrel.		
Half Barrels.	24	4	2	1	Firkin.	
Firkins.	48	8	4	2	1	
Gallons.	504	84	42	21	10½	Gallons.

A Table of the Measure of Cask for Eeles and Salmon.

Herrings

Troy Weight.

Pound.	1	Ounce.						
Ounces.	12	1	Penny-weight.					
Penny-weights.	240	20	1	Grain.				
Grains.	5760	480	24	1	Mite.			
Mites.	115200	9600	480	20	1	Droite.		
Droites.	2764800	230400	11520	480	24	1	Peroite.	
Peroites.	55296000	4608000	230400	9600	480	20	1	
Blanks.	1327104000	110592000	5529600	230400	11520	480	24	Blanks.

A Table of
Troy-Weight.

Troy Weight hath feldom any greater denomination than the pound, yet sometime 2 lb. thereof is called a Maſt allowed for Amber and Gold and Silver Thread of Collen by the Book of Rates. *A Maſt how much.*

Eight Ounces of the Pound Troy make the Weight, called sometime Bes, sometime the Mark Weight in Latin, *Marchus & Marca*. This is beyond Sea more than in England uſed to weigh Silver; every Ounce divided into 20 English, and every English into 32 Grains, called by ſome Aſſes or Azes. *A Mark how much.*

The Goldſmiths divide the Ounce Troy into 24 parts, which they call Carats, Carats, Caracts, Characts, Karaçts or Kareçts, and every Kareçt into 4 Grains. So the pound ſhall have 1152 Grains Kareçt, and the Ounce 96. They alſo divide the Ounce into 150 Kareçts, and every of theſe Carrats into 4 Grains. By the firſt ſort of Carats they try the fineneſs of Bullion, buy and ſell Gold and Silver; by the latter Pearls and Diamonds. An old Authour I have ſeen that parts each of the 24 Kareçts into 12 Grains, but theſe ſeem *Exotick*, and not of *English* Extract. *Legal* at the end of *Thomas* his *Dictionary* divides the Ounce Troy into 16 parts, which he calls Farthing Gold Weights, and allows to every Charact 20 grains, From hence appear Grains of 3 ſorts, *Viz.* *Karaçts by what Names called, how much, and for what uſe.* *Farthing Gold-Weights.*

Grains by the Statute.	} in 1 lb. Troy	{ 7680.	Grains the ſorts.
Grains of Aſſize.			
		{ 5760.	
		{ 7200.	
Grains Caract in 1 lb. Troy.		{ 3456.	
		{ 1152.	

The Grains of Aſſize, and the laſt of the Grains Caract are moſt in uſe with us. Why called Caract, ſee after in the Notes on the *Greek Weight Siliqua*, pag. 96.

The Conſtitution of Meaſures by *Troy Weight* hath been already touched, beſides which, the uſe thereof is to weigh Bread, Bullion, Money, Pearles and Precious Stones, and choice Phyſical Druggs, as Ambergreafe, Bezoar, Civer, Muſk, Unicorns-horn, &c. to all which except Bread and Money, nothing need more to be ſaid. *What to be weighed by Troy Weight*

Bread.

The Statute 51. Hen. 3. Intituled the Aſſize of Bread and Ale, Enacteth, That when the price of a Quarter of Wheat is 1 s. the Waſtel Bread of a Farthing ſhall weigh 6 l. 16 s. 00 d. *Bread the old Aſſize.*

- Bread Cocket of the ſame Corn and Bultel ſhall weigh more than the Waſtel by 2 s.
- Bread Cocket of lower price Corn more than the Waſtel by 5 s.
- Bread made into a Simmel leſs than the Waſtel by 2 s.
- Bread made of the Whole Wheat ſhall weigh a Cocket and a half;
- So that the Cocket weigh more than a Waſtel by 5 s.

Bread of Treet ſhall weigh 2 Waſtels.			
Bread of common Wheat ſhall weigh 2 great Cockers.	1.	s.	d.
When a Quarter of Wheat is ſold for 1 s. 6 d. then Waſtel-Bread of a Farthing white and well baked ſhall weigh	4	10	08
When for 2 s.	3	08	00
and ſo proportionably decreasing at the increaſe of 6 d. in every Quarter of Wheat.			200

Not exact.

But whither through want of knowledge in reversed proportions, or by the mistake of some *Amanuensis* is not certain, yet sure it is that the Assize of Bread set forth in that Statute for some higher prices than 2 s. the quarter of Wheat in all the Printed *Latin*, *French* and *English* Copies extant is erroneous, as was long since rightly observed by Record in his Book of *Arithmetick* called *The Ground of Arts*. Wherefore and for that at the making of the Statute Money by which the Assize of Bread is there reckoned was at the old rate, *Viz.* One pound of Silver Money weighed One pound *Troy-Weight*, which is since much altered, this old Assize grew intricate, and later times have appointed the Assize by the *Troy-Weight*, and not by Money; yet grounded on the Assize in the Statute, accounting 6 l. 16 s. Money according to the Standart then to answer to 6 lb. 9 s. 12 pwt. *Troy*, as indeed it doth.

Old Assize of Bread.

How altered.

Sorts of Bread, in the Statute.

Wassel Bread what.

Bread Cocket what.

Fine and Course

Simnel Bread what.

Bread of the whole Wheat.

Bread of Treet.

Household Bread

Horse Bread.

Sea-Bisket.

Allowance to Bakers.

Corn Cheap when.

old Winchelsea drowned when.

Farthing Loaves, when.

How the Assize at any Rate may be had.

The Statute allows of seven sorts of Bread, though those mostly now in use is White, Wheaten and Household.

Wassel Bread seems to be Rowles or fine Manchet Bread used principally in Victualling Houses to drink with, perhaps from the old word *Wassail*, to drink or banquet, as it were a drinking Bread.

Bread Cocket is White Bread of the best sort drawn through the finest Boultel or Låni-Sieve, or of the best Wheat; This is the first in the latter Assize Books.

Bread Cocket of the lower price Corn is the ordinary Wheat-Bread now usual, drawn through a courser Cocket or Boultel than the former, or made of worse Wheat.

Simnel Bread is a kind of Cake or Bisket-Bread made with some Butter and Spice, still in use in some places.

Bread made of the whole Wheat is sometime called Cribble or fine Ravel Bread, but in the Assize Books, and in most places called Wheaten Bread, made of Flour courser than the White, but not so coarse as the Household Bread.

Bread of Treet seems to be Household-Bread of the best Wheat unravelled, or ravelled through the coursest Boultel, that is only the Husk or Bran taken out of the Meal.

Household-Bread in the Assize-Books is assized like that in the Statute called Bread of Common Wheat, as being common for the whole Household, which then seemed either to be made of the common sort of Wheat or baked as it came from Mill.

Besides these Horse Bread though not ordained by Statute, yet hath been of long continuance and allowed, every loaf thereof is alway, at what price soever Corn be sold, to be of the full weight of the Penny white Loaf, and the Baker is to sell three such Loaves for a Penny, and 13 to the dozen, that is 39 loaves for 1 s.

Sea-Bisket of excellent use for the Sea, because baked without Salt, and well dried, is not assized by the Statute, therefore weighed and sold by the hundred of *Averdupois* weight.

In the Book of Assize of Bread published towards the latter end of *Q. Eliz.* Countrey Bakers were allowed 4 s. upon a Quarter above the middle price of Wheat: But Bakers that dwell in Cities and Towns were allowed 6 s. in regard they were subject to more Scot and Lot than the Countrey-Bakers were, which 6 s. is still generally allowed to Townish Bakers; but by the Orders of the Council Board Dated the last of *January*, 1604. the Countrey-Bakers Penny white-Loaf shall weigh 2 Ounces more than the Bakers of the Towns, their Penny wheaten loaf 3 Ounces, and their Penny household Loaf 6 Ounces.

Former Times had Corn very cheap, either by reason of the Plenty thereof, or scarcity of Money, for in a Book of Presidents remaining among the Records of this Town of *Rye*, pag. 131. is a *Memorandum* entred, That the Year Old *Winchelsey* was drowned, which is there said to be *An. Dom.* 1287. Corn was at 2 s. the Quarter, which at the Rate Money was then, is but 6 s. of our Money now. Whereas the cheapest it hath been known of late years was *Annis Dom.* 1654. and 1655, when good Wheat was sold in some places in *England* for 12 s. the Quarter.

Anciently by reason of the cheapness of Corn, Farthing Loaves of all sorts of Bread assized by the statute were made; but now none less than half-penny Loaves, and these but of white and wheaten-bread, for of household bread no Loaf is to be made under a Penny. Nevertheless by the Assize of the Farthing Loaf is the Assize of the half-penny and penny Loaf come to be had by doubling the weight of the lesser Loaf for the Assize of the greater, and contrariwise by halving the weight of the greater the assize of the lesser Loaf is had, and having the assize of the penny Loaf of any sort of bread at 1 s. per Quarter according to the Statute, the Assize of the same Loaf by the Reversed Rule of Three, as hereafter in the 4th Book, *Of Proportions*, may be seen, is obtained at any other Rate, but by the Law the Assize is not to be altered, but when there is 6 d. increasing or decreasing in the Price of the Quarter of Wheat.

The Statute Affize of Farthing, Half-Penny, and Penny-Bread at the price of Twelve pence the Quarter of Wheat, reduced from the Old Standard of Money to Troy-weight, accompting 20 s. of Money then to be a Pound Troy. The Affize of Bread by Statute.

Bread.	Loaves.	Money.			VWeight.		
		l.	s.	d.	lb.	z.	ptws.
Wastell	{ Farthing	06	16	00	06	09	12
	{ Half-penny	13	12	00	13	07	04
	{ Penny	27	04	00	27	02	08
Fine Cocket.	{ Farthing	06	18	00	06	10	16
	{ Half-penny	13	16	00	13	09	12
	{ Penny	27	12	00	27	07	04
Course Cocket	{ Farthing	07	01	00	07	00	12
	{ Half-penny	14	02	00	14	01	04
	{ Penny	28	04	00	28	02	08
Sinnel	{ Farthing	06	14	00	06	08	08
	{ Half-penny	13	08	00	13	04	16
	{ Penny	26	16	00	26	09	12
Wheaten	{ Farthing	10	11	06	10	06	18
	{ Half-penny	21	03	00	21	01	16
	{ Penny	42	06	00	42	03	12
Treet	{ Farthing	13	12	00	13	07	04
	{ Half-penny	27	04	00	27	02	08
	{ Penny	54	08	00	54	04	16
Household	{ Farthing	14	02	00	14	01	04
	{ Half penny	28	04	00	28	02	08
	{ Penny	56	08	00	56	04	16

From this Legal Basis is deduced the following Table containing the Affize by the Troy Weight of the Penny white, wheaten, and household Loaves being the Bread still in use from 12d. the Quarter of VVheat unto 2 l. the Quarter, omitting some of the small Fractions. The Affize of Bread now in use.

Price of Wheat			Penny White.			Penny Wheaten.			Penny Household.			Price of Wheat.			Penny White.			Penny Wheaten.			Penny Household.		
l.	s.	d.	lb.	z.	pwt.	lb.	z.	pwt.	lb.	z.	pwt.	l.	s.	d.	lb.	z.	pwt.	lb.	z.	pwt.	lb.	z.	pwt.
—	1	0	28	02	08	42	03	12	56	04	16	—	00	0	1	04	10	2	00	15	2	09	00
—	1	6	18	09	12	28	02	08	37	07	04	—	01	0	1	04	02 1/2	2	00	03	2	08	04 1/2
—	2	0	14	01	04	21	01	16	28	02	08	—	01	6	1	03	14 1/2	1	11	12	2	07	09 1/2
—	2	6	11	03	07 1/2	16	11	00 1/2	22	05	14 1/2	—	02	0	1	03	07 1/2	1	11	01 1/2	2	06	15 1/2
—	3	0	9	04	16	14	01	04	18	09	12	—	02	6	1	03	00 3/4	1	10	11	2	05	01 1/2
—	3	6	8	00	13 1/2	12	01	00 1/2	16	01	07 3/4	—	03	0	1	02	14 1/4	1	10	01	2	05	08 1/2
—	4	0	7	00	12	10	05	18	14	01	04	—	03	6	1	02	08 3/4	1	09	12	2	04	16 1/2
—	4	6	6	03	04	9	04	16	12	06	08	—	04	0	1	02	02 1/2	1	09	03	2	04	04
—	5	0	5	07	13 1/2	8	05	10 1/2	11	03	07 1/2	—	04	6	1	01	16	1	08	14	2	03	12
—	5	6	5	01	10 1/2	7	08	05 1/2	10	03	01 1/2	—	05	0	1	01	10 1/2	1	08	05 1/2	2	03	01 1/2
—	6	0	4	08	08	7	00	12	9	04	16	—	05	6	1	01	05 1/2	1	07	18	2	02	10 1/2
—	6	6	4	04	01	6	06	01 1/2	8	08	02	—	06	0	1	01	00 1/4	1	07	10	2	02	00 1/2
—	7	0	4	00	06 1/2	6	00	10 1/2	8	00	13 1/2	—	06	6	1	00	15 1/2	1	07	03	2	01	10 1/2
—	7	6	3	09	02	5	07	13	7	06	04	—	07	0	1	00	10 1/2	1	06	16	2	01	01 1/2
—	8	0	3	06	06	5	03	09	7	00	12	—	07	6	1	00	06	1	06	09	2	00	12 1/2
—	8	6	3	03	16	4	11	14	6	07	12	—	08	0	1	00	01 1/2	1	05	12	2	00	03 1/2
—	9	0	3	01	12	4	08	08	6	03	04	—	08	6	—	11	17	1	05	15 1/2	1	11	14 1/2
—	9	6	2	11	12	4	03	08	5	11	04	—	09	0	—	11	13	1	05	10	1	11	05 1/2
—	10	0	2	09	16 1/2	4	02	15 1/2	5	07	13 1/2	—	09	6	—	11	09	1	05	04	1	10	18 1/2
—	10	6	2	08	04 1/2	4	00	06	5	04	09	—	10	0	—	11	05	1	04	18 1/4	1	10	11 1/2
—	11	0	2	06	15 1/2	3	10	02 3/4	5	01	10 1/2	—	10	6	—	11	01	1	04	12	1	10	03 1/2
—	11	6	2	05	08 1/2	3	08	02 1/4	4	10	17	—	11	0	—	10	18	1	04	7 1/4	1	09	10 1/2
—	12	0	2	04	04	3	06	06	4	08	08	—	11	6	—	10	14 1/2	1	03	02	1	09	09 1/2
—	12	6	2	03	01	3	04	11 1/2	4	06	02	—	12	0	—	10	11 1/2	1	03	17 1/4	1	08	03 1/2
—	13	0	2	02	00 1/2	3	03	00 3/4	4	04	00	—	12	6	—	10	08	1	03	12 1/4	1	08	16
—	13	6	2	01	01	3	01	11 1/4	4	02	02	—	13	0	—	10	05	1	03	07 1/2	1	07	10
—	14	0	2	00	03 1/2	3	00	05 1/2	4	00	06 1/2	—	13	6	—	10	02	1	03	03	1	07	04 1/2
—	14	6	1	11	06 3/4	2	11	00 1/2	3	10	12	—	14	0	—	09	19	1	02	18 1/2	1	07	18 1/2
—	15	0	1	10	11	2	00	16 1/2	3	09	12	—	14	6	—	09	16 1/2	1	02	14 1/2	1	07	12 1/2
—	15	6	1	09	16 1/2	2	08	14 1/2	3	07	13	—	15	0	—	09	13 1/2	1	02	09 1/2	1	07	00 1/2
—	16	0	1	09	03	2	07	14 1/2	3	06	06	—	15	6	—	09	10 1/2	1	02	05 1/2	1	06	00 1/2
—	16	6	1	08	10	2	06	15	3	05	00	—	16	0	—	09	08	1	02	02	1	06	00
—	17	0	1	07	18	2	05	17	3	03	16	—	16	6	—	09	05 1/2	1	01	17 1/2	1	05	00 1/2
—	17	6	1	07	06 1/2	2	04	19 1/2	3	02	13	—	17	0	—	09	02 1/2	1	01	14 1/2	1	05	00 1/2
—	18	0	1	06	16	2	04	04	3	01	12	—	17	6	—	09	00 1/2	1	01	10 1/2	1	05	00 1/2
—	18	6	1	06	05 1/4	2	03	08 1/2	3	00	11 1/2	—	18	0	—	08	18	1	01	07 1/2	1	05	10 1/2
—	19	0	1	05	16	2	02	14	2	11	12	—	18	6	—	08	15 1/2	1	01	03 1/2	1	05	11 1/2
—	19	6	1	05	07	2	02	00 1/2	2	10	14	—	19	0	—	08	13 1/2	1	01	00 1/2	1	05	00 1/2
—	20	0	1	04	18 1/2	2	01	07 1/2	2	09	16 1/2	—	20	0	—	08	11 1/2	1	00	10 1/2	1	04	00 1/2
—	20	6	1	04	00 1/2	2	00	00 1/2	2	08	00 1/2	—	20	6	—	08	09 1/2	1	00	06 1/2	1	04	00 1/2

Money.

* 3. An Angel is 10 s. and so called from the Impression of an Angel upon pieces of Gold of that Value. Yet are there Gold Coins called Ship-Angels of more worth; because since they were Coined Gold is raised. *An Angel why so called.*

* 4. A Noble is 6 s. 8 d. half a Mark, or the third part of a Pound, and for the reason last before-mentioned several pieces of Gold bearing the name of Nobles, are worth more than 6 s. 8 d. as the Angel Noble, first Coined for a Noble, then advanced to an Angel, and now worth more. *Nobles it old Coine worth more now.*

* 5. A Crown is 5 s. the Quarter of a Pound, and the greatest piece of *English* Silver Coin, sometime called the *English* Dollar, from whence down to Farthings are Silver Coins, and besides these some Harpers in value 9 d. a piece, and half Harpers worth 4 d. a piece were Coined by *Q. Elizabeth*, but seem to be *Irish* Money, now almost worn out as well as her Three Farthing pieces. *Crown the greatest Silver Coine in England called sometime Dollar*

Farthings before the advance of the Ounce of Silver to 60 pence were Silver Coins, as appeareth by the Statutes 4 *Hen. 4. Cap. 10. & 14. Hen. 8. Cap. 12.* but since grew inconvenient by their smallness. *Farthings sometime Silver Coines.*

Mites are no pieces of Coines, but a lesser division used about reduction and finding out the Value of Foreign Coines by the *Sterling* Standard; and some for their private use and more curiosity divide the Farthing into 2 *Ques*, the *Q* into 2 *Cees*, the *C* into 2 *Dodkins*, the *D* into 2 *Mites*, that is 16 Mites in one Farthing. *Mites no Coines.*

The Coines of *England* are of Gold and Silver, some of greater value, some lesser, many elder, others later, and so different in Fineness and Weight, that it is hard for any but those whose common Converse is thereabouts, as Mintmasters, Goldsmiths, &c. to give any perfect accompt thereof; and the rather, because they proceed not by the strict Rules of *Arithmetick*, besides any such accompt will be subject to future alteration by addition of new Coines or advancing the Value of the old. *Farthings how divided by some.*

Mr. *Gerard Malynes* sometime a Commissioner about the Mint Affairs, and a Man expert in Coinage in his Book Intituled *Lex Mercatoria*, hath Calculated the Weight and Fineness of several both *English* and *Outlandish* Coines, and from thence in the succedent Tables much light is borrowed. Nevertheless whether because he hath (as he saith) omitted the small Fractions as unnecessary, or because he hath rather inserted what Weight the Coines ought to be according to the Royal Orders at the times when they first issued forth then what they are, I know not, but sure I am, his accompt in several of the Gold Coines doth not agree with the Book called *Perfekt Directions for all English Gold*, Imprinted 1663, nor others, nor yet with the Common Weights, as by often experience trying such as have come to my hands I have found *English Coines of both sorts different in value and fineness:*

Wherefore in the following Table for *English Gold*, besides the Weight after *Malynes* and others according to the more regular proportions of *Arithmetick*, is another Collumn containing the Common weight as the pieces were when Coined (for many by use are worne much lighter, and must have allowance) according to which Weight is the value reckoned; and because the value of Old Coines have since the time of their Coinage been increased not only by Proclamation, but beyond the Rates limited thereby, to wit, even as Merchants have found them valued in Foreign Exchange, or as Goldsmiths have found them worth to melt down: Let therefore the *Sterling* Silver Coines be understood as they anciently were and still are Currant in *England*, but the Gold Coines are differently valued in their respective Collumns, viz. those of *K. James* and *K. Charles* the First, as in their Proclamations, those elder as they were currant with the Goldsmiths Anno 1640, and are now worth since the Proclamation of *K. Charles* the Second Anno 1660. accompting in the one Penny weight of Gold 22 Carats fine worth 3 s. 4 d. and in the other 3 s. 6 d. 2 q. 4 m. omitting in some of the small pieces the odd Farthings and Mites, and proportionably for Gold of other fineness: Nevertheless according to the Proclamations and vulgar currant Exchange, most of the Old Coines were not, nor yet are valued so high, for the Old Spurre Royal by the Proclamation of King *James* 1611. is rated at 16 s. 6 d. yet, with the Goldsmiths long before the Proclamation 1660. was worth 18 s. which is 1 s. 6 d. more than that Proclamation values it, and the like may be observed in others. As to the pieces to the *Pound Troy* where the Fractions hapned to be small and inconsiderable in stead thereof the next nearest is taken and marked with — or + according as it is too little or too much; other things are perspicuous enough by the Tables themselves, and need no explanation. *Old Coinage advanced since their Coinage by Proclamation and otherwise.*

English Silver
Coins their
Fineness Weight
and Value.

The Table of Sterling-Silver-Coins now Currant.

Names of the Pieces.	Kings and Queens.	Fineness.		Weight.				Worth.	
		Ounces	pwt.	pwt.	gr.	m.	dr.	s.	d.
Crown of	Edw. 6. & Eliz.	11	2	1	0	20	00	00	5 0
	James, Charles 1. & 2.	11	0	1	9	08	10	07	5 0
Half Crown of	Edw. 6. & Eliz.	11	2	1	0	00	00	00	2 6
	James, Charles 1. & 2.	11	0	9	16	05	03	2	2 6
Shilling of	Edw. 6. Phil. & Mary, & Eliz.	11	2	4	00	00	00	00	1 0
	James, Charles 1. & 2.	11	0	3	20	18	01	1	1 0
Six pence of	Edw. 6. Phil. & Mary, & Eliz.	11	2	2	00	00	00	00	0 6
	James, Charles 1. & 2.	11	0	1	22	09	00	3	0 6
Old Groat of Henry 8.		11	2	1	20	13	00	2	4 +
Last Groat of Henry 8.		11	2	1	12	00	00	0	4 +
Groat of	Mary & Eliz.	11	2	1	08	00	00	00	0 4
	Charles 1.	11	0	1	06	19	08	1	0 4
Three pence of	Elizabeth.	11	2	1	00	00	00	00	0 3
	Charles 1.	11	0	0	23	04	12	1	0 3
Two pence of	Henry 8.	11	2	0	18	00	00	00	0 2 +
	Elizabeth	11	2	0	16	00	00	00	0 2
Three half pence of	James, Charles 1. & 2.	11	0	0	15	09	16	3	0 2
	Elizabeth.	11	2	0	12	00	00	00	1 1/2
Penny of	Henry 8. & Edw. 6.	11	2	0	09	00	00	00	0 1 +
	Mary, & Eliz.	11	2	0	08	00	00	00	0 1
Three Farthings of	James, Charles 1. & 2.	11	0	0	07	14	20	3	0 1
	Elizabeth.	11	2	0	06	00	00	00	0 3/4
Half penny of	Elizabeth.	11	2	0	04	00	00	00	0 1/2
	James, Charles 1. & 2.	11	0	0	03	17	10	3	0 1/2

English Gold
Coins, their
Fineness Weight
and Value.

The Table of most Sterling Gold Coins yet Currant.

Names of the Pieces.	Pieces to the Troy.	Weight by Malynes, &c.				Common Weight.	Pieces to Fine the Troy.	Value 1640.				Value 1660. low.								
		pwt.s.gr.	m.	dr.	pwt.s.gr.			m.	dr.	Car.gr.	l.	s.	d.	l.	s.	d.	gr.			
Old Double Rose Noble.		23 $\frac{3}{8}$	10	06	08	08 $\frac{4}{8}$	10	00	00	00	24	23 $\frac{3}{4}$	1	16	4	1	18	8	5	
Double Rose Noble of { Henry 8. Edw. 6. Phil. & Mary Elizabeth. }		24	10	00	00	00	9	22	00	00	24 $\frac{1}{2}$ +	23 $\frac{3}{4}$	1	16	0	1	18	4	5	
Great Sovereign of K. James.		24	10	00	00	00	9	16	05	03 $\frac{2}{3}$	24 $\frac{4}{5}$	22	0	1	13	0	1	15	3 $\frac{1}{2}$	
Double Rose Noble of K. James.		26 $\frac{2}{3}$	9	00	00	00	8	21	06	16	27	23 $\frac{3}{4}$	1	13	0	1	15	3 $\frac{4}{5}$	4 $\frac{1}{2}$	
Double Rose Royal or Real.							8	02	03	03	29 $\frac{2}{3}$ +	23 $\frac{3}{4}$	1	10	0	1	12	0	4 $\frac{1}{2}$	
Double Old Sovereign.		27 $\frac{3}{8}$	8	18	08	05 $\frac{1}{2}$	8	00	00	00	30	22	0	1	06	8	1	08	5	4
Best Double Sovereign of Henry		30	8	00	00	00	7	04	00	00	33 $\frac{1}{2}$ —	22	0	1	03	10	1	05	5	4
Double Sovereign of { Edw. 6. Elizabeth. }																				
Double Sovereign of K. James called Unite or Jacobus.		36	6	16	00	00	6	10	16	18 $\frac{1}{2}$	37 $\frac{1}{5}$	22	0	1	02	0	1	03	10	3
Laureat or xxs. piece of James.		39 $\frac{2}{3}$	6	01	09	02 $\frac{2}{3}$	5	20	09	18 $\frac{1}{4}$	41	22	0	1	00	0	1	01	4	3
Twenty Shillings piece of Charles I.		40	6	00	00	00	5	20	09	18 $\frac{1}{4}$	41	22	0	1	00	0	1	01	4	3
Old Rose Noble.		46 $\frac{3}{4}$	5	03	04	04 $\frac{2}{3}$	5	00	00	00	48	23 $\frac{3}{4}$	0	18	2	0	19	4	2 $\frac{1}{2}$	
Spurre Royal of { Henry 8. Edward 6. Philip & Mary Elizabeth. }		48	5	00	00	00	4	23	00	00	48 $\frac{2}{3}$ +	23 $\frac{3}{4}$	0	18	0	0	19	2	2 $\frac{1}{2}$	
Spurre Royal of James		53 $\frac{1}{3}$	4	12	00	00	4	10	13	08	54	23 $\frac{3}{4}$	0	16	6	0	17	7	2 $\frac{1}{2}$	
Double Noble of Elizabeth							4	10	06	16	54 $\frac{1}{5}$ —	23 $\frac{3}{4}$	0	16	0	0	17	1	2 $\frac{1}{2}$	
Old Noble or Noble of Henry		53 $\frac{3}{4}$	4	11	03	06 $\frac{1}{3}$	4	10	00	00	54 $\frac{1}{3}$	23 $\frac{3}{4}$	0	16	0	0	17	1	2 $\frac{1}{2}$	
Rose Royal.							4	01	01	13 $\frac{1}{2}$	59 $\frac{1}{3}$ +	23 $\frac{3}{4}$	0	15	0	0	16	0	2 $\frac{1}{2}$	
Old Sovereign.		54 $\frac{3}{4}$	4	09	04	02 $\frac{1}{4}$	4	00	00	00	60	22	0	0	13	4	0	14	2	2
Best Sovereign of Henry		60	4	00	00	00	3	14	00	00	66 $\frac{1}{2}$	22	0	0	11	11	0	12	8	2
Sovereign of { Edward 6. Elizabeth. }																				
Old Angel Noble or Angel of Henry		69	3	11	09	13 $\frac{1}{2}$	3	08	00	00	72	23 $\frac{3}{4}$	0	12	1	0	12	10	2	2

Names of the Pieces.	Pieces to the lb Troy.	Weight by <i>Malynes, &c.</i>				Common Weight.	Pieces to the lb Troy.	Fine <i>Car. gr.</i>	Value 1640.			Value Allow- 1660. ance.							
		pwts.	gr.	m.	dr.				pwts.	gr.	m.	dr.	l.	s.	d.	l.	s.	d.	gr.
Last Angel Noble of Henry { Edw. 6. Phil. & Mary. Elizabeth.	72	3	08	00	00	3	07	05	00	72 $\frac{2}{3}$ +	23	3 $\frac{1}{2}$	0	11	11	0	12	8	2
First Angel of James.																			
Sovereign of K. James called Double-Brittain Crown.	72	3	08	00	00	3	05	08	09 $\frac{9}{16}$	74 $\frac{2}{3}$	22	0	0	11	0	0	11	9	2
George Noble.						3	00	00	00	80	23	3 $\frac{1}{2}$	0	10	10	0	11	6	2
Last Angel of James.	80	3	00	00	00	2	23	02	05 $\frac{1}{3}$	81	23	3 $\frac{1}{2}$	0	11	0	0	11	9	2
Half Laureat of James.	79 $\frac{1}{3}$	3	00	14	13 $\frac{1}{11}$	2	22	04	21 $\frac{3}{11}$	82	22	0	0	10	0	0	10	8	2
Ten Shilling Piece of Charles I.	80	3	00	00	00	2	22	04	21 $\frac{3}{11}$	82	22	0	0	10	0	0	10	8	2
Angel of Charles.						2	16	14	09	89 +	23	3 $\frac{1}{2}$	0	10	0	0	10	8	2
Half Spurre Royal.	96	2	12	00	00	2	11	10	00	96 $\frac{4}{5}$ +	23	3 $\frac{3}{4}$	0	09	0	0	09	7	2
First Crown of K. Henry.	100 $\frac{1}{2}$	2	09	06	06 $\frac{3}{7}$	2	09	00	00	101 $\frac{3}{7}$	22	+	0	08	0	0	08	5	2
Single Noble of Elizabeth.						2	05	03	08	108 $\frac{2}{3}$	23	3 $\frac{1}{2}$	0	08	0	0	08	6	2
Half Old Noble.	107 $\frac{1}{2}$	2	05	11	15 $\frac{3}{4}$	2	05	00	00	108 $\frac{3}{4}$	23	3 $\frac{1}{2}$	0	08	0	0	08	6	2
Salute.	108	2	05	06	16	2	05	00	00	108 $\frac{3}{4}$	23	3	0	07	11	0	08	5	2
Base Crown of K. Henry, cal- led Rose-Crown.	120	2	00	00	00	1	23	00	00	122 $\frac{2}{3}$	20	0	0	05	11	0	06	4	1
Crown of { Edward 6. Elizabeth.	120	2	00	00	00	1	19	00	00	133 $\frac{4}{5}$	22	0	0	05	11	0	06	4	1
Half Angel Noble of Henry	138	1	17	14	18 $\frac{8}{13}$	1	16	00	00	144	23	3 $\frac{1}{2}$	0	06	00	0	06	5	1
Half Last Angel of Henry.																			
Half Angel of { Edw. 6. Phil. & Mary Elizabeth.	144	1	16	00	00	1	15	12	12	145 $\frac{1}{3}$ +	23	3 $\frac{1}{2}$	0	05	11	0	06	4	1
Half first Angel of James.																			
Brittain Crown of James.	144	1	16	00	00	1	14	14	04 $\frac{2}{3}$	148 $\frac{4}{5}$	22	0	0	05	6	0	05	10	1
Half George Noble.						1	12	00	00	160	23	3 $\frac{1}{2}$	0	05	5	0	05	9	1
Half Last Angel of James.	160	1	12	00	00	1	11	11	02 $\frac{2}{3}$	162	23	3 $\frac{1}{2}$	0	05	6	0	05	10	1
New Crown of James.	158 $\frac{2}{3}$	1	12	07	06 $\frac{6}{11}$	1	11	02	10 $\frac{2}{11}$	164	22	0	0	05	0	0	05	4	1
Crown of Charles I.	160	1	12	00	00	1	11	02	10 $\frac{2}{11}$	164	22	0	0	05	0	0	05	4	1
Two parts of Salute	162	1	11	11	02 $\frac{2}{3}$	1	11	00	00	164 $\frac{4}{5}$	23	3	0	05	3	0	05	7	1
Half Henry first Crown	201	1	04	13	03 $\frac{1}{6}$	1	04	10	00	202 $\frac{6}{7}$	22	+	0	04	0	0	04	2	1
Half Salute	216	1	02	13	08	1	02	10	00	217 $\frac{1}{3}$	23	3	0	03	11	0	04	2	1
Half Rose Crown	240	1	00	00	00	0	23	10	00	245 $\frac{5}{7}$	20	0	0	02	11	0	03	2	$\frac{1}{2}$
Half Crown of { Edward 6. Elizabeth.	240	1	00	00	00	0	21	10	00	267 $\frac{3}{4}$	22	0	0	02	11	0	03	2	$\frac{1}{2}$
Quarter Old Angel Noble.	276	0	20	17	09 $\frac{9}{13}$	0	20	00	00	288	23	3 $\frac{1}{2}$	0	03	00	0	03	2	$\frac{1}{2}$
Quarter Last Angel of Henry.																			
Quart. Angel of { Edw. 6. Phil. & Mary. Elizabeth.	288	0	20	00	00	0	19	16	06	290 $\frac{2}{3}$ +	23	3 $\frac{1}{2}$	0	02	11	0	03	2	$\frac{1}{2}$
Quarter First Angel of James.																			
Half Brittain Crown of James.	288	0	20	00	00	0	19	07	02 $\frac{1}{11}$	297 $\frac{3}{4}$	22	0	0	02	09	0	02	11	$\frac{1}{2}$
Quarter Last Angel of James.						0	17	15	13 $\frac{1}{3}$	324	23	3 $\frac{1}{2}$	0	02	09	0	02	11	$\frac{1}{2}$

Many of the Physical Doses are weighed by the Pound of 12 Ounces, and every Ounce is divided into 8 Drams, 1 Dram into 3 Scruples, and 1 Scruple into 20 Graines. *Physical Doses how weighed.*

Mettals more base than Gold or Silver with Course Druggs and divers sorts of Goods and Merchandizes as before noted are bought and sold by *Avoirdupois* Weight, the ordinary Hundred whereof contains 112 Pounds, and the Pound 16 Ounces, which Ounces are less than the Ounce *Troy*, though the Pound be bigger; because that Pound is divided but into 12 Ounces, each of which bear proportion to the Ounce *Avoirdupois* as 1 to 1 $\frac{1}{16}$ for 1 Ounce *Troy* makes 1 $\frac{1}{16}$ Ounce *Avoirdupois*, and 1 Ounce *Avoirdupois* is but $\frac{1}{16}$ of 1 Ounce *Troy*, as saith my *Dutch* Authour, and accordingly he makes the Pound *Troy* to be but $\frac{1}{16}$ of the other Pound, and 1 lb *Avoirdupois* to equal 1 $\frac{1}{16}$ lb *Troy*. *Course Druggs and base Mettals how weighed.*

Herewith also agreeth *Dalton* and *Malynes* before named in their making 56 lb *Avoirdupois*, and 67 lb 8 $\frac{3}{4}$ *Troy* justly accord, though elsewhere both of them unhappily mistake to count 7 lb *Avoirdupois* equal to 102 $\frac{1}{2}$ *Troy*, which is 8 $\frac{1}{2}$ lb , for then should 56 lb *Avoirdupois* be 68 lb *Troy*. Others affirm 16 $\frac{1}{2}$ *Avoirdupois* equal to 14 $\frac{1}{2}$ 12 *pwt.s.* *Troy*, and then shall 56 lb *Avoirdupois* be equal to 68 lb 1 $\frac{1}{2}$ 12 *pwt.s.* *Troy*. Nevertheless some think anciently the Pounds admitting the like Number of Graines differed no more than the weight of Wheat and Barley one to another, seeing 1 lb *Avoirdupois* contains 7680 Grains or Barley-Corns, and so many Grains of Wheat are found in 1 lb *Troy*, if every Pennyweight be multiplied by 32 Grains, or Wheat Cornes according to the Old Statutes. *By some thought to differ no more than Wheat and Barley.*

Common greater and smaller Divisions than the Hundred of Avoirdupois Weight.

A Table of
Avoirdupois-
Weight.

Tonn.	1	C. Hundred.	1	$\frac{1}{2}$ C. Half Hundred.	qr. Quarter.	lb. Pound.	3. Ounce.	3. Dram.	3. Scruple.	Grains.
Hundreds.	20	1								
Half Hundreds.	40	2	1							
Quarters.	80	4	2	1						
Pounds.	2240	112	56	28	1					
Ounces.	35840	1792	896	448	16	1				
Drams.	286720	14336	7168	3584	128	8	1			
Scruples.	860160	43008	21504	10752	384	24	3	1		
Grains.	17203200	860160	430080	215040	7680	480	60	20		

Custom hath made familiar the use and knowledge of Stones, Nails, Cloves, Tods, and such like denominations, though frequented but to weigh some sort of Commodities, and to which of them to allot more Allowance or Tare than 12 on the 100 or less, the Experienced Merchant well knows. However for satisfaction of the Curious Inquisitor, take a Breviat of such as have come to hand in perusal of the Statutes, the Book of Rates (according to which the King receiveth his Customs), and other approved Authours and Experience.

By Avoirdupois Weight.

Allom the Hundred, Stone.

Ashes the Last Barrel.

Barillia the Barrel.

Beef, the Nail, Score.

Butter the Wey.

Cheese the Wey, Clove.

Cinnamon as Allom.

Glass the Seam, Stone.

Hay, the Load, Truss.

Hemp the Stone.

Lead the Fodder.

The Old Weight

Allom, 1 Hundred, $13\frac{1}{2}$ Stones, 1 Stone 8 Pounds, by the Ordinance *Compositio de Ponderibus*, which allows but 108 lb. to the Hundred.

Ashes, Called Pot-Ashes, also Soap-Ashes, 1 Last 12 Barrels, 1 Barrel 2 Hundred, by the Rates Inwards.

Barillia, Or Saphora, 1 Barrel 2 Hundred weight, by the same Rates.

Beef, 1 Nail 8 Pounds of Common use. Some places sell by the Score, each Score 20 Pounds.

Butter in Cask is affized by Statute, as is seen before in Measures, but besides of *Suffolk* and *Essex* Butter, the Wey is usually reckoned alike to their Wey of Cheese.

Cheese by the Statute of 9. K. Hen. 6. Chap. 8. One Wey is to contain 32 Cloves, and 1. Clove 7 Pounds. *Suffolk*-Cheese by the Assize Book 1597, and usage ever since, notwithstanding 8 Pounds to the Clove, and *Essex*-Cheese 10 Pounds to the Clove, and both 32 Cloves to the Wey. But by some the Wey of *Essex*-Cheese doth contain 42 Cloves, and the Clove but 8 Pounds. Both which agree to make the Wey of *Essex*-Cheese 336 lb. of *Suffolk* 256 lb. whereas by the Statute a Wey is but 224 lb. as before.

Cinnamon, by the Ordinance *Compositio de Ponderibus* is to have the same weight as before noted of Allom.

Glass, by the same Ordinance containeth 1 Seam, 24 Stones. 1 Stone 5 Pounds.

Hay, by Custom 1 Load 40 Trusses, 1 Truss 56 Pounds, which make 20 Hundred weight to the Load, yet most times it passeth with 18 Hundred.

Hemp, is commonly sold by the Stone, which by the Statute of 21. Hen. 8. Cap. 12. is especially ordained to contain 20 Pounds. Nevertheless in *Rye* a Stone of Hemp is 32 lb. and so hath been time out of mind.

Lead, the Common Accompt 1 Fodder 19 1/2 Hundreds, 1 Hundred 112 Pounds. By the Book of Rates Outwards to 1 Fodder is allowed 20 Hundreds. By the Ordinance abovesaid, 1 Load 30 Formells, 1 Formel 6 Stones wanting 2 lb. every Stone 12 lb. and 1 Pound 25 Shillings Sterling. So was the Formel then 70 Pounds, a Weight now grown obsolete.

Meal commonly sold by Weight, 1 Bushel 2 Tovit or Half Bushels, 1 Tovit 2 Pecks, 1 Peck 2 Gallons, 1 Gallon 7 Pounds. So the Bushel must weigh 56 lb. and hereto agree the old and later Books of Assize, yet thereby is the Bushel more than the Bushel by Statute, for 56 lb. or Pints of *Avoirdupois* Weight exactly answers to 67 lb. 8 3/4. *Troy* Weight, whereas the Bushel by Statute is to contain but 64 lb. or Pints *Troy*.

Nutmegs, Pepper and Spice, as *Allom* and *Cinnamon*, by the Ordinance above mentioned

Raw Silk, of *China*, *Morea*, Long and Short, 1 Pound, 24 Ounces.

Silk Nubs, or Husks of Silk, 1 Pound 21 Ounces, both by the Book of Rates Inwards.

Wool, hath the Weight established by 31. *Edw.* 3. *Cap.* 8. and other Statutes, according to the *Lunar* Year of 13 Moneths, and 28 Dayes to the Moneth, making one Sack 26 Stones, and 1 Stone 14 Pounds, which makes the Sack 364 lb. other denominations may be seen in the following Table.

Nutmegs, &c.
the Old Weight.
Raw Silk the
Pound.
Silk Nubs the
Pound.
Wool the Weight
The Sack how
much.

Last.	1	Sack.				
Sacks.	12	1	Wey.			
Weycs.	24	2	1	Todd.		
Todds.	155	13	6 1/2	1	Stone.	
Stones.	312	26	13	2	1	Clove.
Cloves.	624	52	26	4	2	1
Pounds.	1268	364	182	28	14	7

A Table of
Wool Weight.

A Pack of *Wool* contains but 240 lb. that is, 2 Hundred Weight, and 16 lb. over, less by 124 lb. than the Sack of *Wool* by the Statute.

Yarne, called *Irish Yarne*, by the Book of Rates Inwards is accompted, 1 Pack 4 Hundreds, 1 Hundred 120 lb.

Besides the Weights allowed as before-mentioned, there hath been allowed at the Kings Custom House for Tare (which is the Weight of the Cask or Wrappers, where Goods are packed up) as followeth.

Difference between the Pack and Sack of Wool.
Yarn the Pack Hundred.
Tare, What Allowance for it at the Custom-House.

Upon Butts of Currance per Cent.	14
Caritels of Currance per Cent.	16
Quarter Rowles of Currance per Cent.	18
Prunes, 6, or 7 C.	84
Prunes 10 C. and upwards	112
Raisins Solis, per Cent.	12
Malaga Raisins, per Piece	3 1/2
Figgs the Barrel	10
The 3/4 Barrel	8
The 1/2 Barrel	6
The 1/4 Barrel	4
Mather the Bale	28
Bales of Raw-Silk from	
<i>Aleppo</i> , with Cotton Legee, the Bale	34 1/2
<i>Ardus</i> , the Bale	32
<i>Smirna</i> , the Bale	14
<i>Messina</i> , the Bale	8
Bales of Grogran-Yarne from	
<i>Aleppo</i> , the Bale	28
<i>Smirna</i> , the Bale	16
Bales of Silk from	
<i>Naples</i> , the Bale	14
<i>Bologna</i> , the Bale	30
Spanish Tobacco the Barrel	28
The Half Barrel	20
Sugar Chests	part.
Sugar in Fatts, 6 C. weight.	84
Goods packed up in Paper.	
For Paper and Packthread, per Cent.	2

What allowed
by Merchants.
Cloff what.
Hundred at
Londonderry.
Forraign Geo-
deticals.

Moreover there is an Overweight allowed by Merchants called Tret, which is 4 lb. upon every Hundred of 112 lb. And also 2 lb. upon every Scale of 3 C. weight, which is called Cloff, but in many Places if not conditioned for, will not be allowed. At Londonderry in Ireland 140 lb. is reckoned for 1 Hundred weight.

The English Accompt of Measures and Weights passed, an Eye may be cast now on the Forraign Accompt of smaller Geodeticals, whether Ancient, or Modern, and the Credit of both must depend on the respective Authours out of which they are here Collected.

Of the Hebrews
Greeks and
Latins to what
compared.
Jews Value of
their Gold.

Ancient Measures and Weights to avoid prolixity are referred only to Hebrew, Greek, and Latine as before, most of which are here compared to our Winchester Measure, and Troy Weight, and the Money valued by our Sterling Coin at the rate of 5 s. the Ounce of Silver, and 3 l. the Ounce of Gold, though some say the Jews valued their Gold but 10 times as much as their Silver. And this is one cause why some Authours differ in the value they put on their Hebrew Coins.

A Table of the
Hebrew Mea-
sures.

Hebrew Measures.

Hebrew Measures.	Long	(a) <i>Estbang</i> , A Fingers Breadth, an Inch.
		(b) <i>Tophach</i> , A Palm or Hands Breadth, 4 Fingers, or Inches.
		(c) <i>Zereth</i> , A Span.
		(d) <i>Pagnam</i> , A Foot, or 12 Inches.
		(e) <i>Ammah</i> , A Cubit { Common, Half a Yard. Holy, A Yard. Kings, Half a Yard and 3 Fingers. Geometrical, Three Yards.
		(f) <i>Tsugad</i> , A Pace, Five Feet.
		(g) <i>Orgyia</i> , A Fathom, Six Feet.
		(h) <i>Chebel</i> , A Cord, Line, or Rope to measure Land with.
		(i) <i>Kaneh</i> , A Reed, common 6 Cubits, 6 Cubits and a Palme.
		(k) <i>Stadium</i> , A Furlong, 125 Paces.
	Broad	(l) <i>Cibrath haarets</i> , Half a Dayes Journey, &c.
		(m) <i>Milliarium</i> , A Mile, 1000 Paces.
		(n) <i>Parasang</i> , 30 Furlongs.
		(o) <i>Noph</i> , A Clime, or Tract of Land 60 Feet every way.
		(p) <i>Maanath</i> , An Acre, in length 240, in breadth 120 Feet.
	Dry.	(q) <i>Kab</i> , A Quart, &c.
		(r) <i>Omer</i> , Three Pints and an half.
		(s) <i>Seah</i> , A Gallon and an half.
		(t) <i>Ephab</i> , Four Gallons and a Pottle.
		(u) <i>Lethec</i> , Two Bushels, 6 Gallons and a Pottle.
	Liquid	(w) <i>Homer</i> , } Five Bushels and Five Gallons. Cor,
		(x) <i>Log</i> , Half a Pint.
		(y) <i>Hin</i> , Three Quarts, &c.
		(z) <i>Bath</i> , Four Gallons, and an Half.

Hebrews begin
their Dry Mea-
sures with Bar-
ley, and Wet
with Eggs.
Shiur used for
an Accompt.
Estbang, the
Opinions thereof
Zithe how ta-
ken.
Tophach how
reckoned.
Cith used for
what.
Zereth how
much.

(a) *Weemse* in his *Christian Synagogue*, tells us, the beginning of their Dry Measure was Barley, and their Wet Eggs, and therefore saith he, An Accompt is called *Shiur*, from *Shiur* Barley. And one *Estbang* by him and others, contained the breadth of 6 Barley-Cornes in their greatest thickness. Others again but the space that 2 laid end to end, or 4 laid close side by side, will lye in. And in round reckoning (though not exactly) passed for an Inch. For 4 Fingers make 3 of our Inches, as most generally accompt. *Junius* on *Ezek.* 40. 5. and *Ser.* 52. 21. *Holyoke* on the Latine word *Pollex*, saies an Inch is 1 1/2 Fingers breadth, if so, then should 3 Inches be 4 1/2 digits, or Fingers breadths. Some of the Rabins call a Fingers breadth *Zithe*.

(b) The lesser Palm or Hands breadth, 4. *Estbangs*, *Exod.* 28. 16. and 37. 12. *Ezek.* 40. 5. 2 *Chron.* 4. 5. may be accompted 3 Inches English Measure. With some of the Rabins, *Cith*, is used for the Measure of the Wrist, to the Roots of the Fingers, which is somewhat more than 4 *Estbangs*.

(c) The greater Palme 3 *Tophachs*, *Exod.* 28. 16. *Isa.* 40. 12. properly a Span, and by the 70 rendred *Σπθαυή*, in *Ezek.* 43. 13. containing the length between the Thumb and the Top of the little Finger stretched out.

(d) *Pagnam*, 4 *Tophachs*, or 16 *Estbangs*, *Peter Martyr* in 1 *Kings* 6.

(e) A Cubit

(c) A Cubit, some say, is the length from the Elbow to the Wrist, others to the top of the longest Finger, some making it the 4th, others the 6th part of a Man, allowing some 2 Foot, most 1 ½ Foot, to the Common Cubit or Cubit of a Man so called, *Deut.* 3. 11. to which the Great Cubit is reckoned double, 1 *King.* 7. 15. with 2 *Chron.* 3. 15. See it called the Great Cubit, *Ezek.* 41. 8. and the Cubit of the first Measure, 2 *Chron.* 3. 3. The Kings Cubit in *Herod. lib.* 2. in *Descrip. Babyl.* mentioned to be 3 Fingers longer than the Common Cubit. The Geometrical Cubit, *Origen Hom.* 2. in *Gen. Augustine de Civit. Dei, lib.* 15. cap. 27. take for the Measure used in Building Noah's Ark, And to this Rabbi *Cambi* in his Comment on *Ezek.* 46. 2. cited by *Arias Montanus de Mensuris Sacris*, comes near assigning 1000 Emoth or Cubits to make a Mile, And some say, this is the Cubit used in Egypt. *Gomed, Judg.* 3. 16. taken by Interpreters for the same with. *Ammah* a Cubit.

*Ammah the
Sorts of Cubits.*

*Gomed how
rendred.*

(f) *Tsafad*, or *Tsaad* mentioned often, but because never measured in the Text, as to the certainty must remain unknown. Several Authours make Two sorts of Paces, the *Minor* of 2 ½ Feet, a Step, or Half a Remove of the Body; the *Major* of 5 Feet, a Stride or a Pace by removing both Leggs from the Heele at the first Stand to the Toe at the last.

*Tsafad, uncer-
tain.*

*Two sorts of
Paces.*

(g) A *Fathom* in Greek *ὄσ πλά*, greatly questionable if ever any Measure with the *Hebrems*, because not once mentioned in the *Old Testament*, and but twice in the *New*, in one Verse, *Viz. Acts* 27. 28. is as much of a Rope or Line as a Man can include between the tops of his longest Figures, when the Arms are stretched out at length in a right line, and so uncertain according to the length of the Armes fathoming, but generally taken for 2 Yards or 6 Feet *English* Measure. Some promiscuously taking it for the Pace give it but 5 Feet. Others render it in the *Latine*, *Ulna*, but then must not be taken for our Ell, which is but 3 Feet 9 Inches. The *Fathom* is mostly used at Sea to measure their Ropes and Soundings, wherein they do not strictly take the length from Finger to Finger, but so much as shall be included holding the Rope in the Hands extended between the Thumbs and Fore-Fingers.

*Fathom how
much.*

*When rendred
Ulna must not
be taken for our
Ell.*

*Fathom used
at Sea.*

(h) Most confess their Ignorance in the length of the *Chebel*, *Psal.* 16. 6. taken Metonymically for the Inheritance it self.

*Chebel, uncer-
tain.*

(i.) Used to measure Buildings, as the *Chebel*, Lands, expressed *Ezek.* 40. 5. to contain 6 Cubits, and an Handsbreadth, but *Tremelius* on the place takes the Reed to contain so much of the Kings Cubits, though the Common Reed, by the *Targum* and several others is accounted but 6 Cubits just. *Salel* a *Rabinal* word for a Reed of 6 Cubits, and *Rus* for 70 Reeds, to be found in their Writings.

*Kanch of what
use.*

The length.

*Salel and Rus
how taken.*

(k) A *Furlong*, (*quasi*, *Furrowlong*, because in Champion Countreys their Furrows were usually very long) not mentioned in the *Old Testament*, a Measure brought in with the *Græcian* Monarchy as seemeth, because first met with in 2 *Macc.* 12. 9. continued till the *New Testament* Times, and there often the compute of distances at Sea and Land, with *Pliny, lib.* 2. cap. 23. *Isidore* and others made to be 125 Paces, and so must be the Eighth part of an *Italian* Mile, which contains 1000 Paces, though the same *Pliny* will have 7 ½ Furlongs make a Mile. There is no doubt but *Stadium* hath admitted of different acceptations, as the Controversie between *Pliny* and *Diodorus Siculus* testifies, So also *Stadium* because of the different content thereof in several Countreys with Authours is respectively to be taken, besides the *Italian* which contained as before 125 Paces, or 625 Feet, sometime for the Olympique at 120 Paces, or 600 Feet, and sometime for the *Pirbique Stadium*, at 200 Paces, or 1000 Feet.

*Stadium the
Furlong from
whence.*

The Length.

*Difference
thereof.*

(l) Half a Dayes Journey, a Dayes Journey, a Sabbath-Dayes Journey, and a Space less than any of them, a Bow-shot, all left indeterminate in the *Old Testament*, the *New* *Acts* 1. 12. *John* 11. 18. accompts the Sabbath-Dayes Journey about 15 Furlongs, which doubled in a Journey forward and backward made about 4 Miles. So *Fuller, Pil. Sight of Palestine, Book* 1. p. 43. 44.

*Cibrath, &c.
differently ta-
ken.*

(m) The *Italian* Mile consisting of 1000 Paces, hath set the Name *Mile* from the *Latine Mille* in *English* 1000. Nevertheless in several Countreys more than 1000 Paces go to make up a Mile, and the *German* Mile is 4 times as much as the *Italian*, and though *Buxtorf* renders the word *Cibrath*, *Miliare*, yet we find no Mile mentioned by Translators of the *Old Testament*, *Matt.* 5. 41. argues it rather of *Roman* Extraction than *Jewish*, and *Kimchi lib. Rad.* thinketh that *ב* in *Cibrath* is but *Servile*, and that the Root is *Barab*, or as some write *Berab*, which *Tremelius, Gen.* 35. 16. and 48. 7. and 2 *King* 5. 19. readeth it *Exiguum terræ Spatium*, and our Translation, a little way. Nevertheless some Learned Men conceive *Berab* answers to the proportion of a *Roman* Mile, but *Barab* properly signifying a Dinner or Meale. Others will when applied to Journeys take it for such a space of Ground, as usually is travelled or conveniently may be gone in Half a Day, between Meal and Meal, or Bait and Bait.

*Miles of differ-
ent length.*

*Barab Berab
what.*

(n) This

Parasang how
taken.

(n) This seems to be a *Persian* Word and Measure, taken for 30 Furlongs, *Herodor. lib. 2. Ramus Geometry* set forth by *Bedwell* in *English*, by others for a *German* Mile. *Elias* in *Thisbi* mentioneth a *Persah*, which he saith was the Great Mile, and contained 4 lesser.

Noph how
rendred.

(o) *Noph*, rendred by *Buxtorfe*, A Clime, or Tract of Land, *Psal. 48. 2. Noph*, also the name of a City in Scripture, but the content thereof not there found, other Writings, as *Holyokes Dictionary*, &c. reckons it 60 Feet every way, yet may be a Plot of ground big enough for a small Town.

Maanath di-
versly taken.

(p) Used 1 *Sam. 14. 14.* and made by several fitly to correspond with *Jugerum* in the *Latine*, but differ in the quantity, some reckon it 200 Foot every way, others 240, in length, and 120 in breadth. *Quint. 1. 28. Isidor. 15. 15.*

Kab what it
contained.

(q) The fourth part of a *Kab*, 2 *Kings 6. 25.* for want of more accurate Correspondencies may be taken for our Half Pint, and so the *Kab* for our Quart. *Buxtorf* out of *Rab. Alphes. Tract. de Paschate. cap. 5. fol. 176.* with whom divers conclude, that the fourth part of a *Kab*, and a Log are of a like quantity, and that each contained 6 Eggs, viz. as much as will fill Six Hen Egg-shells of the ordinary, or middle size, and thence called by some a *Sextary*, and so accordingly the *Kab*, 24 Eggs, or 4 Logs.

Called a
Sextary.
Omer how
much.

(r) An *Omer*, was the Dayly Ordinary of a Man, and the tenth part of an *Ephah*, *Exod. 16. 16. 36.* more than 3 $\frac{1}{2}$ Pints our Measure, and not full a Pottle, 43 $\frac{1}{2}$ Eggs.

Seah the Con-
tent.

(s) A *Seah*, noted *Gen. 18. 6.* and 2 *King. 7. 1.* commonly estimated by Writers at 6 *Kabs*, that is 2 Hins, or 144 Eggs, or about a Gallon and an Half our measure. *Godwin* in *Moses and Aaron, lib. 6. cap. 9.*

Ephah the
Content.

(t) An *Ephah* contained 3 *Seahs* or 18 *Kabs*, in Eggs 432 about 9 Pottles *English* Measure, it was the tenth part of an *Homer*, *Ruth 2. 17. Ezek. 45. 11.*

Lether its con-
tent.

(u) *Lether*, in *Hosea, 3. 2.* is reckoned to be the Half of an *Homer* or 5 *Ephahs*, and consequently 90 *Kabs*.

Homer and
Cor how much.

(w) An *Homer*, being 10 *Ephahs*, *Ezek. 45. 11.* is thought by some to be the ordinary Burden of an Ass, but with us a good Horseload, at the Rates aforesaid 5 Bushels and 5 Gallons, our Measure. The *Cor* was equal to the *Homer*, common to measure both Liquids and Dry. *Ezek. 45. 14. Luke 16. 7.*

Log the Con-
tent.

(x) The *Log* was the smallest Measure for Liquids, we find mentioned, see *Levit. 14. 12.* and before in the *Kab*.

Hin how much.

(y) The *Hin* often in Scripture, in quantity Three Quarts our Measure or thereabouts, 3 *Kabs*, or 72 Eggs. This Measure was divided into the Half, Third, Fourth, and Sixth parts, *Numb. 15. 6, 9. and 28. 5. Ezek. 4. 11.*

Bath and
Solomons Sea
their Contents.

(z) The *Bath*, *Ezek. 45. 14.* is made alike to the *Ephah*, so must the Molten Sea, 2 *Chron 4. 5.* holding 3000 Baths, contains 210 Quarters, 7 $\frac{1}{2}$ Bushels, or 13500 Gallons our Measure

Nebel how
translated.

As for *Nebel*, translated *Jer. 13. 11.* a Bottle, few reckon it a Measure except *Epiphanius* who saies it contains 150 Sextaries.

A Brief view of most of the Hebrew Measures aforesaid, with their English Content may be taken in the following Table.

The Table of Hebrew Long Measures.

A Table of Hebrew Long Measures compared with the English.

	1	8	6	833 $\frac{1}{3}$	1000		5000		60000	Milliarium
	Mile.	1	1	104 $\frac{1}{2}$	125	h	625		7500	Stadium
	Milliarium.	Furlongs	1	1 $\frac{1}{2}$ $\frac{3}{4}$	1 $\frac{1}{2}$ $\frac{7}{8}$	lis	9 $\frac{1}{4}$	s.	111	Kaneh
Mill.	1	Stadium	Reeds	1	1 $\frac{1}{2}$	ng	6	re	72	Orgya
Stadium.	8	1	Kaneh.	Fathoms	1	E	5	fu	60	Tfagad
Kaneh.	540 $\frac{2}{3}$ $\frac{5}{7}$	67 $\frac{2}{3}$ $\frac{1}{7}$	1	Orgya	Paces.	1	1 $\frac{1}{2}$	ca	13	Ammah c.
Orgya.	833 $\frac{1}{3}$	104 $\frac{1}{2}$	1 $\frac{1}{2}$ $\frac{3}{4}$	1	Tfagad	Cubits	1	M	12	Pagnam
Tfagad.	1000	125	1 $\frac{1}{2}$ $\frac{7}{8}$	1 $\frac{1}{2}$	1	Ammah c.	Foot	1	9	Zereth
Ammah Com.	3333 $\frac{1}{3}$	416 $\frac{2}{3}$	6 $\frac{1}{2}$	4	3 $\frac{1}{2}$	1	Pangam	Palmes	3	Tophach
Pagnam.	5000	625	9 $\frac{1}{4}$	6	5	1 $\frac{1}{2}$	1	Zereth.	Inches	
Zereth.	6666 $\frac{2}{3}$	833 $\frac{1}{3}$	12 $\frac{1}{3}$	8	6 $\frac{2}{3}$	2	1 $\frac{1}{3}$	1	Tophach	
Tophach.	20000	2500	37	24	20	6	4	3	1	
Eftbang.	80000	10000	148	96	80	24	16	12.	4	

The Table of Hebrew Dry and Liquid Measures.

A Table of Hebrew Concave Measures compared with the English.

	5 $\frac{5}{8}$	11 $\frac{1}{4}$	22 $\frac{1}{2}$	45	90	180	360	720	Homer, Cor.
	Bushels	5 $\frac{5}{8}$	11 $\frac{1}{4}$	22 $\frac{1}{2}$	45	90	180	360	Lethec.
	HomerCor	Tovits	2 $\frac{1}{4}$	4 $\frac{1}{2}$	9	18	36	72	Ephah, Bath.
Homer, Cor.	1	Lethec	Pecks	1 $\frac{1}{2}$	3	6	12	18	Seah.
Lethec.	2	1	Ephah, Bath	Gallons	1 $\frac{1}{2}$	3	6	12	Hin.
Ephah, Bath.	10	5	1	Seah	Pottles	1 $\frac{4}{5}$	3 $\frac{3}{5}$	7 $\frac{1}{5}$	Omer.
Seah.	30	15	3	1	Hin	Quarts	2	4	Kab
Hin.	60	30	6	2	1	Omer	Pints	1	Log, $\frac{1}{4}$ Kab.
Omer.	100	50	10	3 $\frac{1}{3}$	1 $\frac{2}{3}$	1	Kab	Half Pints	
Kab.	180	90	18	6	3	1 $\frac{4}{5}$	1	Log	
Log, $\frac{1}{4}$ Kab.	720	360	72	24	12	7 $\frac{1}{5}$	4	1	
Eggs.	4320	2160	432	144	72	43 $\frac{1}{5}$	24	6	

Hebrew Weights.

The principal Weights in use among the Jews were Talents, Pounds, Shekels, Hebrew Drams, and other small divisions of the Shekel, all which may be further seen in the Weights. Account of their Money.

A Table of the
Hebrew Coines
and their Value.

Hebrew Coines.

		l.	s.	d.		
Hebrew	Money	Silver	(a) Shekel, of the Sanctuary in { Weight $\frac{1}{3}$ Troy. ————— } Value to Sterling Money. ————— }	00	02	06
			(b) King's Shekel, Half the Sanctuary Shekel, called Bekah. ———	00	01	03
			(c) Third part of a Shekel. —————	00	00	10
			(d) Zuz, Fourth part of a Shekel. —————	00	00	07½
			(e) Gerah, Agorah, Keshtah. —————	00	00	01½
		Gold	(f) Zahab ————— } each $\frac{1}{3}$ Troy in value 12 Silver }	01	10	00
			Golden Siclus, or Shekel } Shekles of the Sanctuary ————— }			
			(g) Adarkon. ————— } each half so much. ————— }	00	15	00
			Dracmon, or Darcon. } ————— }			
	Sums of Money	(h) Maneh, or Mina, or a Pound, valuable in { Gold 100 Shekels. } 25 $\frac{1}{3}$ Troy. }	75	00	00	
		{ Silver, 60 Shekels. }	07	10	00	
		{ 30 $\frac{1}{3}$ Troy. }				
		(i) Chichar, or Talent, { 3000 Shekels } valuable in { Gold — } 4500 00 00 in Weight { 125 lb. Troy } { Silver — } 375 00 00				

Shekel how in-
scribed and
stamped.

Whence the
word.

Silverling
what.

Bekah how
much, and for
what paid.

Third part of a
Shekel.

Zuz, how much.

Gerah, or
Agorah the
Image of a
Lamb thereon.

Zahab a Gold
Coine how
reckoned.

A Darkon or
Drakmon
seem. a Persian
Coin.

Maneh, Mina,
how much.
Difference
about it.

(a) This piece of Coin (is often mentioned in Sacred Writ, and looks like the Standard of all the rest) on the one side shewed the Vessel in which the *Manna* was, inscribed with this Perigraph *Shekel Israel*, on the other side, the Rod of *Aaron* that budded, with this Inscription, *Jerusalem Kodshash*, in English, *The Holy Jerusalem*, called *Shekel* from *Shakal* to weigh, for Money at the first seems to be a Merchandise exchanged or given for other Commodities, as *Gen. 23. 16.* after the *Chaldee* called *Silgha*, and commonly with the *Hebrews*, *Keseph*, i. e. *Silver*, and being put absolutely rendred a *Silverling*, or piece of Silver, by Expositors; as usually as a *Shekel* when the quality is not mentioned is taken for a *Silver Shekel*.

(b) First mentioned, *Gen. 24. 22.* paid by all as a Yearly tribute, *Exod. 30. 13. 15. 2 Chron. 24. 6. 9.* towards the Repair of the Tabernacle first, and after of the Temple, between this and the Sanctuary Shekel, some mention a third sort of Shekel, called the Common Shekel, valued at 20 d. of our Money, of which I find nothing but uncertainty.

(c) In *Nehem. 10. 32.* noted as a yearly tribute given of the Jews by a Civil Decree to the second Temple.

(d) 1 *Sam. 9. 8.* spoken of, and equal to the Old Attick Dram, and Roman Penny, as several say, and by some called Zuz, Zuzza, Zur, and Zura.

(e) Agorah, rendred Gerah by the Chaldee Paraphrase, and also Megna, or Megha, in that Tongue, by the Arabians, Megah, Greeks, Obolus, and in English, a Piece of Silver, 1 *Sam. 2. 36.* called also in the Hebrew, Keshtah, because signed with the Image of a Lamb, See *Gen. 33. 19. Joshua 24. 32.* Twenty of these made a Shekel. *Exod. 30. 13.*

(f) The Zahab, and Golden Shekel in Rider's Dictionary undervalued at 15 s. for though Hunt in his *Handmaid to Arithmetick* out of *Brerwood de nummis*, mentioneth the Weight but 2 Attick Drams, yet he valueth it at 30 s. Alsted in his *Encyclopædia of Arithmetick* makes it 4 Drams, which accompted $\frac{1}{3}$ Troy, must yield double that value of 15 s. and most Authours agree that the Gold Shekel was equal in weight with the Silver Shekel, so the difference must be only in the value of the Metall. Some to keep up the Credit of differing Authours conceive there were 2 sorts of Golden Shekels as well as Silver, the one double to the other.

(g) These Golden Adarkons, and Drakmons seem to be Persian, or Coines of some other Nations, and Currant, not Coined in Judea, not read of till after the Captivity, for though the word be used 1 *Chron. 29. 7.* yet both Books of the Chronicles, as most take it were penned by Ezra, after his return from Babylon, in whose Book mention is made thereof, as in *Ezra 2. 69. and 8. 27. &c.* in Greek called Drachme, and in English, rendred Drams. Alsted, in his *Encyclopædia of Arithmetick* makes the value equal to a Ducat of Hungary. But Hunt who throughout his Book hath much lost himself in his Method; forgot himself in valuing 2 Attick Drams in the Zahab at 30 s. and 2 Attick Drams in the Adarkon (which he saith is the weight thereof) at 15 s.

(h) The Maneh in Gold by comparing 1 *King 10. 17.* with 2 *Chron. 9. 16.* is found to be 100 Shekels, which Buxtorf and others understand not of the Holy but the Royal Shekel. The Maneh in Ezek, 45. 12. seems to be 60 Shekels, and hereto several agree, but some think it was now increased 10 Shekels more than of old, and call it, The New Maneh, valued

valued nevertheless with most after the *Holy Shekel*, in Silver at 7 lb . 10 s. *Buxtorfe* tells us also of a *Maneh* of 25 *Holy Shekels*. The *Assemblies Annotations* on *Ezek.* 45. 12. make 3 sorts of *Manehs*, viz. Common 15, the Kings 20, and the Holy 25 *Shekels*, but the word *Maneh* is Singular and not Plural in the *Hebrew Text* there, as noting all those divisions to make but one *Maneh*.

(i) The *Chichar*, by some wrote *Kichar*, and commonly translated *Talent*, contained 3000 *Shekels*, as may be collected from *Exod.* 38. 25, 26. where the Silver Collected is expressed to be 100 Talents, and 1775 *Shekels*, v. 25. and the Persons that paid it at half a *Shekel* a piece, v. 26. numbered to be 603550, whereof the half is 301775; which divided by 3000, the *Shekels* in one *Talent*, makes 100 Talents, and leaves the odd *Shekels* remaining 301775(100

Talent, how much.

The *Talent* thus valued after the *Holy Shekel*, makes the Number of Talents mentioned in some Texts of Scripture, especially 1 *Chron.* 22, and 29 *Chapters* amount to such Massy Sums, that some think the Talents are to be reckoned at the rate of the other *Shekel*; and others, not improbably that the *Jews* had a piece of Money or Plate of Gold of small value (as may be observed anciently in *Homer Iliad*, lib. 23.) called a *Talent*. And *Fuller* in his *Pisgah Sight of Palestine*, Book 3. p. 356, 357. shews whereon such an Opinion may be strengthened, and that the *Talent* mentioned in some Scriptures may be rather this than the other.

Doubts thereof.

Talent, a piece of Coin.

The Account both of the Hebrew Weight and Money, with their Value in Sterling-Money and Troy-Weight, is contracted into the following Table.

A Table of Hebrew Gold and Weights.

Hebrew Gold and Weights.

Chichar, or Talent.	1 Maneh.					
Maneh, or Pound.	old	new	1		Zahab.	
Zahab, or Shekel.	60	50	new	old	1	Adarkon.
Adarkon, or Dram.	3000	60	50		2	1
Troy-Weight.	6000	120	100		$\frac{1}{2}$	$\frac{1}{4}$
Sterling-Money.	125 lb .	2 $\frac{1}{2}$ lb .	2 lb 13		$\frac{1}{2}$	$\frac{1}{4}$
	4500 l.	90 l.	75 l.		11. 10s.	15 s.

Hebrew Silver and Weights.

A Table of Hebrew Silver and Weights.

Chichar, or Talent.	1 Maneh.					
Maneh, or pound.	old	new	1		Shekel.	
Shekel.	60	50	new	old	1	Bekah.
Bekah.	3000	60	50		2	1
Third parts.	6000	120	100		$1\frac{1}{2}$	Third part.
Fourth Parts, Zuzins, or Drams.	9000	180	150		3	1
Gerahs.	12000	240	200		4	$1\frac{1}{3}$
Troy-Weight.	60000	1200	1000		20	10
Sterling Money.	125 lb .	2 $\frac{1}{2}$ lb .	2 lb 13		$\frac{1}{2}$	$\frac{1}{4}$
	375 lb .	75 l.	61. 5s. 2s. 6d.		15. 3d.	10d.

The like Method may be observed in viewing the Measures, Weights and Monies of the old *Greeks*, who imitating the *Hebrews* kept some like theirs, altered and added others.

Greeks, their Geodæticals.

Græcian Long Measures.														
Schoenes	Parasang.	Dolich.	Mile.	Hippacon.	Diaulus.	Furlong.	Plethron.	Fathome.	Pace.	Cubit.	Pygon.	Pygme.	Foot.	Span.
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
7½	7½	7½	7½	7½	7½	7½	7½	7½	7½	7½	7½	7½	7½	7½
15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
30	30	30	30	30	30	30	30	30	30	30	30	30	30	30
60	60	60	60	60	60	60	60	60	60	60	60	60	60	60
375	187½	75	50	25	12½	6¼	1	1	1	1	1	1	1	1
6250	3125	1250	833½	416½	208¼	104¼	16½	1	1	1	1	1	1	1
7500	3750	1500	1000	500	250	125	20	1	1	1	1	1	1	1
25000	12500	5000	3333½	1666½	833¼	416½	66½	4	4	4	4	4	4	4
30000	15000	6000	4000	2000	1000	500	80	4	4	4	4	4	4	4
33333	16666½	6666½	4444½	2222½	1111¼	555½	88½	5½	5½	5½	5½	5½	5½	5½
37500	18750	7500	5000	2500	1250	625	100	6	6	6	6	6	6	6
50000	25000	10000	6666½	3333½	1666½	833¼	133¾	8	8	8	8	8	8	8
51545½	27272½	10909½	7272½	3636½	1818¼	909¼	145½	8½	8½	8½	8½	8½	8½	8½
60000	30000	12000	8000	4000	2000	1000	160	9½	9½	9½	9½	9½	9½	9½
150000	75000	30000	20000	10000	5000	2500	400	24	24	24	24	24	24	24
500000	300000	120000	80000	40000	20000	10000	1600	96	96	96	96	96	96	96
450000	225000	90000	60000	30000	15000	7500	1200	72	72	72	72	72	72	72

A Table of Long Measures of the Greeks, and the English Inches therein.

Schoen how different.

A Schoen, in Greek Σχοῖνος, generally taken with Herodot. lib. 2. to contain 60 Furlongs, Pliny lib. 12. cap. 14, reckons it 40, some say 32, others but 30, an Egyptian Measure as some think, Scapula in Verb.

Parasang. Vide antea.

A Parasang, See before in the Hebrew Measures p. 84.

Dolich how taken.

A Dolich, some will have 24 Furlongs, but the Common Account is but 12, Scapula in Verb.

Mile, Vide antea.

A Mile, called μίλιον, Mat. 5. 41. by birth a Latine, though in use a Word and Measure with the Greeks, Hebrews, Syrians, &c. See before in Hebrew Measures, p. 84. also Leigh Crit. Sacra gr.

Hippacon the Length, and Diaulus.

An Hippicon, commonly taken for 4 Furlongs.

A Diaulus, for half an Hippicon.

A Furlong, See before in the Hebrew Measures, Stadium, p. 83.

Plethron uncertain.

A Plethron, Scapula from Plutarch reports an Acre from Suidas to be of a Furlong, or an 100 Feet, from Hesychius, a Measure of 10000 Feet. of others 100 Furlongs, such uncertainty there is in the Measure or discrepancy in the Authours. In the Table followed Suidas, at 100 Feet, which occasion the following Numbers in the Table differ from those in Alsted and some other Authours.

A Furlong

A *Fathom*, See before in the *Hebrew Measures*, *Orgyia*, p. 83.
A *Pace*, in Greek *βῆμα*, See before in the *Hebrew Measures*, *Tsafad*, p. 83.
A *Cubit*, after the measure of the Common Cubit with the *Hebrews*. See before in their Measures *Ammah*, in Greek *πῆχυς*.
A *Pygon*, taken with *Hesychius* to be a Measure containing the space from the Elbow to the Fingers bent, called by some *Palmipes*, of a Foot, and a Palm, being 20 Fingers breadth, *Scapul. in verb.* and others.
A *Pygme*, used *Mark 7. 3.* in measure taken for the length from the Elbow to the Fingers closed, as the Hand is contracted when it is called a Fist, *Leigh Crit. Sacra.* *Greek.* 2 Fingers breadth shorter than the *Pygon*.
A *Foot*, in Greek *πῦς*, the same with the *Hebrew Pagnam*, though *Hunt*, upon what Authority I know not, will have the *Greek πῦς* less than the *Roman Foot*, $\frac{1}{2}$ Inch, and greater than the *Hebrew Pagnam* almost $\frac{1}{4}$ Inch. But *James Capel* a Man of far more exactness in his Treatise *De Mensuris intervallorum*, makes the *Attick Foot* and the *English* to agree as near as 75 to 76, and the *Roman* and *English Foot* as 18 to 19. And the Learned *Willebrand, Snellius* of *Leiden* in his *Eratoſthenes Batavus* (who some think comes nearer the Truth) makes the

English

Roman

Old Greek

}

Foot agree as

484.

500.

521.

A *Span*, in Greek *σπθαυή*, in *Latine*, *Palmus major*, and *Dodrans*, by *Poll. lib. 2.* *Hesych.* and others alwaies accompted 12 Fingers breadth, like the *Hebrew Zereth*, answering to 9 of our Inches. One calls this Measure a *Graciary*.
An *Orthodoron*, *Poll. lib. 2.* calls a *Palm*, some others a *Span*, shorter by a Fingers breadth, than the *Span*, or greater *Palme*.
A *Lichas* is generally reckoned for the length between the Thumb and the Extent of the Fore-finger, shorter than *Orthodoron* by a Fingers breadth. Some make it the same with *Dichas*, but *Cooper* in his *Dictionary* makes *Dichas* but 8 Fingers breadths, when most agree *Lichas* is a *Span* with the Thumb and Fore-Finger as before.
A *Palest*, in Greek *Παλαίστη*, also *Δάκτυλ*, is the less *Palme*, agreeing to the *Hebrew To-phach*, 4 Fingers breadths, answering to 3 of our Inches.
A *Daetyl*, Digit or Fingers breadth: See *Etſbang* in the *Hebrew Measures*.
The *Greeks* had few if any Land-Measures of length and breadth notable, save what they borrowed of the *Hebrews* and *Latines*, *Plethron* before spoken of, is often rendred *Jugerum* the Old *Latine* word for an *Acre*, which gives occasion to some to think, that the Measures aforesaid, or many of them were considered both in breadth and length, as necessity served to make use of them. Wherefore passing over what might be further said of those, the next that come in order to be seen are Measures of length, breadth, and depth.
Whether by confounding the *Attick*, and *Roman Sextaries*, or the Pounds Mensural or Ponderal, or the *Attick* and *Georgick Measures*, or by what other occasions, I know not; but sure I am, it is hard to reconcile Authors one to another, and some to themselves about the capacious Measures of the *Greeks*, and being not willing to spare so much time, or tumifie these Papers, I have given much credit to the Accompt set down by *Alsted* out of *Daniel Angelocrator. lib. de Ponderibus.* and from him and others wherein most general agreement is to be found, have collected what follows.
Greek Measures of capacity may be considered, as *Indigenital* or of most use and chief note among them. Or, 2. *Exotick*, or used but in some particular places. Or Thirdly, *Hippiatrical*, or used about the Cure of Beasts.
Indigenital, or Proper *Græcian Measures* are again considered as useful for things dry only, or liquid, or both, and those either *Attick* or *Georgick*.

Fathom,
Pace and Cu-
bit, *Vide*, an-
tea.

Pygon, how
taken.

Pygme, the
length.

Pous, a Foot,
like Pagnam.

Spithama for
how much ta-
ken.

Orthodoron,
the length.

Lichas, how
taken.

Palest, the
Length.

Daetyl, what.

Plethron, oft
rendred in La-
tine Jugerum.

Authors hard-
ly reconciled a-
bout the Græ-
cian Measures.

Capacious
Measures of the
Greeks of 3.
sorts.

Indigenital,
Attick, or
Georgick.

Græcian Indigenital Capacious Measures.

Dry	Common	Liquid.
Kypſele.	Sextary.	Metretes.
Medimnus.	Kotyle.	Amphora.
Modios.	Oxybaph.	Chous.
Choenix.	Kyath.	Tetarton.
	Concha.	
	Mystrum.	
	Chemes.	
	Cochlear.	

A Table of the Attick Measures of the Greeks compared with the English.

The Table of Attick Measures compared with the English.

Kypfele.	I	Medimnus } Metretes }											
Medimnus } Metretes }	6	I	Modion.										
Modios.	36	6	I	Chous.									
Chous.	72	12	2	I	Choenix.								
Choenices.	288	48	8	4	I	Sextary.							
Sextaries.	432	72	12	6	$1\frac{1}{2}$	I	Kotyle.						
Kotyles.	864	144	24	12	3	2	I	Tetarton.					
Tetartons.	1728	288	48	24	6	4	2	I	Oxybaph.				
Oxybaphs.	3456	576	96	48	12	8	4	2	I	Kyath.			
Kyaths.	5184	864	144	72	18	12	6	3	$1\frac{1}{2}$	I	Concha.		
Conchas.	10368	1728	288	144	36	24	12	6	3	2	I	Mystrum.	
Mystras.	20736	3456	576	288	72	48	24	12	6	4	2	I	Cheme.
Chemes.	25920	4320	720	360	90	60	30	15	$7\frac{1}{2}$	5	$2\frac{1}{2}$	$1\frac{1}{4}$	I
Cochlears.	51840	8640	1440	720	180	120	60	30	15	10	5	$2\frac{1}{2}$	2
Pints or Pounds Troy.	648	108	18	9	$2\frac{1}{4}$	$1\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$

A Table of the Georgick Measures of the Greeks compared with the Attick.

The Table of Georgick Measures compared with the Attick.

Medimnus } Metretes }	1	Amphora.										
Amphoras.	2	1	Chous.									
Chous.	8	4	1	Choenix.								
Choenices.	48	24	6	1	Sextary.							
Sextaries.	72	36	9	1½	1	Kotyle.						
Kotyles.	96	48	12	2	1½	1	Tetarton.					
Tetartons.	192	96	24	4	2½	2	1	Oxybaph.				
Oxybaphs.	384	192	48	8	5½	4	2	1	Kyath.			
Kyaths.	576	288	72	12	8	6	3	1½	1	Concha.		
Conchas.	1152	576	144	24	16	12	6	3	2	1	Mystrum.	
Mystras.	2304	1152	288	48	32	24	12	6	4	2	1	Cheme.
Chemes.	2880	1440	360	60	40	30	15	7½	5	2½	1¼	1
Cochlears.	5760	2880	720	120	80	60	30	15	10	5	2½	2
Attick Co- chlears.	12600	6480	1620	270	180	135	67½	33¾	22½	11¼	5½	4¼

Kypfele the content.
Medimnos both reckoned.

Kypfele, or after the *Latine*, *Cypfele*, by *Scap.* out of the *Annotat. Schol. Aristoph.* is reckoned for a Corn-Measure, and by *Plum* to contain 6 *Atticke Medimnos*.
Medimnos, or *Medimnus* both *Attick* and *Georgick*, *Suidas*, *Scapula*, *Legat*, and several others agree to contain 48 *Choenices* or 72 *Sextaries*, and *Pellybins lib. 4.* affirms the

The *Hemimedimnus* to be 24, *Choenices* accordingly; but because the *Georgick Coenix* was bigger than the *Atticke*, the *Medimnus* was alike proportional.

Metretes, or *Metreta*, a Liquid Measure, rendred in Latine sometime *Cadus*, sometime *Amphora*, (but corruptly *Amphora* being another Measure) *Legat* and some others make equal with the *Atticke Medimnus*, but that all other *Georgick* Measures should be greater than the *Atticke*, and only the *Metretes* equal, seems unlikely, used *John* 2. 6. in *English* there rendred a Firkin, but how justly, *quare*.

Amphora, or *Amphoreus*, properly a *Georgick* Liquid Measure, and was half the *Georgick Medimnus* or *Metretes*, by *Alsted*, though *Schrevelius* mentions it as an *Atticke* Measure, and saies it contained 3 Urnes, 20 whereof made a *Roman Cule*.

Modios, or *Modion*, both Name and Measure seem borrowed from the Latine *Modius*, used in *Mat.* 5. 15. *Mark* 4. 21. *Luke* 11. 33. ordinarily Englished a Bushel, but in quantiry far less than the *English* Bushel. Neither may the Latine *Modius* and Greek *Modios* agree exactly, though each should contain a like number of *Sextaries*, if the *Sextaries* be different. *Alsted* reckons it but 8 *Atticke Coenices*, that is 12 *Sextaries*, as in the Table above. Great Annotations 16 *Sextaries*, and by some, a Pint less than our Peck.

Chous, wrote often *Cbus*, sometime *Choas*, and confusedly *Congius* for a *Roman* Measure of that Name, was both *Atticke* and *Georgicke*, that contained 6 *Atticke Sextaries*, and this 9 *Georgicke Sextaries*, as most agree.

Choenix, mentioned *Rev.* 6. 6. taken for a Measure serving a Servant with Food enough for a day, whence that saying of *Pythagoras*, *Super Choenice non Sedendum*, intending the provident care that should be taken for the future. The *Attick Choenix* contained by severall 1½ *Atticke Sextaries*, the *Georgicke Choenix* 2¼ *Atticke Sextaries*, whereby the *Georgicke Sextary*, and by consequence, all the lesser *Georgicke* Measures are proportionally greater than the *Atticke*, and accordingly the former Tables are computed. But there are others, and of good note too, that reckon the *Choenix Georgicke* to contain but 2 *Sextaries Atticke*. Some mention a Triple *Choenix*, as *Bilibras*, *Quadrilibras*, and *Quinquelibras*.

Sextary, Sometime *Xesta*, from the Greek *ἕξαις*, the sixth part of an *Atticke Chou*, made 2 *Kotyles*, but the *Georgicke Sextarie* though bigger made but 1½ *Georgicke Kotyle*, a Measure translated a Pot, *Mark* 7. 4. and though usurped sometimes for the *Roman* Measure of that Name, yet upon more exquisite search, questionable; since some affirm 11 *Atticke Sextaries* made 12 *Roman*. *Alsted* and several others make the *Attick Sextarie* contain half the *Augustane* Measure and 2 Ounces over, that is 18⅓ *Mensural*. Some reckon it 20⅓, others 24⅓, that is 1½ Pound, at 16⅓ to the Pound, as 18⅓ at 12⅓ to the Pound. Others will have it 13⅓ 7 pwt. 18 gr. *Troy*. Some 1½ Pint our Measure. Others but half a Pint, and equal to the *Hebrew Log*.

Kotyle Atticke is half the *Sextary*. *Scap.* out of *Dioscorides* & *Heraclitus* make it equal with the *Roman Hemina*, and if so, then must the *Roman* and *Atticke Sextaries* be equal. And with *Thuc. apud. Athen. lib.* 11. is made to contain 9 *Mensural* Ounces. Of the *George Kotyle* see above, with the *Latines* wrote *Cotyle*.

Tetarton, was properly a Liquid Measure in Latine *Quartarius*, being the Quarter of the *Atticke Sextary*, but the *Georgicke Sextary* is 2½ *Georgicke Tetartons*.

Oxybaph, was a Vessel to pour Vinegar in to dip Meat into at the Table, as the Latine *Acetabulum*, if *Georgicke*, was the Eight part of the *Choenix*, but if *Atticke*, the Twelfth.

Kyath, in Latine *Cyathus* 1½ whereof whether *Atticke*, or *Georgicke* made 1 *Oxybaph*, nevertheless in quantity proportioned to the respective *Sextaries*. A *Kyath* was used at *Athens* for a little Drinking Cnp.

As for the *Concha*, *Mystrum* and *Cheme*, (measures more Minute than the *Kyath*) the Tables follow *Alsted*, *Scapulus*, and others, yet there are not wanting that speak of the uncertainty of them as Measures, make 2 sorts of them a greater and a less, and divide the *Kyath* otherwise than above.

As *Malines*, in his *Lex Mercatoria*, thus.

- 1 Kyath { 1 Great Concha.
- 2 Small Conchas.
- 3 Great Mystras.
- 4 Small Mystras.
- 5 Chemes.
- 10 Dragma, Cochlears.

Legat, at the end of *Thomas* his Dictionary, thus,

- 1 Oxybaph. = Greater Concha.
- 1 Kyath. = Lesser Concha.
- 1 Kotyle = { 16 Greater Mystras.
- 20 Lesser Mystras.
- 1 Kotyle. = { 20 Greater Chemes.
- 30 Lesser Chemes.

The lowest Rank in the Table of *Georgick* Measures accompts the Number of *Atticke* *Cochlears* or *Spoonfuls* in every of the said *Georgick* Measures at the rate of $2\frac{1}{4}$ of the one for $1\frac{1}{2}$ of the other. And the lowest rank in the Table of *Atticke* Measures, values their Content with the *English* Pint or Pound of $12\frac{3}{4}$ proportionally to $1\frac{1}{2}$ lb for the *Atticke* *Sextary*, wherein at present I am best satisfied.

A Table of
Exotick Mea-
sures of the
Greeks compa-
red with their
Atticke.

Græcian Exoticke Measures compared with the Atticke.

	Kypsele.	Med.	Mod.	Chous.	Choen.	Sext.	Kot.	Tetar.	Oxy.
1 <i>Achana Persica</i> —————	7	3							
2 <i>Metreta Syria</i> —————	0	1	4						
3 <i>Artaba Persica</i> —————	0	1	0	0	3				
4 <i>Kypros</i> —————	0	1	0	0	0				
5 <i>Ægyptia Artaba</i> —————	0	0	5	0	0				
6 <i>Medimnus Kyprius</i> { <i>Salam</i> 0 ——— 0 ——— 5 ——— 0 ——— 0									
{ <i>Papho</i> 0 ——— 0 ——— 4 ——— 1 ——— 0									
7 { <i>Collathum Syrium</i> } ——— 0 ——— 0 ——— 2 ——— 0 ——— 0 ——— 1									
{ <i>Modios Pondicus</i> }									
8 <i>Ponticus Cyprus</i> —————	0	0	2	0	0	0			
9 <i>Sabitha Syria</i> —————	0	0	1	1	2	1			
10 <i>Mares Ponticus</i> —————	0	0	1	1	1	$\frac{1}{2}$			
11 <i>Kophinus</i> —————	0	0	1	1	0	0			
12 <i>Modios Kyprios</i> —————	0	0	1	0	3	$\frac{1}{2}$			
13 { <i>Kampsaces</i> } ——— 0 ——— 0 ——— 1 ——— 0 ——— 0 ——— 0									
{ <i>Tetarpe Laconice</i> }									
14 <i>Dadix</i> —————	0	0	0	1	2	0			
15 { <i>Aphin</i> } ——— 0 ——— 0 ——— 0 ——— 0 ——— 4 ——— 0									
{ <i>Topium</i> }									
16 <i>Choenix Syria</i> —————	0	0	0	0	2	1			
17 { <i>Capitha</i> } ——— 0 ——— 0 ——— 0 ——— 0 ——— 2 ——— 0									
{ <i>Mares</i> }									
18 <i>Inion</i> —————	0	0	0	0	0	1			
19 <i>Elenius</i> —————	0	0	0	0	0	0	0	1	
20 <i>Gabcnon</i> —————	0	0	0	0	0	0	0	0	0
21 <i>Alabastron</i>									

Achana-Persica what.
Metreta of *Syria* the content.
Artaba-Persica the content.

Artaba of *Ægypt* the content by *Hierome*.

Kypros the content.

Artaba of *Ægypt* the content by *Hierome*.

Collathum and *Pontick Modios* the content.
Pontick Cyprus the content.
Sabitha the content.
Kophinus the content.

Modios Kyprios the content.

1. A *Persian* Corn Measure, as *Hesychius* testifieth.
2. Expressed by *Legat* to contain 120 *Sextaries*, which is all one with 1 *Medimnus* & *Modios*.
3. Most from the Authority of *Herodotus*, lib. 1. pag. 49. agree the *Persian Artaba* was 3 *Choenices* greater than the *Atticke Medimnus*, And whence *Hunt* in his Table of *Græcian* Liquid Measures should make it less then the *Metreta* by 3 *Chous* is to measure known. *Hierome* on *Isaiah* 5. cap. writes the *Ægyptian Artaba* was 20 *Modios*.
4. Equal to the *Atticke Medimnus* saith *Legat*, after the Latine wrote *Cyprus*.
5. The same Author tells us with *Fannius* this Measure is but $3\frac{1}{3}$ *Modios*, with *Epiphanius* equal to the *Atticke Medimnus*, as also the *Median Artaba*.
6. The *Medimnus* at *Papho* is less by half a *Modion* than that at *Salamina*, though both *Kyprian* Measures.
7. Both the *Collathum Syrium* and the *Pontick Modios* are counted to contain alike 25 *Atticke Sextaries* in our Measure, as say the Great Annotations a Peck and a Portle others 12 Ounces a quarter and a half more.
8. The *Pontick Cyprus* after *Epiphanius* is 2 *Modios* as above.
9. *Sabitha Syria*, is accompted to contain 22 *Atticke Sextaries* all one as above.
10. *Mares Ponticus*, *Epiphanius* delivers to contain 20 *Alexandrian Sextaries*, which if different from the *Atticke*, the Content above must be corrected accordingly.
11. *Kophinus*, a Poetick Measure both of Liquid and Dry, according to *Legat*, contains 3 *Corgios*, and in our Measure 1 Gallon, half a Pint, 3 Ounces $\frac{1}{2}$, but if the *Corgius* be 5 Pints $1\frac{1}{4}$ $\frac{3}{4}$ *English* as he saith *Kophinus* must contain more than a Peck by a Pint $3\frac{1}{4}$ Ounces, and near a Peck, if by *Corgius* the *Atticke Chous* be understood. Wherefore of the certainty, quære.
12. *Modios Kyprios*, or after the Latine *Modius Cyprinus*, is reckoned to contain 17 *Atticke Sextaries*; of our Measure by the Great Annotations, a Peck and a Pint; by *Legat*, but 14 Pints, 5 Ounces, a quarter and half.
13. *Kampsaces*

13. *Kampfaces*, and the *Tetarpe Laconicè* are equal, the one being 12 *Sextaries*; the other 24 *Kotyles*, seeing 2 *Kotyles* make but 1 *Sextary*, but some make the *Kampfaces* but 4 *Sextaries*. The *Tetarpe Laconicè* seems to be the quarter of the *Laconian Metretes*. Kampfaces and Tetarpe, their Contents.

14. *Dadix*, by *Pollybius* and others containeth 6 *Choenices*. *Malines* calls it a *Boetick Measure*. Dadix the Content.

15. *Aphin*, an *Ægyptian Measure*, containing 4 *Choenices*, and of the same capacity doth *Hesychius* account the *Topium*, but with whom in use he saith not. Aphin and Topium their Contents.

16. The *Syrian Choenix*, is supposed to be the same which with *Fannius* is set down at 4 *Sextaries*. Choenix Syria the Content.

17. The *Capitha*, a *Persian Measure* contained 2 *Attick Choenices*, and the *Mares* was equal thereto, containing 6 *Kotyles*, and a *Boetick Measure*, as some say. Capitha and Mares their Contents.

18. *Inion*, with the *Ægyptians*, as *Legat* saith, was the name of a *Sextarie*, which with the *Alexandrians* contained 2 $\frac{1}{16}$ of *Oyl*, as *Epiphanius* hath it. But if by Pound he intend, the *Roman Libra*, or the *Greek Mna*, it must not be taken for our Pound; since some affirm neither of them weighed 11 Ounces *Troy*. Inion how taken.

19. *Elenius*, being the quarter of the *Sextarie* seems only another name for the *Tetarton*. Elenius the Content.

20. *Gabenon* was all one with the *Oxybaph*, or *Aretabule*. Gabenon and

21. *Alabastron*, contained one of their Pounds of *Oyle*. Alabastron the Contents.

Græcian Hippiatrical Measures seem for the most part to keep the Names of the *Atticke Measures*, though the Divisions and quantities differ, in *Alsted* thus found.

12 Ounces hath 2 *Oxybaphs*, 1 *Oxybaph*, 3. *Kyaths*. 1 *Kyath* 4 *Mystras*. 1 *Mystrum* 2 *Cochlears*. Hippiatrical Measures of the Greeks.

In one Ounce 8 Drams, 1 Dram 3 Scruples, &c.

Legat, out of *Absyrtus*, pag. 34. and *Hierocles* pag. 35. mentions the *Choe* to contain 10 Ounces of Liquid Measure, which if Mensural, then was the *Choe* lesser than the *Hippiatrical Kotyle*, but if Ponderal equal; for he saith the *Roman Mensural Pound* (to which the *Hippiatrical Kotyle* was equal) contained so much Oyle, as 10 Ponderal Ounces weighed. Choe mentioned by Legat.

In like manner as the Measures, so the Weights among the *Greeks* are differently to be taken; as they are *Attick*, *Physical*, *Hippiatrick*, *Indigenital*, or *Exotick*. Of which see further the following Tables and Notes on the same. Weights of the Greeks as their Measures of divers sorts.

Græcian Attickè Weights.

A Table of the Atticke Weights.

		Pounds. Minas.	Ounces. Uncias.	Drams. Drachmas.	Scruples. Grammata.	Lupines. Thermes.	Keratias. Siliquas.	Aereolos. Chalkos.	Graines. Sitar.	Minutes. Leptas.
Talent	Greater.	80	1000	8000	24000	48000	72000	144000	288000	576000
	Lesser.	60	750	6000	18000	36000	54000	108000	216000	432000
Mina.	New		12 $\frac{1}{2}$	100	300	600	900	1800	3600	7200
	Old.		9 $\frac{3}{8}$	75	225	450	675	1350	2700	5400
a		Uncla. Ounce.		8	24	48	72	144	288	576
		b	Drachm. Dram.		3	6	9	18	36	72
		c	Gramma. Scruple.		2	3	6	12	24	48
		d	Obolus.		1 $\frac{1}{2}$	3	6	12	24	48
		e	Therme. Lupine.		2	4	8	16	32	64
		f	Siliqua. Keration.		2	4	8	16	32	64
		g	Chalkus. Aereolus.		2	4	8	16	32	64
		h	Sitar. Graine.		3 $\frac{1}{2}$	7	14	28	56	112
		i								
		k								

A Table of Weights used by the Græcian Physitians.

Græcian Physical Weights.

	Ounces.	Drams.	Scruples.	Carobseeds.					
	Uncias.	Drachmas.	Grammata.	Obolos.	Lupines.	Keratias.	Aereola.	Graines.	Minutes.
Mina.	16	128	384	768	1152	2304	4608	9216	32256
Litra.	12	96	288	576	864	1728	3456	6912	24192
aa	Uncia.	8	24	48	72	144	288	576	2016
	Ounce.								
		Drachm.	3	6	9	18	36	72	252
		Dram.							
			Gramma.	2	3	6	12	24	84
			Scruple.						
				Obolus.	1½	3	6	12	42
					Lupine.	2	4	8	28
						Siliqua.	2	4	14
						Keration.			
						Carobseed.	2	4	7
						gg	Æreolum.		
								Sitar.	3½
								Graine.	

l

A Table of Weights used by the Græcian Farriers.

Græcian Hippiatrical Weights.

	Ounces.	Denarions.	Drams.	Scruples.	Obolos.
Mina.	15	84 3/8	112 1/2	337 1/2	675
Litra.	12	67 1/2	90	270	540
aaa	Ounce.	5 5/8	7 1/2	22 1/2	45
		Denarion.	1 1/3	4	8
		bb	Dram.	3	6
			cc	Scruple.	2

ll

Notes on the Table.

- Mna the sorts how much.
- Mna of the Physitiam.
- Mna of the Farriers.
- Oungia, Uncia not the English Ounce.
- Denarion how much.
- Drams the Names and sorts.
- Scruples how called.
- a The Mna of 100 Drachms is called Solons, Mna, because thought to be constituted by him, sometime turned into Latine by Mina, often by Libra, though Libra be 4 Drachms lighter; the Roman Libra being but 96 Attick Drachms. The old Mna of 75 Drachms now obsolete for Memory sake hath found room in the Table.
- aa. The Physitians, as by Dioscorides and Galen appears. used a Mna, or Pound of 16 Ounces, and a Litra or other Pound of 12 Ounces, conceived all one with the Roman Libra consisting of 96 Drachms as this did, and by Interpreters commonly rendered Libra, and seldom or never Mna, and Mna, and Litra, as also Libra, commonly Englished a Pound.
- aaa. The Hippiatricks had a Mna of 15 Ounces, and a Litra of 12.
- b. Oungia, in Latine Uncia, must not be taken for our Ounce, but for one of their Ounces, arising by the division of their Pound into Drams differently according to the quantity of Drams in one Pound.
- bb. Among the Hippiatrical Weights there was a Denarion of 4 Scruples, 5 1/2 whereof made one of their Ounces.
- c. Drachme, Drachma, and Dragma, in Greek and Latine, in English a Dram, is the eight part of their Ounce, whether the Pound had 12 or 16 Ounces therein. By Alsted made to equal the German Weight Quintlein. Some call a Dram Resolus, some Holke, from the Greek δολιχον.
- cc. The Ounce Hippiatrick, that divided as well the Mna of 15 Ounces, as the Litra of 12 Ounces; had but 7 1/2 Drams in it.
- d Drams of all sorts were parted into 3 Scruples. A Scruple in Greek sometime Gramma, sometime Grammata, in Latine Scripulum, Scriptulum, and Scrupulum.

e. Obolus

e. *Obolus*, Sometime a Weight, sometime a piece of Money commonly rendred an Half-penny, because alwaies was the half of a Scruple.

f. *Lupine*, in *Greek Thermos*, was a Weight equal in poise to the *Lupine*, which is a Seed growing in a Cod like to a Pease, and both Plant and Seed bear that name. And seeing there are many sorts as *Parkinson Theater of Plants*, pag. 1073. which sort of *Lupine* is meant is uncertain, probably, the Middle White, which are most in use, bigger than the Yellow, and not so big as the great Blew, and from the nearness in Weight thereto, if not exactness might be so called.

g. *Siliqua*, in *Greek Keration*, a Weight alike heavy to the *Carobseed* or Sweet Bean, common in many Countreys subject to the *Gracian* Empire. Sometime called *Carat* or *Caract*, from whence the word still in use with us.

h. *Chalkos*, in *Latine Aereolus* and *Aereolum*, *Aereolus* was also a piece of Brasse Money currant in Antient times among those Countreys of the *Gracian* Dominion.

i. *Sitar*, a Grain of Corn from *Σίτος*, *Frumentum*, likely to have been the Original of their Weight. 2 whereof made 1 *Chalkos*.

k. *Lepton*, from *Leptos*, in *Latine Minutum*, and *Minutia*, supposed to be some small Scale of the Rinde or Bark of some Tree, 3 ½ ballanced the *Sitar*

l. Besides these in the Table of Physical Weights, some Books mention the *Affarion*, allowed for 2 Drams which is ¼ of an Ounce. Also the *Exagion*, wrote sometime *Stagion*, sometime *Agion*, for brevity, which was the *Roman Sextula*, the Sixth part of their Ounce, whereof 12 made the *Litra*. Likewise *Orobis* which was a graine of a Wild Vetch. And *Phaïke* a *Lentill*, but whether Weights or no is not worth the Inquiry.

ll. As the other Weights are divided into lesser Divisions than the *Obolus*, so no doubt but the *Hippiatrick* also were, and may accordingly be done, when occasion serves. The *Obolus* of all sorts admitting the like smaller Denominations.

Gracian Exotick Weights.

A Table of the Exotick Weights of the Greeks.

Talents	{ Mentioned by <i>Vitruvius</i> , supposed to be the <i>Thracian</i> , or <i>Bizantium</i> Talent ————— }		120	} Libras.
	{ Several mentioned by <i>Hesychius</i> ————— }		100	
			125	
			165	
			405	
Talent of	{ Old } <i>Sicilian (m)</i> —————		1150	} Minas.
	{ New } —————		24	
	{ <i>Alexandria</i> ————— }		12	
	{ <i>Aegina</i> } —————		12000	} Attick Drams
	{ <i>Corinth</i> } —————		10000	
	{ <i>Egypt</i> ————— }		8000	
	{ <i>Babylon</i> ————— }		7000	
	{ <i>Rhodium</i> ————— }		4500	
	{ <i>Euboicum</i> ————— }		4000	
	{ <i>Syria</i> ————— }		1500	
Mna	{ <i>Alexandria</i> ————— }		20	} Uncias.
	{ <i>Ptolemaica</i> ————— }		18	
	{ <i>Egyptia</i> ————— }		1	Obolus.

(m) The *Sicilian* Old and New Talent is thus reckoned by *Legat* before-mentioned, but *Rider* and another Author make them pieces of Money, and of a far smaller Value, set afterward among the *Gracian* Coines.

A Table of
Græcian Coines
and their Va-
lue.

		l. s. d.		
Græcian	Money	Brafs	Mite	00 00 00
			Aereolus	00 00 00
			Quadrans { Oboli	00 00 00
			Assis	00 00 00
			Assarius	00 00 00
			Semiobolus	00 00 00
			Danaces	00 00 00
			Obolos { Attick	00 00 01
			Aeginean	00 00 02
			Diobolus	00 00 02
			Triobolus { Attick	00 00 03
			Aeginean	00 00 06
			Cistophorus	00 00 04
			Tetrobolus	00 00 05
			Drachma { Attick	00 00 07
			Aeginean	00 01 00
			Siglus, Sardinian and Persian	00 00 10
			Didrachma	00 01 03
			Tridrachma	00 01 10
			Stater { Attick	00 02 06
			Corinthian	00 01 08
			Macedonian	00 02 09
			Semistater { Attick	00 07 06
			of Darius	00 07 06
			Stater { Attick	00 15 00
			of Darius	00 15 00
			Stater Macedonian	00 18 04
			Stater Cizycen	01 01 00
			Tetra-Stater	03 00 00
			Mna Attick	03 02 06
			Rhegium	00 00 03
			Sicilian { New	00 01 10
			Old	00 03 09
			Neapolitan	00 03 09
			Syrian	04 17 06
			Euboicum	12 5 00
			Rhodium	14 0 12
			Attick { Less	18 7 10
			Greater	25 0 00
			Babylonian	21 8 15
			Egyptian	25 0 00
			Aeginean	3 12 10
			Corinthian	3 12 10
			Alexandrian	37 5 00

Mite how cal- A Mite in Latine *Minutum* and *Minutia*, in Greek *Lepton* used *Mark* 12. 42. and
led, the Value. from thence proved to be half the Quadrant, (but not half of our Farthing) by the
Syriack Interpreter, and *Alsted* reputed, of the *Assarion*. It weighed $\frac{1}{2}$ Barley Corn
as some say, and was currant for as much as $\frac{1}{2}$ of our Penny, but some will have it
twice as much.

Aereolus the Aereolus, and Aereolum, several agree to be the weight of 2 Graines, and call it
Value. Chalkos, and according to the Attick Weight, weighed 7 of their *Leptas*, or *Mites*,
and was valuable with $\frac{1}{4}$ of our Penny, being the 36th part of their Dram, worth
7 $\frac{1}{2}$ d. Sterling.

Quadrans the The Quadrans, commonly translated a Farthing, *Mat.* 5. 26. *Mark* 12. 42. in
Sorts and Value. Greek *κοδράντις*, by some is taken either for the Fourth part of the *Obolus*, or the *Assis*;
not a Farthing. and so accordingly valued as before. The latter was the double of the Mite. The
Ass of the Ro- Ass was the 10th part of the Roman Penny, which being 7 d. made the Ass $\frac{1}{2}$ d.
mans how and consequently the 4th part thereof $\frac{1}{4}$ d.
much.

Assarius the Assarius, or Assarium, *Holyokes Dictionary* makes the 4th part of the Assis equal
difference about to the Quadrans; but *Leigh* makes it equal to the Assis it self; for he saith, it is the
it how called 10th part of the Roman Penny, the 96th part of the Attick Stater that is with us but 1
and translated. Farthing. Others make it more, and say it was 1 Farthing and an Half, which I ra-
ther

ther incline to, and so have set it down, of old called *Affar*, and by the *Rabins*, *Iffor*, *Mat.* 10. 29. translated a Farthing. *Alsted* likewise makes *Affarium* worth 1 *Cruciat* or *Creutzer*, that is 3 Farthings *Sterling*, all one with the *Roman Ass*, counting 40 *Creutzers* to a *Stater*, which is 2 s. 6 d. and so 10 to the *Drachmal Denary*.

Semiobolus, is $\frac{1}{12}$ of an *Attick Dram* (of which below) that is 2 $\frac{1}{2}$ Farthings our Money, or $\frac{1}{6}$ of a *Peny*.

Semiobolus
how much.

Danaces, in Greek $\Delta\acute{\alpha}\nu\alpha\kappa\eta$, $\Delta\acute{\alpha}\nu\alpha\kappa\eta$, and $\Delta\acute{\alpha}\nu\alpha\kappa\eta$, *Charon's Ferriage-Piece*, which the *Barbarians* used to put into the Mouths of Dead Persons to pay *Charon* for their Carriage over the River *Styx* into the *Elisian Fields*. If it be an *Obolus* as *Lucian* calls it, it is worth 1 $\frac{1}{2}$ d. our Money. But if of the same weight which the *Greek* and *Arabian* Writers call the *Arabian Danich*, weighing $\frac{2}{3}$ of an *Obolus*, is $\frac{1}{3}$ of a *Peny*, that is 3 q, and $\frac{1}{3}$ of a Farthing our Money.

Danaces, used
to be put into
the Mouths of
Dead Persons.

Obolus, is twofold; the *Attick* which is $\frac{1}{12}$ of their *Dram* in Value with us 1 $\frac{1}{2}$ d. the *Aeginean* almost double the other, *Viz.* 2 d. and $\frac{1}{12}$ of a *Peny*. *Holyoke* thinks *Obolus* came from *Obelos*, which sometimes signified a *Dart* used in War, as being stamped with the like Form: Or was so called from the Oblong Form thereof, or from the Image of some *Obelisk*, or *Spire* coined thereon.

Obolus, the va-
lue, why so
called.

Diobolus, was $\frac{1}{6}$ of the *Attick Dram*, or double the *Attick Obolus*, and had on the one side *Jupiters Face*, and on the other an *Owle*.

Diobolus, how
stamped, the
Value.

Triobolus, was both *Atticke* and *Aeginean*, that just $\frac{1}{2}$ *Dram*, the other 3 *Aeginean Oboli*, that is 6 $\frac{1}{4}$ d. in Value.

Triobolus, the
Value.

Cistophorus, so called from the Form of a *Coffer* or *Chest* thereon, valued in *English* Money 4 $\frac{1}{2}$ d. and a quarter of a Farthing; by *Holyoke* and others who set not down the weight thereof.

Cistophorus
how stamped,
the Value.

Tetrobolus, had *Jupiters Face* on the one side, and 2 *Owles* on the other, contained 4 *Oboli*, or $\frac{1}{3}$ of the *Attick Dram*, worth with us 5 d.

Tetrobolus the
Value, what
Print thereon.

Drachma, or *Drachme*, used *Luke* 15. 8, 9. Sometime *Arguris*, in *English* a *Dram*, *Acts* 19. 19. a *Silverling*, a Piece of Money common with the *Athenians*, bearing the Image of *Minerva's Candle* burning, in weight $\frac{1}{8}$ of an Ounce, and accordingly valued at 7 $\frac{1}{2}$ d. *Sterling* at the rate of 5 s. *Sterling* the Ounce. This was called the *Attick Drachme*, and was all one as very many conceive with the *Roman Peny*. The *Aeginean Dram* was heavier, and so worth more the weight 1 $\frac{2}{3}$ $\frac{1}{3}$ *Attick*, Value 12 $\frac{1}{2}$ d. *English*.

Drachma, the
Names, Sorts,
and Values.
The Image
thereon.

Siglus, was of *Exotick Extract*, and weighed 1 $\frac{1}{3}$ $\frac{1}{3}$ or 4 *Attick Scruples*, may be valued at 10 d.

Siglus the
Value.

Didrachmum, called also by the *Athenians Bous*, or *Boos*, because there was an *Ox* stamped thereon, whence the Proverb, *Bos in Lingua*, as the *English*, *The Angels blind their eyes*, applyed to them that are bribed to speak, or blinded in Judgment, equal to 2 *Drams*, or $\frac{1}{2}$ their *Silver Stater*, and was $\frac{1}{4}$ of our Ounce *Troy*, and worth 1 s. 3 d. all one with the *Hebrew Bekah*, paid by the *Jews* to the Sanctuary, and Temple, till *Cæsar* changed it into *Tribute-Money* for his own *Coffers*, *Mat.* 17. 24. and afterwards by Vertue of a Decree made by *Vespasian* paid towards the *Roman Capitol*.

Didrachmum
how stamped
and called.
Proverbs,
whence.

Didrachmum
Exacted by
Cæsar.

Tridrachmum, was 3 *Drams Attick*, and valued with us accordingly.

Tridrachmum
the Value.

Stater of Silver, was either *Attick*, having on the one side *Minerva's Head*, and an *Owle* on the other, worth with us 2 s. 6 d. weighing $\frac{1}{2}$ Ounce; Or *Corinthian*, which was not full 3 *Attick Drams* and worth but 1 s. 8 $\frac{1}{2}$ d. Or *Macedonian*, which was bigger than either, and worth 2 s. 9 $\frac{1}{2}$ d. The *Attick Stater* was double the *Didrachmum*, and so served for *Tribute-Money* both for *Christ* and *Peter*, *Matth.* 17. 27. and is sometimes called the *Tetradrachmum*, because it contained 4 *Drams*.

Stater of Silver
how stamped;
the Value and
Sorts.

The *Semistater of Gold*, both the Common *Attick*, and that of *Darius* Coine are reckoned equal either of them 1 $\frac{1}{2}$ s. valuable with us, 7 s. 6 d.

Tetradrachmum.

The *Staters* also in Weight equal 2 *Drams* of our *Troy Weight*, or $\frac{1}{4}$ $\frac{1}{3}$ worth 1 s. That of *Darius* is reported to have the Image of *Sagitaris* thereon.

Semistater of Gold the Value.
Staters the
Sorts and
Values.

The *Macedonian Stater* weighed of *Attick Weights* 2 $\frac{1}{2}$ s. 2 ob. 2 *Siliq.* worth proportionally with us 1 s. 4 d.

Of *Darius*.
Macedonia.
Cizycen.

The *Stater of Cizycen* or *Cizycus*, a City in *Greece* 2 $\frac{1}{2}$ $\frac{1}{3}$ *Attick*, was valuable in *Sterling Money* at 1 l. 1 s. 0 d.

The *Tetra Stater*, seems to some no piece of Coine, but signifies only Four *Staters*, worth 3 l. *Sterling*.

Tetra Stater.

Mna Attick, containing 100 *Drams*, 96 whereof being equal to the *Pound Troy*, make the whole *Mna* in *Silver* at the rate as worth with us 3 l. 2 s. 6 d.

Attick Mna
how much in
Account.

Talents the
Account of
them.

The Sorts and
Weight.

Talents of the Lesser sort, and improperly so called, seem to me rather Pieces, than Sums of Money. That of *Rhegium* a Town in *Italy*, currant in *Greece*, was in Value but 3 $\frac{1}{2}$ d. of the New and Old *Sicilian Talents*, the Old was double the New, and the biggest worth no more than that of *Naples*, to wit 3 s. 9 d. *English Money*.
Talents of the Greater sort, and indeed deserving that name, were divers as before noted with their respective Value in our Silver Money, according to their weight of the *Attick Drams* to which they are compared, being some of them *Exotick* as here followeth.

		lb.	3.
Syrian	1500	15	7 $\frac{1}{2}$
Euboicum	4000	41	8
Rhodes	4500	46	10 $\frac{1}{2}$
Attick Less	6000	62	6
Babylon	7000	72	11
Attick Great	8000	83	4
Egyptick		104	2
Aegina	10000	125	0
Alexandria	12000		

Drams Attick < Troy.

Geodæticals
of the Latines
and Romans.
Measures of the
Latines.

The Third and last sort of the Ancients whose *Geodæticks* are to be seen, are the *Latines*, and their Successors the *Romans*.
Alsted fits us with Tables for the Long and Superficial Measures, and another which he calls *Geometrical*; wherein the main differences between the other Long Measures, and these are about the Mile and Furlong. A Fourth Table also he hath for division of the Inch, all which here follow.

A Table of the
Long Measures
of the Latines.

Latine Long Measures.

	Furlongs	Decempede.	Passes.	Steps.	Cubits.	Palmipede.	Feet.	Palme.	Inches.	Digit.
Mile	8	500	1000	2000	3333 $\frac{1}{3}$	4000	5000	20000	60000	80000
a Furlong.	62 $\frac{1}{2}$	125	250	416 $\frac{2}{3}$	500	625	2500	7500	10000	
Decempede.		2	4	6 $\frac{2}{3}$	8	10	40	120	160	
b Pass.			2	3 $\frac{1}{3}$	4	5	20	60	80	
Step.				1 $\frac{2}{3}$	2	2 $\frac{1}{2}$	10	30	40	
c Cubit.				1 $\frac{1}{3}$	1 $\frac{1}{2}$	6	18	24		
Palmipede.					1 $\frac{1}{4}$	5	15	20		
d Foot.						4	12	16		
e Palme.							3	4		
f Inch.								1 $\frac{1}{3}$		
g										b

A Table of the
Measures for
Land used with
the Latines.

Latine Superficial Land-Measures.

	Centuries.	Jugera.	Modes.	Verfes.	Climes.	Act.	Feet.
Saltus.	4	400	800	1152	3200	24000	11520000
i Centurie.		100	200	288	800	6000	2880000
Jugrum.			2	2 $\frac{1}{2}$ $\frac{2}{3}$	8	60	28800
k Mode.				1 $\frac{1}{2}$ $\frac{1}{3}$	4	30	14400
l Verfe.				2 $\frac{2}{3}$	20 $\frac{1}{2}$		10000
m Clime.					7 $\frac{1}{2}$		3600
n Act.							480

Latine Geometrical-Measures.

A Table of the Long Measures of the Latines used on special Occasions.

	Miles.	Furlongs.	Cubits.	Feet.	Palmes.	Digits.	Graines.
Parasang, or Schoone.	3	30	12000	18000	72000	288000	1728000
	Mile.	10	4000	6000	24000	96000	576000
	aa	Furlong.	400	600	2400	9600	57600
			Cubit.	1½	6	24	144
				Foot.	4	16	96
					Palme.	4	24
						Digit.	6
							bb

The Division of an Inch.

A Table of the Division of the Inch according to Alsted.

	Drams.	Scruples.	Obolos.	Siliquas.	Points.	Minutes.	Moments.
Inch.	8	24	48	144	288	576	1152
gg	Dram.	3	6	18	36	72	144
		Scruple.	2	6	12	24	48
			Obolus.	3	6	12	24
				Siliqua.	2	4	8
					Point.	2	4
						Minute.	2

a. aa. Hereby it seemeth the Latines had 2 sorts of Miles, viz. The Common consisting of 8 Furlongs, every Furlong 625 Feet, that is 5000 Feet in the Mile; and a Mile called Geometrical, or used in accompt upon Special Occasions, consisting of 10 Furlongs, every Furlong 600 Feet which made the Mile 6000 Feet. In the first reckoning the Mile was shorter, and the Furlong longer than in the second *Milliare*, or *Milliarium* in the Latine for a Mile came from *Mille* 1000, as was said before. Whence the word. Parasang and Schoen. Vide antea. Decempede the length. Paces and Steps the sorts and lengths.

Of the Parasang or Schoene, Furlong, Pace and Cubit, see before in the Hebrew and Greek Measures.

b. A Decempede, some call a Perch, but because they agree it was but 10 Feet long, and so signified by the very name; it cannot be taken for our Perch, which is 6 Feet longer, as before. Some mention a Decempede of 12 Feet.

c. A Step, in Latine, *Gressus*, and *Gradus*, here taken for half a Pace, or 2½ Feet, and not to be Englished a Degree, which terme is most proper for the 360th part of a Circle. Alsted counts upon 3 sorts of Paces or Passes, each of a double difference, thus.

		Feet.	Palmes.
Simple	of the first difference	2	8.
Double		4	16.
Simple	of the second difference	2½	10. This the Grade, or Step.
Double		5	20. This the Pace Geometrical.
Simple	of the third difference	3	12.
Double		6	24.

That double of the first difference be called *Ulna Communis*, or the Common Ell, to difference it from the Cubit of 1 Feet, which he sometime calls *Ulna*. The double of the third difference he calls *Ulna agrestis*, seu *Orgyia*, the Countrey Ell or Fathom. Ulna the sorts.

Palmipes, the Length.

Foot of the Latines and Romans.

how called.

The parts thereof.

Palmes of 2 sorts.

Inch how much the names thereof.

Uncia diversly rendred.

Digit, the Length.

Saltus how taken.

Jugerum, the Content.

Greater than the English Acre.

A Table of the Roman Juger divided by Alsted.

d. A *Palmipes* may be seen before in the *Greek Pygon*, the *Latine* Name shews the Content thereof.

e. To what hath been said already on the *Hebrew Pagnam* and *Greek Pous*, may be added, that little difference with any certainty being observed the *Roman* or *Latine* Foot may be parallel'd with the *English* Foot. The *Romans* called their Foot sometimes a *Pound*, and 2 Foot *Dupondium*, and divided several of their Land Measures into 12 parts called *Unchia*, or *Inches*, of which below at g, and such Inch into 24 *Scruples*, using like Names as for *Weights*.

f *Palmes* are of 2 sorts, though but one set in the Tables, a Greater answering to the *Hebrew Zereth*, and *Greek Spithame*, containing 3 Lesser *Palmes* or 12 *Digits*, The Lesser *Palme* which is placed in the Table contains 4 *Digits* answerable to the *Hebrew Tophach*, and *Greek Paleste*.

g. gg. And Inch in *Latine*, *Pollex*, rendred sometimes a Thumb, because many times of the same breadth, equal to a Digit or Fingers breadth, and a third part of a Digit; The parts of which Inch follow in the 4th foregoing Table into Imaginary Moments. *Uncia*, when relating to Measure is translated an Inch, when to weight an Ounce, sometime wrote *Ouncia*, but whether corruptly, or that it contains 3 Digits or 2 Thumbs, making thereby 1 Thumb or Inch, $1\frac{1}{2}$ Digit, as *Malines* and *Thomas* say, is further to be quæried. *Vide plus at k* on the parts of the *Jugerum*.

h. hh. A Digit or Fingers breadth, answering to the *Greek Dactyle*, and *Hebrew Etsbang* is there spoken of, and here in the *Geometrical* Table made to contain 6 Graines; but in the upper Table to be reckoned only the breadth of 4 Graines of Barley.

i. *Saltus*, Sometime taken for a Grove or Forest, here for a piece of Land, 4 *Centuries*, or 400 *Jugera*, every *Centurie* being 100 *Jugers*.

k. *Jugerum*, commonly translated an Acre, must alwaies be understood for the *Roman* and not *English* Acre, being far larger as containing 28800 Square Feet in the *Area* thereof arising from the Multiplication of 240 Feet in length, and 120 in breadth, when as the *English* Acre containeth but 2640 Feet, which is the Product of 160 Pecks multiplyed by $16\frac{1}{2}$ the Feet in one Perch as before was declared. *Alsted* divides the *Roman Juger* into 12 parts which he calls Inches, and every Inch into 7 parts, as followeth.

	¹	²	³	⁴	⁵	⁶	⁷	Longer.	Shorter.	Arct.	
	Inches.	Semiuncias.	Siliquas.	Sextulas.	Drams.	Semifextulas.	Scruple.	Obolos.			
Juger.	12	24	48	72	96	144	288	576	240	120	28800
	Inch.	2	4	6	8	12	24	48	60	40	2400
		Semiuncia.	2	3	4	6	12	24	40	30	1200
		¹ / ₂	Siliqua.	1 ¹ / ₂	2	3	6	12	30	20	600
			¹ / ₄	Sextula.	1 ¹ / ₂	2	4	8	20	20	400
				¹ / ₈	Dram.	1 ¹ / ₂	3	6	20	15	300
					¹ / ₈	Semifextula.	2	4	20	10	200
						¹ / ₁₂	Scruple.	2	10	10	100
							¹ / ₄	Obolus.	10	5	50
								¹ / ₈	Side.	Side.	Square Feet

Modus how called the Content.

Versus, the Content

Clima, how taken.

Act how called the Content.

l. *Modus* is half a *Juger*, called often *Actus quadratus*, containing 14400 Feet Square, and was so called from the Square Form thereof, being every way 120 Feet.

m. *Versus*, used by *Pliny* for a Square Plot of Ground 100 Feet every way.

n. *Clima*, for the *Hebrew Noph*.

o. An *Act*, called *Actus Minimus*, the least or lesser *Act*, for distinction from *Actus quadratus*, which is 30 times bigger than this lesser *Act*, that a Square, and this an Oblong or Long Square, one side whereof was 4, and the other 120 Feet, or proportionally 10, that the *Area* might be 480 Feet.

Roman

Roman Capacious Measures.

Dry	Common	Liquid.
<i>Modius.</i>	<i>Sextary.</i>	<i>Cule.</i>
<i>Modiolus.</i>	<i>Hemin.</i>	<i>Amphora.</i>
	<i>Acetab.</i>	<i>Urne.</i>
	<i>Cyath.</i>	<i>Congius.</i>
	<i>Ligula.</i>	<i>Quartary.</i>

Capacious Measures of the Romans.

The Table of Roman Dry and Liquid Measures.

A Table of the Roman Capacious Measures.

	Amphoras	Urnes	Modius	Congius	Modiolus	Sextaries	Heminas	Quartaries	Acetables	Cyaths	Ligulas
Cule.	20	40	50	100	240	960	1920	3840	7680	11520	46080
^a Amphora.	2	3	8	12	48	96	192	384	576	2304	
^b Urne		1½	4	6	24	48	96	192	288	1152	
^c Modius			2½	4	16	32	64	128	192	768	
^d Congi.				1½	6	12	24	48	72	288	
^e Modiolus.					4	8	16	32	48	192	
^f Sextarie.						2	4	8	12	48	
^g Hemina							2	4	6	24	
^h Quartarie								2	3	12	
									1½	6	
										4	

i. k.

- a. *Culeus*, and sometime *Coleus*, *Culeum* and *Culleum*, in *Latine* taken also for a Sack, or such like, wherein *Paricides* were wont to be put, and so cast into the River *Tyber*, by the Old Law of the *Roman*. Some mention *Doleum*, and say it contained a *Cule* and an half. *Cule how taken.*
b. *Amphora*, some say was of a Curick Form, and therefore called *Quadrantal*. *Punishment of Paricides.*
c. *Urna*. *Sennertus* in his *Influitions of Physick*, lib. 5. par. 3. sect. 1. cap. 4. affirms to be $\frac{1}{2}$ the *Italian*, $\frac{1}{3}$ of the *Attick Amphora*, making thereby the *Greek* half as big again as the *Italian Amphora*. *Doleum the Content.*
d. *Modius*, Englished a Bushel, was spoken to among the *Greek Measures*, but whereas there it was made to contain 12 *Attick Sextaries*, here, upon the Authority of *Holyoke*, *I egat*, and others, it is made of 16 *Roman Sextaries*. *Amphora how called.*
e. *Congius*, was of a like number of *Roman Sextaries* as the *Greek Chous*, of *Atticke Sextaries*, which may be the Reason why they are sometime taken the one for the word were needless for other. *Urna the Content.*
f. *Modiolus*, mentioned in *Plautus*, but without mention of its capacity, yet *Alsted* makes it the Quarter part of their *Modius*, a diminutive of *Modius*, the very word betrays it, and less than the *Semi-Modius*, or Half Bushel very probably, or else another the same. *Modius Vide antea.*
g. The *Sextary*, and so downward to the *Cyath* are divided alike to the *Attick Measures*. This *Sextary* was called *Italicus Sextarius*, to difference it from the *Greek Sextary*, and all δ *Urbicus*. q. d. the City *Sextarie*, with respect to *Sextarius Castrensis*, which was a *Sextarie* used in the Army, and double to the other. *Congius the Content.*
h. *Hemina* is sometime called *Cotyla*, and *Cotyla Romana*. *Alsted* mentions *Cotyla Italica*, which he saith is 12 *Mensural Ounces*, this seems to be some New, and not the Old, which himself reckons but at 9. *Hemina how called.*
i. Between the *Cyath* and *Ligula*, *Sennertus* placeth a *Mustrum*, which he calls a Com- mon little Spoo- ne, containing half a *Cyath*, as the *Greek Concha*, and may not be con- founded with the *Greek Mustrum*. *Mustrum of Sennertus.*
k. *Ligula*, a *Lingel*, as some *English* it, rather a *Spoon* or *Cochlear*, of which *Sennertus* makes 4. sorts. *Ligula what.*

The Sorts.

The Least containing— $0\frac{1}{2}3$ of a thing of a middle Weight.
The next bigger— 13 .
The Great— $1\frac{1}{2}3$ or 23 .
The Greatest— $0\frac{1}{2}3$.

Roman Measures their account by weight according to Sennertus.

This *Ligula* then may be reckoned for the *Attick Mystrum*, for as 4 of them made one *Kyath*, so 4 *Ligulas* make one *Roman Cyath*.
Sennertus before named accompts the Content of the *Roman* Measures by Weight of Oyle, Wine, or Water, and Honey, as followeth, save only I have proportioned the Weight of the *Ligula* according to the former Table, at the rate of $\frac{1}{4}$ of the *Cyath*, and have inferted the *Modius* and *Modiolus*, which being dry Measures, *Sennertus* omitteth. *Malines*, p. 29. of his *Lex Mercatoria* saies the *Romans* did accompt $10\frac{2}{3}$ Ponderal for $12\frac{2}{3}$ Mensural, and so the *Sextarie* at $18\frac{2}{3}$ should be $21\frac{3}{5}$ and not $21\frac{1}{2}$ as is there set.

	Oyle.			Wine or Water.				Honey.				
	lb	3	3	lb	3	3	3	lb	3	3	3	
Cule.	1440	0	0	1600	0	0	0	2160	0	0	0	} at $\frac{2}{3}$ per lb
Amphora.	72	0	0	80	0	0	0	108	0	0	0	
Urne.	36	0	0	40	0	0	0	54	0	0	0	
Modius.	24	0	0	26	8	0	0	36	0	0	0	
Congius.	9	0	0	10	0	0	0	13	6	0	0	
Modiolus.	6	0	0	6	8	0	0	9	0	0	0	
Sextarie	1	6	0	1	8	0	0	2	3	0	0	
Hemina.	0	9	0	0	10	0	0	1	1	4	0	
Quartary.	0	4	4	0	5	0	0	0	6	6	0	
Acetable.	0	2	2	0	2	4	0	0	3	3	0	
Cyath.	0	1	4	0	1	5	1	0	2	2	0	
Mystrum.	0	0	6	0	0	6	2	0	1	1	0	
Ligula.	0	0	3	0	0	3	1	0	0	4	$1\frac{1}{2}$	

A Table of Roman Weights.

The Table of Roman Weights.

	Minas	Libras	Uncias	Semiun	Duellas	Sicilicas	Sextulas	Denarios	Drams	Quinars	Scruples	Quadr	Sext	Obol	Siliq	Graines
Talent.	75	152	1500	3000	4500	6000	9000	10500	12000	21000	36000	42000	63000	72000	216000	864000
Mina.	$1\frac{2}{3}$	20	40	60	80	120	140	160	280	480	560	840	960	2880	11520	
Libra.		12	24	36	48	72	84	96	168	288	336	504	576	1728	6912	
^a Uncia.			2	3	4	6	7	8	14	24	28	42	48	144	576	
^b Semiuncia.			$1\frac{1}{2}$	2	3	$3\frac{1}{2}$	4	7	12	14	21	24	72	288		
^c Duella.			$1\frac{1}{2}$	2	$2\frac{1}{2}$	$2\frac{2}{3}$	$4\frac{1}{2}$	8	$9\frac{1}{2}$	14	16	48	192			
^d Sicilicum.			$1\frac{1}{2}$	$1\frac{3}{4}$	2	$3\frac{1}{2}$	6	7	$10\frac{1}{2}$	12	36	144				
^e Sextula.			$1\frac{1}{2}$	$1\frac{1}{3}$	$2\frac{1}{3}$	4	$4\frac{2}{3}$	7	8	24	96					
^f Denarius.			$1\frac{1}{2}$	2	$3\frac{1}{2}$	4	6	6	6	20	82					
^g Dram.			$1\frac{3}{4}$	3	$3\frac{1}{2}$	$5\frac{1}{4}$	6	6	18	72						
^h Quinar.			$1\frac{5}{7}$	2	3	$3\frac{1}{7}$	10	$10\frac{2}{7}$	41							
ⁱ Scruple.			$1\frac{1}{8}$	$1\frac{3}{4}$	2	6	24									
^j Quadrans.			$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$
^k Sextans.			$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$
^l Obolus.			3	3	3	3	3	3	3	3	3	3	3	3	3	3
^m Siliqua.			4	4	4	4	4	4	4	4	4	4	4	4	4	4

a. Libra

a. *Libra* called also *As*, by Translators commonly rendred a Pound, was divided into 12 Ounces, and for every number of Ounces under 12, a Proper Name used, As, *Roman Libra how called and divided.*

Deunx	11	} Ounces.
Dextans and Decunx.	10	
Dodrans	9	
** Bes, Bessis, and of old Des.	8	
Septunx.	7	
Semis, Semissis, Semissius, Selibra, and Simbella.	6	
Quincunx.	5	
Triens.	4	
Quadrans, and Triunx.	3	
Sextans	2	
Uncia.	1	

Malines, p. 24. of his *Lex Mercatoria*, divides *Pondus*, which he calls the Old Pound of the Romans, into *The Division of Pondus by Malines.*

- 64 Denarios.
- 128 Quinarios.
- 256 Sestertios.
- 640 Asles.
- 1280 Semilibella's
- 2560 Teruncios.

A Reason is wanting why *Legat* makes the *Roman Libra* of 12 $\frac{2}{3}$ but 10 $\frac{1}{3}$ *Troy*, since if he reckon by the Number of Graines (the Original of Weights) at 5760 Graines of *Affize* in the Pound *Troy*; it can be but 10 $\frac{2}{3}$ just; for 10 times 6912 the Graines in a *Roman Pound*, and 12 times 5760 are equal. But if he count the $\frac{1}{2}$ *Troy* at 7680 Graines according to the Statute at 32 Graines of Wheat to a Penny Weight, the *Troy Pounds* will be 13 $\frac{1}{3}$ *Roman*. *Legat questio ned.*

** *Bes*, is the Mark Weight, two thirds of the Pound, *Malines* p. 24. aforesaid *Bes*, how divided, makes the *Bes*, or old Mark of the *Romans* to be divided into.

- 16 Loot, or Tetrads.
- 23 $\frac{1}{2}$ Tridrams.
- 32 Didrams.
- 64 Drams.
- 96 Obolos, or Treobolas.
- 128 Triobulos.
- 384 Obolos.
- 768 Miobolos.
- 3840 Moments.

b. *Semiuncia*, or the Half Ounce is sometimes called *Affarion*, and *Affarius*, and by *Alsted*, *Lotho*, answering to a *Germane* Weight of that Name. *Semiuncia how called.*

c. *Duella*, being double to the Weight of the *Sextula* is sometimes called *Bina* *Duella* how much. *Sextula.*

d. *Sicilicum*, or *Sicilius*, and by Abbreviation *Siclus* is $\frac{1}{4}$ of an Ounce. *Sicilicum how much.*

e. *Sextula*, used promiscuously with *Sextans*, and understood by Import of the Name to be the Sixth part. *Sextula used for Sextans.*

f. *Denarius*, a Penny-weight, the seventh part of the Ounce, whether used to weigh any thing but Money as other the Divisions thereof, somewhat questionable, See among the Money. *Alsted* compares the *Drachmal Denarius* to the *German* Weight *Quintlein*. *Denarius the Weight.*

g. *Quinar* was half the Pennyweight, and a piece of Money set afterward among the *Roman* Coines. *Quinar both Weight and Coine.*

*. Between the *Quinar* and *Scruple*, some mention a Weight called *Tremissis*, containing 32 Graines, being the 18th part of the Ounce. *Tremissis how much.*

b. *Quadrans*, here is $\frac{1}{4}$ of the Penny weight, and so called *Quadrans Denarii* to distinguish it from *Quadrans Librae*, which was 3 $\frac{1}{3}$. *Quadrans what.*

i. *Sextans*, called *Sextans Denarii* to difference it from *Sextans Librae*, was the sixth part of the Penny-weight, and sometime called *Sextula*. *Sextans the weight thereof.*

k. *Obolus*, or Half a *Scruple*, called sometimes *Simplium*, weigheth 12 Graines. If there be another *Obolus*, as some say, which was the third part of a *Quinar*, it seems: *Obolus how called, the weight thereof.* It

Between the *Obolus* and the *Siliqua*, some mention a *Cerates*, which they say contains 6 Graines, and so is $\frac{1}{2}$ the *Obolus*, or $\frac{1}{4}$ of the *Scruple*.

*A Table of
Roman Coins
and their Va-
lues, before the
translation of
the Imperial
Seat.*

The

The Brass *Uncia*, misprinted in *Rider*, at $\frac{1}{16}$ *Assis*, for $\frac{1}{16}$ part of 3 Farthings cannot be $\frac{1}{16}$ of our Penny; counting 4 Cees to a Farthing as the doth *Rider misprinted.*

So also is *As*, at *ob. q.* for *ob. qa.* for *As* being the 10th part of the *Denarius* must be 2 Farthings, 10 times 3 making 30 Farthings which is $7\frac{1}{2}$ d. the value of the *Denarius*. To the Brass *As* was the Silver *Libella* equal in value.

Obolus, being $\frac{1}{4}$ of the Roman Penny, is called by *Celsus*, *Sextans*.

Sestertius, Englished a *Sestertian* was $\frac{1}{4}$ of the Roman Penny, and being of the *Masculine Gender* was differenced from the other being of the *Neuter Gender*, and in Numbring by these *Sestertias* these 3 Rules are to be observed.

Obolus. how called. Sestertius the Account thereof.

1. If the Numeral Noun agree in *Case*, *Gender* and *Number* with the *Sestertian*; it signifies barely just so much as was pronounced, as *Decem Sestertii* is 10 *Sestertians*.
2. If the Numeral Noun of another *Case* be joyned with the *Genitive Case Plural* of *Sestertius*: It noteth so many Thousands, as *Decem Sestertiūm* (for *Sestertiōrum*) is Ten Thousand *Sestertians*.
3. If an *Adverb* be put without any Numeral joyned, as *Decies*, *Vigesies*, &c. or joyned with *Sestertiūm* the *Genitive Case Plural*; there is understood by it so many Hundred Thousand, as *Decies Sestertiūm*, is Ten Hundred Thousand *Sestertians*.

Alsted delivers it thus.

From 1 *Sestertian* to 1000 in the *Masculine Gender*, as *Unus Sestertius*, *Decem Sestertii*, &c. is 1 *Sestertian*, 10 *Sestertians*.

From 1000 to 100000 in the *Neuter Gender* and *Plural Number*, as *Singula Sestertia* 1000 *Sestertians*, *Bina Sestertia*, 2000 *Sestertians*, &c.

From 100000 upward, all expressed adverbially and in the *Genitive Plural*, as *Semel Sestertiūm* 100000, *Decies Sestertiūm* 1000000, &c.

Victoriatus was so called, because stamped with the Image of *Victory*, and *Quinarus* because equal in value to 5 Brass *Asses*, or Half the *Denarius*.

Victoriatus how stamped.

Bigatus, some call *Quadratus*, had the Print of a *Cart* or *Chariot* on it, and was of value equal with *Denarius*.

Quinar, how much.

Denarius, *q. f. Dena aris*, because it contained 10 *Asses*, rendred a Penny, *Mat. 18. 28. and 22. 19.* at the old rate was $\frac{1}{16}$ of an Ounce, and at the New $\frac{1}{16}$, and at this rate all the other Coines are valued in the Table. This is sometime called the *Drachmal Denary* for distinction sake. Some make 3 sorts of Pence, the heavier weighing $1\frac{1}{4}$ Attick Dram, the Meane of 1 Dram, and the Least lighter than 1 Dram by $\frac{1}{16}$ of an Ounce, or thereabouts. Some say one was $\frac{1}{2}$ of the Roman *Uncia*, the Mean $\frac{1}{4}$, and the Lighter $\frac{1}{8}$. *Budens* makes the *Attick Dram* and *Roman Penny* of the same Weight and worth, wherewith most agree, and accordingly each in the foregoing Tables are valued at $7\frac{1}{2}$ d. after 5 s. the Ounce.

Bigatus, the Print and Value.

Denarius, the Value. The Sorts.

The Golden *Denarius* mentioned in *Holyoke* at 2 s. 4 $\frac{1}{2}$ d. *Sterling* I have omitted; as not satisfied in the Weight, nor certain of such a Coine.

Golden Denarius.

The Golden *Amient*, seems the Eldest and Greatest, a Piece Coined by the *Consuls*, therefore called *Consularis*, weighed 2 $\frac{1}{2}$ Drams.

Amient how called.

The *Imperatorius*, or Piece of the Emperors Coine 2 Drams.

Imperatorius. Drachmal.

The *Drachmal* 1 Dram, and the *Triens* $\frac{1}{4}$ of the *Imperatorius*.

After *Constantine* removed his Seat to *Bizantium*, now called *Constantinople*, a City after his own Name; we read of *Follis* in *Eusebius*, a Brass Piece, as *Lamprid*, or of *Iron*, as *Eustathius* saith, so called because it represented a Leaf in Latine *Folium*, and was $\frac{1}{16}$ of the Silver Simple *Siliqua*.

Follis, what, why so called.

The Silver *Ciliqua* or *Ceratum* was double. The Simple $\frac{1}{2}$ of the *Milliarisium*, valued 5 d. The Great called *Cerates*, 1 Dram equal to the Penny $7\frac{1}{2}$ d.

Cerates of 2 sorts.

Milliarisium, weighed 2 Drams.

Milliarisium the weight.

Constantines Piece of Gold was called *Romanus Solidus*, at the proportion of 7 s. 6 d. for a Dram of Gold must weigh 1 $\frac{1}{2}$ Dram.

Romanus Solidus.

These continued currant till *Valentinian*, who made his Coine somewhat heavier.

Valentinian's Piece of Gold by some is called *Sextula*, and being valued at 10 s. *Sterling*, must weigh 1 $\frac{1}{2}$ Dram.

Valentinian's Sextula.

Of which the $\left\{ \begin{array}{l} \text{Semissis} \\ \text{Tremissis, or Triens} \\ \text{Scruple} \end{array} \right\}$ being $\left\{ \begin{array}{l} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{4} \end{array} \right\}$ was $\left\{ \begin{array}{l} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{4} \end{array} \right\}$ of a Dram.

Parts thereof.

Sportula, say some, was a Lawyers Fee, or an Almes distributed by Princes among the People. See *Selden's History of Tythes*, chap. 4. p. 37, 38.

Moderne Geodeticals.

Russian Vorst.

Some of the Principal, not all of the Modern Geodeticals follow with the Kingdoms and Countries where used.

Geodeticals of the smaller sort, with the Antient *Hebrews*, *Greeks* and *Latines* Emedullated; it is time to say something of others more Moderne. A Perfect List whereof is not yet come to hand, and those that are, very difficult to reconcile one with another to any exact Computation, though sometimes the difference be inconsiderable, and in round reckoning pass with Authours one for another, without much sensible Error. As the *English* Mile is often joyned with the *Italian*, yet as aforesaid this contains but 1000 Paces, and that 1056. So the *Russian Vorst*, wrote also *Verst*, and sometime *Worst*, commonly called a Mile, and counted of *English* or *Italian* Measure; though *Fletcher* in his History of *Russia*, Chap. 1. tells us it wants a quarter. And so *Johnson* in his *Atlas* accompts *Heylin* without help of the *Printers Errata's* is irreconcilable to himself in his *Cosmography*, p. 511. who counts 2260 *Vorsts* to 3690 *Italian* Miles, and not 4 lines of 4400 *Vorsts* to 3300 Miles.

Out of the Variety and incertainty of Authours to undertake the discovery of the different Measures, Weights and Monies of all places; were endless. Wherefore amongst the Moderne, some of the Principal may suffice. And to spare the often Writing the Names of the Kingdoms and Countreys wherein most of the Provinces and Cities herein mentioned lye: Take the brief Accompt following; which by help of the figures annexed will easily direct the Reader to find them out when he comes at them.

Parts of the WORLD.

Parts of the
World, and
some of the
Kingdoms and
Countries
therein.

1.	2.	3.	4.
Europe.	Asia.	Africa.	America.
1 Denmark	1 Anatolia	1 Egypt	1 Mexico
2 England	2 Arabia	2 Barbary	2 New Spain,
3 France	3 Armenia	3 Isles	or,
4 Germany	4 Chaldea	4 Terra Nigritay	Nova Spagnia
5 Greece	5 China		3 Peru
6 Ireland	6 India		
7 Italy	7 Oriental Isles		
8 Low Countries	8 Palestina		
9 Poland	9 Persia		
10 Portugal	10 Russia		
11 Russia	11 Syria		
12 Scotland			
13 Slavonia			
14 Spain			
15 Sweden			

1. 1. Denmark	Heylegerhaven.	
	Holstein	Hamburgh, or Hamborough.
		Lubecke.
	Islands	Fameren.
		Scandia-Elbogen, or Nellebogh.
		Seland { Coppenhagen, or Haffen.
		Elfinure, or Elsingnore
	Juitland	— Ebbeltorfe
	Norway	— Bergen, or Barrow.

Denmark
some Places no-
ted therein.

1. 2. England	Berkshire	Reading.
	Cambridgeshire	Cambridge
	Chestire	Westchester
	Devonshire	Dartmouth.
		Exceter.
		Plymouth.
	Essex	Colchester.
	Glocestershire	Bristol.
		Glocester.
	Hamshire	Winchester.
	Herefordshire	Hereford.
	Kent	Canterbury.
	Lancashire	Lancaster.
		Manchester.
	Middlesex	London.
		Westminster.
	Norfolke	Great Yarmouth.
	Northumberland	Newcastle upon Tyne.
	Oxfordshire	Oxford.
	Somerfetshire	Bridgewater.
		Dunster.
		Taunton.
	Suffolke	Dunwich.
	Suffex	Rye.
		Winchelsea.
	Warwickshire	Coventry.
	Westmorland	Kendal.
	Wiltshire	
	Worcestershire	Worcester.
	Yorkshire	York.

England, some
of the Counties
and Cities
there.

France
Ireland
Germany.
Hungary
*Some of the
Places in the
Book after-
ward referred
to.*

1. 3. France.	Aquitaine.	Gascony	Coniac	1. 4. { Germany Hungary or Switzerland
			Oleron	
		Guiennæ	Baionne	
			Bordeaux	
			Condet	
			Libourne, or	
		Xantoigne	Lisborne	
			Bruage	
		or Santoine	Rochel	
			Santonum, or	
	Anjou	St. Antoine		
		Angiers		
	Berry	Tours		
		Bourges		
	Bourbon	Clermont		
		Tureme		
	Bretagne	Morlaix		
		Nantes		
	Borgoingue	St. Malo		
		Auxere		
	or Burgundy	Province		
		Valence		
	Champagne	Vienna		
		Clermont		
	Daulphine	Paris		
		Aquismort, or		
	Isle of France	Aigues Mort		
		Montpellier		
		Narbon		
		Tholouse		
	La Beaufie	Blois		
		Orleans		
	Lionois	Lyons		
		Caen, or		
	Normandy	Cane		
		Dieppe		
		Rouen, or		
		Roan		
	Nivernois	Nevers		
		Abbeville		
	Picardy	Amiens		
		Calais		
		Boulogne		
		Roy		
	Perigort	Angouleme, or		
Engoulesine				
Poictou	Poictiers			
	Mirebeau, or			
Provence	Mirabel			
	Aix, or Ay			
	Avignon			
	Aurange, or			
	Orange			
	Marfeilles			
1. 6. Ireland	Dublin			
	London-Derry.			

1. 4.	Alfatia	Colmar	
		Rufach	
		Strausburgh, or	
		Strasburgh	
		Thann	
		Weiffenbergh	
		Austria	Vienna
			Baden, Durlach
		Baden	Munchen
			Paffaw, or
	Bavaria	Patavia	
		Regensburgh, or	
	Bohemia	Ratisbone	
		Saltsburgh	
	Brandenburg	Prague	
		Brandenburgh	
	Carinthia	Frifach	
		Cleue	
	Cleveland	Wefel, or	
		Wifel	
	Collenland	Collen	
		Rhineburgh	
	East Frifland	Emoden	
		Bamberg, or	
	Franconia	Babemberg	
		Frankford, on the	
		Moene.	
		Nurenburgh	
	Gulich	Wurtspurg, or	
		Wurtzburg	
		Aix, Aken, Aquen,	
		or Achon	
	Hungary	Gulich, Juliers, or	
		Guliers.	
	Lorraine	Pooßen, or	
		Presburgh	
	Lufatia	Mets	
		Verdun	
	Lunenburgh	Bautzen, or	
		Botzen	
	Mecklenburgh	Lunenburgh	
		Domyn, or	
	or Macklenburgh	Dammin	
		Rostock	
	Mark	Wifmar	
Tremone, or			
or March	Dortmond		
	Werden		
Mentz	Koningsbergh		
	Mentz		
Meydeburgh	Meydeburgh, or		
	Magdeburgh, or		
	Mageburgh		
	Eifted, or Aichftad		
Palatinate	Heydelburgh		
	Norenborgh, or		
	Norenbergh, or		
	Nurenbergh		
Pomerania	Spiers		
	Wormes		
	Coftin		
	Gripfwald		
Straeßondt	Ockermond		
	Stetin		

Germany and Switzerland Greece, Italy, several Places of Note in them after-ward referred to.

B	Saxony	Turingia. Meissen.	Erdford	Church-Lands.	Compagnia di Roma	Rome	
			Halo, or Kala		Ducato Spoletano	Narnia,	
			Jene	Estate of Urbin		Negropont	
			Friberg				Pesara
	Silesia		Lipfich			Urbino	
			Meyfen			Marca Anconitana	Ancona
			Mansfield			Rechanati	
			Bresslaw				Boloignia, or
			Ausburgh, or			Bononia	
			Auspurgh				Cervia
			Bibrach	Romandiola, } or Romagnia }		Cefena	
			Brifach, or			Faenza	
			Erifgow			Forli, or Furli	
			Conitance			Ravenna	
			Friburgh			Rimini, or	
			Kempton, or			Rimano	
	Suevia, or Almaine		Campidona	Territory of Ferrara		Carpi	
			Norlingen			Ferrara	
			Offen, or Offner	Genoa		Genoa, or Genes	
			Ravenfpurgh			Luca	Luca
			Scaffhaufen, a			Friuli	Trieste
			Canton of the			Hiftria	Cape de Iftria
			Switz	Venice		Piran	
			Ulme			Marca-Trevigiana	Bergamo
			Upper Baden			Brefcia	
			Bafil			Crema	
			Berne			Padua	
			Friburg			Trevifo	
	Switzeeland, or Helvetia		Laufanna			Verona	
			Lucern			Vincencia, or	
			Soloturn			Vincenza	
			Switz				
			Zurich				
			Confluentz				
	Triers		Triers	Mantua		Mantua	
			Upper Wefel			Millan	Como
			Inpruch			Cremona	
			Trent			Modena	Millan
	Tyrol		Ereme			Pavia, or Papia	
			Homberg			Montferrat	Modena
			Ofenbrigh, or			Alba	
			Ofenbridg			Parma	Cafal, or St. Va9
	Westphalia		Ravenfburgh			Mirandula	
			Fribergh			Parma	Placentia, or
			Naffaw			Plaiffance	
			Weilborough, or			Tufcaine	Florence
	Weteraw		Wiffelborough			Ligorne	
			Tubing				Massa
	Wirtenbergh		Wiberg			Prato, or Perato	
				Corfica			
						Sardinia	Marfala, or
						Maflila	
						Sicilia	Meffena, or
						Meffana	
							Nicofia
						Palermo	
							Syracufa
	Islands.						
				Kingdom of Naples.		Aquila	
						Abruzzo	Lanciano, or Lanfan
						Apulia, or Puglia	
						Rhegium	
						Calabria	Otranto
						Terra de Otranto	Naples
						Rivelli	
						Terra di Lavoro	St.Fumia, or Fiume

Principal Provinces of the Allobroges, &c.	Piedmont—	Turin	C	Henault—	Ach	Low- Coun- tries. Se- veral of the Pla- ces after- ward re- ferred to.
	Savoy—	Chambery			Halle	
		Lunebourg			Valenciennes	
	Geneva, or	Jenfer			Covyn, or	
	Walfland—	Sedun			Convennes	
	Grifons—	Curienfis			Amsterdam	
		Arras			Briel	
	Artois—	Perne			Delfe	
		St. Omer			Dort	
		Anrwerp			Enckhuyfen	
Brabant and Marquifate of the Empire		Arifhot, or	Holland—		Hague	
		Afchot			Harlem	
		Barrow, or			Horne	
		Bergen op Zome			Leiden	
		Bolduc, Boifeleduc,			Medenblick	
		Hertogenbofh, or			Naerden	
		Shertogenbofh			Rotterdam	
		Bruffels, or Bruxels			Schoonehaven	
		Dieft			Texel	
		Lovaine			Waffenar	
Cambray—		Maeftrecht	Limbourg—		Carpen	
		Machlyn, or Mechlyn			Limbourg	
		Malines			Affelt	
		Cambray			Hoeÿ, Hoyer, or	
		Alft, Ailft, or Aloft			Huy	
		Audenarde, or			Liege	
		Oudenarde			Looth	
		Axele			Mafiers	
		Bridges, or Bruges			Sedah, fet	
		Cafel			by fome in	
Flanders—		Cortrycke, or	Luxembourg, or		France.	
		Courtray			Namen	
		Damme			Campan	
		Deynfè, or Deyfè			Deventer	
		Dixmude			Swdoll, or Zwell	
		Doway, or Douay			Utrecht	
		Dunkerck			Utrecht	
		Gaunt			Harlingen	
		Graveling			Zuricza	
		Honfchotten			Goës, or	
1. 8. Low Countries.		Hulft	Zeland		Tergoes	
		Ippe, or Ypre			Romerfwal	
		Lille, or Ryffel			Armuyden, or	
		Loo, or Lowe			Ramme	
		Meenen, or Meenen			Camfere, or	
		Newport			Vere	
		Orchies, or Orfies			Flufhing	
		Oftend			Middleburgh	
		Popering			Groll	
		St. Amand			Zutphen	
1. 9. Poland		Sclufe, or Sluys	Zutphen—		Liefland, or	Poland the like.
		Tournay, or			Livonia	
		Dornir			Riga	
		Walfland, or			Vilna, or	
		Land Van Waes			Wilde	
		Winocksborough, or			Cracow	
		Winocksbergh			Warfaw	
		Dam, or Damme			Coningsbergh	
		Groeninghen			Dantzick	
		Arnhem			Melvyn	
1. 10. Portugal		Bomel	Portugal—		Thoren	Portugal the like.
		Batenborgh			Lisbon	
		Ghent			St. Tubal, or	
		Gueldres			Serubal	
		Nimnegent, or			Mufcovie	
		Nimneghen			Edenburgh	
1. 11. Russia			1. 12. Scotland			Russia, and Scotland the like.

1. 13. Slavonia.	Dalmatia	Ragusa, or	2. 1. Anatolia, or Asia the less	Asia specially so called	Ephesus	Slavonia, Spain; Sweden; the places of Note in them and in Asia, Africa also in the Book after- ward refer- red to:
		Raguza			Smyrna	
	Liburnia	Sebenico		Bithynia	Bursa, or Prusa	
		Spalato			Alexandria, or	
		Zara		Cilicia	Scanderoon	
		Zegna, or			Cyprus	
		Sequia		Islands	Rhodes	
		St. Lucar.			Scio	
	Andaluzia	Sevill		2. 4. Chaldea	Babylon	
	Arragon	Saragossa		2. 6. India	Decan	Goa
1. 14. Spain.	Biscay	Bilboa			Malabar	Calicut
	Castile	Burgos			Narsinga	Maliapur, or
		Madrid	2. 7. Orient Isles	Molucco	St. Thomas	
		Medinadel				
		Campo	2. 8. Palestine	Judea	Rama	
		Salamanca				
	Catalonia	Barcelona	2. 11. Syria	Comagena	Aleppo	
		Girone		Coele-Syria	Damascus	
	Corduba	Xeres, or		Phoenicia	Acon, Achri, or	
	Gallicia	Sherry			Acre	
		Baione			Barutti, or	
	Granada	Almeria		Syria	Berytus	
Granada		Tripoli				
Guipuscoa	Malaga	Alcario	Antioch			
	St. Sebastians		Alexandria			
Islands	Toloso		Arcadia	Cairo, or Caire		
	Cadiz		Forfori			
	St. Mary		Sciba			
Leon	Majorca		Zaidin			
	Leon	3. 2. Barbary	Algiers	Zeroi		
Murcia	Villa Viciosa			Algers	Capo d' Aguer or Capo d' Algier	
	Carthagenia		Fesse	Oran		
Navarre	Viano		Morocco	Fesse		
	Victoria, or			Fesse		
Toledo	Vellica		Tunis	Sus		
	1. 15. Sweden.			Toledo	3. 3. Isles	
Valentia	Medina Coeli	Terra Nigritarum	Tripoly	Tripoly		
	Allicante		3. 4. Terra Nigritarum		Tunis	Tunis
	Valentia				Una	
Boden		Canaries				
			Madera			
Finland		3. 3. Isles	St. Thomas	Cabo Verde		
Gothland	Ostrogoth			Guinea		
	Smalandia	3. 4. Terra Nigritarum				
Lapland	Westrogoth					
Liefland, or	Revell, or	3. 4. Terra Nigritarum	St. Thomas	Cabo Verde		
	Livonia					Rivallia
Narva, or						
Nareca						
	Helsingen					
	Suderman					
Sweden	Westman					
	Upland, in which is the City Stockholme					

The English are not alone in the knowledge and use of the Inch, Foot, Ell, Pace, Fathom, Furlong, Mile, &c. for besides the Hebrews, Greeks and Latins before spoken of; most Nations especially of Europe, use the same as occasion serves. Some nearly correspondent, some vastly different, and some Countries have divers Measures of the same Names. Hence is observable, that although most of the Europeans reckon distances of places by Miles, yet there are no agreement between them. And in Germany it self are

Long Measures
in England
used elsewhere,
But different.

German Miles
the sorts.

are 3 farts. The Common Mile which is 4 *Italian* Miles, and yet the least of the 3. A Mean which is 5, and the Great (called sometimes the *Great Saxony*) Mile, being 6 *Italian* Miles. And besides these *Alsted* mentions a Mile that containeth 22800 Feet.

Spanish Miles. Spain, by several hath $\frac{1}{2}$ Mile to a *Dutch* Mile; but their Common Account is by Leagues called with them *Leguas*, whereof 17 $\frac{1}{2}$ make 60 *Italian* Miles. *Alsted* speaks of a *Spanish* Mile to be 21000 Feet.

French Leagues. France, where they reckon by Leagues, hath no less than 4 farts, as *Cotgrave* in his *Dictionary* hath well observed. As,

The Sorts. *Lieue de Bourgongne*, 50 *Portées*, every *Portée*, 12 *Cords*. Every *Cord* 30 Feet, most in use in *Burgundy*.

Grande Lieue, as the common sort of *German* Miles, every *Lieue* 4 *Italian* Miles.

Moyennes Lieues, as those of *Dauphine* or *Languedoc*, containing 3 *Italian* Miles.

Petites Lieues, as those of *Italy*, and of this sort, the *Fiery Collumne*, commonly called the *Dutch Waggoner* in his *Sea Charts* often joynes as one with the *English* Mile.

French Mile. A *French* Mile saith *Cotgrave* is 2 of ours.

Flemish Mile. *Malines*, notes the *Flemish* Mile of 2 farts. One 1000 Rods of 20 Feet to the Rod, which is alike to the Common *Germane* Mile. And the other of 1400 Rods of 15 Feet to the Rod. And the *Holland* Mile he saith is 2000 Roads, and that 5 of them are 4 *Flanders* Miles. *Alsted* mentions the *Belgick* Mile to be 18000 Feet, and by this account the Mile of *Flanders* must be 2500 Roads, but how long the *Holland* Road is he saith not.

Swedish Leagues. *Lenca Suevica*, after the *Latine*, or the *Swedish* League called *Scandica* is 5000 *Passes*, or 5 *Italian* Miles.

Florence Mile. *Florence*, *Malines* reckons 3000 Braces for a Mile, which at the rate of 122 $\frac{1}{2}$ Braces for 60 *English* Ells comes 5510 Feet, 2 Inches, and somewhat over.

Ægyptian Schoenes. In *Ægypt* they reckon by *Schoenes* of various Magnitude, some 60, some 40, some 20 Furlongs. Some will have the *Schoene* equal to a *Spanish* Mile. See before in *Greek Measures*. The *Ægyptian* Cubit, saith *Malines* is 6 of our Cubits.

Russia Vorst vide antea Of the *Russian Vorst*, and *Persian Parasang* see before.

Chinian Stade. The *Furlong* or *Stade* of *China* is almost twice as long as ours, for 12 of ours are but 6 $\frac{1}{4}$ of theirs by *Johnson* in his *Atlas*; who for the most part the following Table, follows.

Long Outlandish Long Measures compared.

Long Forain Measures compared to a Degree.

In	Dalacia	7	Miles, make 1 Degree of a Great Circle in the Earth.
	Westrogoth	8	
	Saxony Great	10	
	Almaine	12	
	Ostrogoth		
	Suderman		
	Upland		
	Westman	15	
	Germany		
Smalandia			

In	Spain	17 $\frac{1}{2}$	Leagues	make 1 Degree of a Great Circle in the Earth.
	Finland	18	Miles	
	Cajan			
	Livonia	20	Leagues	
	France			
	Helsingen	22	Miles	
	Botnia			
	Lapland			
	Scotland	50		
	Italy	60		
England				
Russia	80	Vorsts		
China	250	Furlongs		

Grimstone, p. 715 tells us that 69516 *Diez* of *China* Measure make almost 1000 *Spanish Leagues*. And that the *Chinois* have 3 Measures to Survey withal, which they call *Lij*, *Pu*, and *Icham*. The *Lij* comprehends as much Space as may be assigned to a Mans Voice thrust forth with all his force in a Calme Season upon a fair Plain, 10 of these *Lij*'s makes 1 *Pu*, which is a great *Spanish League*, and 10 *Pu*'s make 1 *Icham*, or a whole Journey.

The Modern Furlongs, Fathoms, and Paces of the *Europeans* differ little if at all from the Ancients. The Furlong 125 Paces, the Pace 5 Feet, or two of the lesser Paces.

The Fathom in *France* called *Toise* ordinarily is as ours 6 Feet, the Kings Fathom 7 Feet 4 Inches. For Woods and Forests by the Custom of *Orleans* for $5\frac{1}{2}$ Feet. The *Burgonian* Fathom is $7\frac{1}{2}$ Feet.

The Ordinary (called also the Royal) *Pied*, or Foot, there is 12 Inches, that of *Clermont* 11, of *Engoulesme* longer than the Ordinary. The Foot used about *Bordeaux* to measure Land called *Pied de Terra* is longer than the ordinary by $\frac{1}{8}$ in $\frac{1}{4}$. *Pied de Ville* there used for Timber, Stone, &c. longer than the Ordinary by $\frac{1}{8}$ of an Inch. The *Petit Pied* is shorter than the Ordinary.

Several affirm, that the Foot used in several Countries is different, as of 10, 12, 16. &c. Inches, and they of different bigness. *Capel* and *Snellius* before-mentioned have compared them together, of whom the latter is by some reckoned most exact. The Tables of both follow.

Capel's Comparison of the Foot to the Foot of Toledo in Spain, supposed to be divided into 120 parts.

Capel his Comparison of the Foot of Toledo with others.

The Foot of	Heidelberg	in	Germany	137	Parts
	Tuscany		Italy	138	
	Sedan		France	139	
	Rome		Italy	144	
	Athens		Greece	150	
	London		England	152	
	Paris		France	160	
	Syria		Asia	166	
	Egypt		Africa	171	
	Judea		Palestine	180	
	Babylon		Chaldea	200	

Snellius his Comparison of the Foot, to the Foot of Rome in Italy, or Rheinland, or Leiden, which he saith are all one, supposed to be divided into 1000 parts.

Snellius his Comparison of the Foot of several Places with others.

The Foot of	Toledo	} in <	Spain	864	} Parts.
	Mechlin		Brabant	890	
	Strasburgh		Germany	891	
	Amsterdam		Holland	904	
	Antwerp		Brabant	} 909	
	Louvain		Brabant		
	Bavaria		Germany	924	
	Copenhagen		Denmark	934	
	Goes		Zeland	954	
	Middleburgh		Zeland	960	
	London		England	968	
	Norembergh		Germany	974	
	Zurichzee		Zeland	980	
	The Ancient Greek		1042		
	Dort	} in <	Holland	1050	
	Paris		France	1055	
	Briel		Holland	1060	
	Venice		Italy	1101	
	Babylon		Chaldea	1172	
	Alexandria		Egypt	1200	
	Antioch		Syria	1360	

Palmes, &c.
like the old.
Spanish Xeme.

Inch of Spain
and France.
French Line.
Superficial
Measures of the
Moderne.
Arpentiers of
France.
The Sorts.

The *Palmes* greater and less Inches and Digits, for the most part have the same Divisions as the Ancient. The *Spanish Xeme* is half a Foot, or 6 Inches. Their *Cotus* half of their greater *Palme*, or 4 Inches, both *Spain* and *France* allow 16 digits to their 12 Inches, called by that *Pulgada*, by this *Poulce*.

France hath a little Measure called a *Ligne*, or Line, whereof 12 to 1 Inch. Superficial Measures of Land having length and breadth are subject also to the different Laws of divers Countries, whether they measure by Lines, Cords, Rods or Perches, &c. And accompt their Measures by Acres, Arpentiers, Bunderen, &c.

France reckoning by Arpentiers useth not fewer than 10 sorts, as *Cotgrave* accompts, *Viz.*

Arpent, that is ordinary is 100 Perches Square of 18 Feet to the Perch.
Arpent de Bois is 2 1/2 Roods, 1 Rood 40 Perches, 1 Perch 24 Feet, 1 Foot 24 Inches.

Arpent de Bois, de Bourgoigne, is 440 Perches.
Arpent de Clermont is 100 Verges in most places, in some but 70, at 26 Feet to the Verge.

Arpent de Dunois, is 100 Perches at 20 Feet to the Perch.
Arpent de Nevers is 4 quarters square, 1 quarter 10 Fathoms, 1 Fathom 6 Feet.
Arpent de Paris, is 100 Perches Square, at 22 to the Perch, and in some places about *Paris* it contains 25 Feet, and in others at the ordinary rate 18.

Arpent de la Perch, 100 Perches, 1 Perch 24 Feet, 1 Foot 13 Inches.
Arpent de Poictou is 80 Paces Square.
Arpent Romain, is 240 Feet long, and 120 Feet Broad. This is like to the *Latine Juger*, of which before.

French Sep-
tier of Land.
Their Muir
and Mine.

A *Septier* of Land he saith is much about the *Arpent*.
A *Muid* of Land is 12 *Septiers* or *Arpents*.
A *Mine* of Land in *La Chastellenie de Bulles*, contains 50 Verges of 24 Feet to the Verge.

The Sorts.
Scruple of
Land.
Spanish
Jugada
Fancca
Stadale.
Moderne Long
Measures.

A *Mine* of Land in *Clermont*, 60 Verges of 22 Feet to the Verge.
A *Mine* of Land in the *Seigneurie de Remy*, 80 Verges of 22 1/2 Feet to the Verge.
A *Scruple* of an *Arpent* is 1/4 of an *Arpent* or 10 Feet Square every way.
In *Spain*, 1 *Jugada* is 50 *Fanecas*, 1 *Fancca* is of Land sowed with Barley 400 Square *Stadales*, of Land with Wheat 600. And 1 *Stadale* is 11 Feet. A Square *Stadale* is 121 Feet.

Aulnes of
France how
different.

Merchants have their *Ells*, *Aulnes*, *Braces*, *Cannes* or *Canes*, *Varras* or *Varas*, *Pitchy*, &c. for Commodities proper to be measured thereby after the Laws and Usages of several Countries. And by Traders is diligently to be sought out, because in several Countreys, though some Common Measures be of most general use, yet in fundry places in one and the same Countries shall measures of the same Name, Nature and use be different, as in *France*, the Common *Aulne*, or that called *Aulne du Roy* is 3 Feet 7 Inches and 8 Lines. But that of *Bordeaux* 4 1/2 Feet almost, that of *Dijon* and *Province*, but 2 1/2. That used of *Merchants* for Silks half an Inch shorter than the Common, that of *Paris* but 3 Feet and 1/2 of an Inch, and about 1/3 of a Foot. Some Customs bespeak it formerly 3 Feet 8 Inches and 4 Lines long. And allowing 100 *Ells* of *Antwerp* to agree with 60 *Ells* or 75 Yards at *London*, the correspondency thereof with other places follow out of *Malines* his *Lex Mercatoria*.

100 Ells of
Antwerp com-
pared with the
Measures of
other places.

	Townes and Cities	Directions as before	Merchandise Measured	Contents or Quantities	Names of the Measures.
The 100 Ells of Antwerp make at	Abbeville	1 3		84	—Ells
	Achrie, or Acon	2 11		115	—Pichy
	Acon, or Aken	1 8		104	—Ells
	Adler			33	—Canes
	Aleppo	2 11		108	—Pichy
	Alexandria	2 1		124	—Pichy
	Aman	2 11	as Aleppo		
	Amsterdam	1 8		101	—Ells
	Ancona	1 7		107	—Braces
	Andaluzia	1 14		83	—Varras
	Antwerp	1 8	for Silks	98	—Ells
	Arago	1 14		43	—Cannes
	Archipelago	1 5		100	—Pichy
	Artois	1 8	all the Province	98	—Ells

Affelt	1	8 as <i>Acon</i> in the <i>Netherlands</i>		
Audenarde	1	8 as <i>Antwerp</i> for Silks	98 $\frac{2}{3}$	Ells
Avignon	1	3	60	Ells
Ausburgh, or } Ausborough }	1	4 for { Linnen Cloth	125 127	Ells
Barrow, or } Bergen }	1	1 Uncertain, for they measure by the bigness of your head with a Rope for		1 Ell
Barcelona	1	14 as <i>Arragon</i>		
Bafill	1	4	125	Ells
Bautson	1	4 for { Cloth Silks	111 120	Ells
Bergamo	1	7	101 $\frac{2}{3}$	Braces
Bolduc, or } Hertogenbosh }	1	8	102	Ells
Bologna	1	7 as <i>Ancona</i>		
Brabant	1	8 all the Province (except such Places as are herein excepted) like <i>Antwerp</i> for Silks.		
Breme	1	4	122 $\frac{1}{2}$	Ells
Brescia	1	7 as <i>Bergamo</i>		
Breslo	1	4 as <i>Bautson</i>		
Bruges, or } Bridges }	1	8 { in the Shops but for Linnen	98 $\frac{2}{3}$ 94 $\frac{1}{4}$	Ells
Brussels	1	8 as <i>Bolduc</i> .		
Bauti, or Baruti	2	11	111 $\frac{1}{3}$	Pichy
Bursa	2	1	114	Pichy
Cadiz	1	14 for { Cloth Silks	81 108	Varras Ells
Calabria	1	7 as <i>Adler</i>		
Cambray	1	8	96	Ells
Candia	1	5	108	Pichy
Capo d' Algier	3	2	136 $\frac{1}{2}$	Covados
There 1 Cane is 12 Cavados.				
Carpi	1	7 as <i>Ancona</i>		
Cassel	1	8 as <i>Bolduc</i>		
Castile	1	14	85 81	Varras
Some allow but There 1 Varra is 4 Quarters, 1 Quarto 2 Palmes.				
Cefena	1	7 as <i>Ancona</i>		
Collen	1	4	120	Ells
Conninsbergh	1	9	125	Ells
Constantinople	1	5 { For Canvers	113 80	Pichy
Corfu, an Island	1	5	116 $\frac{2}{3}$	Braces
Covin	1	8	70	Ells
Crema } Cremona }	1	7 as <i>Bergamo</i>		
Damascus	2	11 as <i>Bruti</i>		
Damme	1	8 as <i>Antwerp</i> for Silks		
Dantick	1	9	122	Ells
Deyfe and Dieft	1	8 as <i>Bolduc</i>		
Domyn	1	4 as <i>Breme</i>		
Doway	1	8 as <i>Cambray</i>		
Dunkirk	1	8 as <i>Antwerp</i>	100	Ells
Embsen	1	4 as <i>Breme</i>		
Erdford	1	4	165	Ells
Ferrara	1	7 as <i>Ancona</i>		
Flanders	1	8 all the Province, as <i>Brabant</i>		
Flushing	1	8	104	Ells
Florence	1	7 for { Woollen Silks	116 122 $\frac{1}{2}$	Ells Braces

100 Ells of
Antwerp com-
pared with
the Measure of
other Places.

France, the Kingdom, except the Places herein excepted. See *Paris*.

Frankford

100 Ells of
Antwerpc
compared with
the Measure of
other Places.

Of Geodeticals.

Lib. II. Part

The 100 Ells at Antwerp make at	Frankford	1	4 as <i>Ausborough</i>			
	Gaunt	1	8 as <i>Antwerp</i> for Silks			
	Gelderland	1	8	104 $\frac{1}{2}$	Ells	
	Genes	1	7	Silks (104, Palmes for 34 $\frac{1}{2}$ Ells)	122	Braces
				Woollen Cloth at 9 Palmes the Cane	288	Palmes
				Linnen Cloth at 13 Palmes the Cane	32	} Canes
			29			
	Geneva	1	7	60	Stabb	
	Goes	1	8	97	Ells	
	Granada	1	14 as <i>Andaluzia</i>			
	Gripfwool	1	4 as <i>Breme</i>			
	Halle	1	4	105	Ells	
	Hamborough	1	1 as <i>Breme</i>			
	Harlem	1	8 in the Market for Linnen	94 $\frac{1}{2}$	Ells	
	Henault	1	8 in the { Market	94 $\frac{1}{2}$	} Ells	
			{ Shops	98 $\frac{1}{2}$		
	Hertogenbof	1	8 See <i>Bolduc</i>			
	Holland	1	8 in most places of the <i>Province</i>	103 $\frac{1}{2}$	Ells	
	Honschotten	1	8 as at <i>Dun kerck</i>			
	Hoye	1	8 as at <i>Bolduc</i>			
	Ipre	1	8 as <i>Antwerp</i> for Silks			
	Ifrica }	1	7 for {	Woollen Cloth	101 $\frac{1}{2}$	} Braces
	Itria }			Silk and Cloth of Gold	108	
	Lanfan	1	7 as <i>Adler</i>			
	Larta, or Laarta	1	5 as <i>Alexandria</i>			
	Lavalona	1	5	111	Pichy	
	Lepanto	1	5	113	Pichy	
	Liege	1	8	114	Ells	
	Lipfich	1	4 for {	Silks and Linnen	105	} Ells
				some fay but 104 $\frac{1}{2}$		
				Cloth	120	
	Lisbon	1	10	by some but 60	62	} Varras
				also	83	
				and for Silks	100	
	Lisle	1	8 as <i>Cambray</i>			
	London	1	2 for {	Linnen with the Palme and Thumb measured into it	60	Ells
				Woollen, the Thumb	75	Yards
				Frize at 1 ; for a Yard	50	} Goads
				some fay	59	
				Roan Canvas, whereof the Centener is 120, being 10 Cords, of 12 Ells to a Cord	61	
	Loo, or Lowe }	1	8 as <i>Bolduc</i>			
	Louvaine }					
	Lubeck	1	1 as <i>Collen</i>			
	Luca	1	7	120	Braces	
	Lyons	1	3 for {	Linnen	60	} Ells
			Silks	94 $\frac{1}{2}$		
	Maeſtricht	1	8 as <i>Acon</i> in the <i>Netherlands</i>			
	Malaca, or }	1	14 as <i>Adler</i>			
	Malaga }					
	Mantua	1	7 as <i>Ancona</i>			
Maroco, or }	3	2 as <i>Capo d' Algier</i>				
Moroco }						
Marſeilles	1	3 for {	Woollen Cloth	33 $\frac{1}{2}$	} Canes	
		Silks	36			
Mafiers	1	8 as <i>Cambray</i>				
Mafilla	1	7	34 $\frac{1}{2}$	Canes		
Meanen	1	8 as <i>Cambray</i>				
Melvyn	1	9 as <i>Dantſicke</i>				
Meydeborgh	1	4 as <i>Halle</i>				
Meyſen	1	4 as <i>Lipſich</i>				

Middleburgh

100 Ells of
Antwerpe
compared with
the Measure of
other Places.

The 100 Ells of Antwerp make at

Middleburgh	1	8	{ in the Market for Linnen— otherwise —————	94½ } Ells
Millan	1	7	for { Linnen ————— Silks —————	100 } Braces
Mirandula	1	7	as Ancona	120 } Braces
Modena	1	7	as Ancona	141 } Braces
Munster	1	4	—————	65 Ells
Namen	1	8	as Acon in the Netherlands	
Nantes	1	3	as Abbeville	
Naples	1	7	—————	{ 116 } Cannes
Narva	1	15	—————	33½ } Arfins
Negropont	1	7	as Ancona	125 Arfins
Nigropont	1	5	as Lepanto	
Norenborgh	1	4	as Lipsich	
Ockermonde	1	4	—————	106 Ells
Offner	1	4	—————	{ 119 } Ells
Orfies, or	1	8	as Cambray	130 } Ells
Orchis	1	8	as Cambray	
Ofenbridg, or	1	4	—————	63 Ells
Ofenborgh	1	4	—————	
Overysfel	1	8	as Gelderland	
Padua	1	7	for { Cloth ————— Silks —————	101½ } Braces
Palermo	1	7	as Masilla	83½ } Braces
Paris			One Cane, 4 Pichy France, and most part of all that Kingdom ————— According to others ————	34½ Canes 59 Ells 57 Aulnes
Parma	1	7	—————	{ 91 } Braces
Perato	1	7	as Ancona	109½ } Braces
Pesaro	1	7	for { Cloth ————— Silks —————	107 } Braces
Picardy	1	3	as Abbeville	103 } Braces
Piran	1	7	as Istrica	
Prague	1	4	as Bautson	
Provence	1	3	—————	36 Cannes
Puglia	1	7	for { Cloth ————— Silks —————	31 } Cannes
Raguza	1	13	as Luca	33 } Cannes
Rama	2	8	—————	
Ravenna	1	7	as Corfu. Some say ————	115 Pichy
Rechanari	1	7	as Bergamo	113 Braces
Regenburgh	1	4	—————	78½ Ells
Revel, or Rivalle	1	15	as Coningsbergh	
Rhode	2	1	as Adler	
Riga	1	9	as Coningsbergh	
Rochel	1	3	as Paris	
Rome	1	7	{ for Woollen Cloth ———— —————	33 } Cannes
Romerfwal	1	8	—————	105½ } Cannes
Rostock	1	4	—————	99 Ells
Rouen	1	3	{ The Centener of Ells being 112, that is 28 to a quarter— According to some ————	119 Ells 58 Ells 52 Aulnes
Salonici	1	5	—————	109 Pichy
Sapi	1	5	as Archipelago	
Saragossa	1	14	—————	33 Cannes
Scio, or Sio	2	1	as Corfu	
Scotland, most part of that Kingdom, where they reckon 120 to the 100			—————	72 Ells
Sebenico	1	13	—————	112 Braces
Sevil, or Sivil	1	14	as Andalusia	

100 Ells of
Antwerp com-
pared with
the Measure of
other Places.

The 100 Ells of Antwerp make at	Sicilia	1	7 as Palermo		
	Sluys	1	8 as Antwerp for Silks		
	Sterin	1	4 as Ockermonde		
	Stockholme	1	15 as Conninsbergh		
	Toledo	1	14 as Castile. Hunt faith	88	Varras
	Tournay	1	8	108	Ells
	Trevifo	1	7 as Bergamo		
	Tripoli	3	2 as Alexandria		
			1 Cane, 4 Pichy		
	Tripoli	2	11	112	Pichy
	Valentia	1	14	73	Cannes
	Venice	1	7 as Istria		
	Vere	1	8	94 $\frac{1}{2}$	Ells
			{ Long Measure	86	Braces
	Verona	1	7 { Short Measure	104 $\frac{1}{2}$	
			{ For Cloth of Gold	108	
	Vincenza	1	7 for { Woollen Cloth	98 $\frac{3}{4}$	Braces
			{ Silks	80 $\frac{1}{2}$	
	Vienna	1	4 for { Linnen	77 $\frac{1}{2}$	Ells
			{ Cloth and Silks	85 $\frac{1}{2}$	
	Ulme	1	4 {	120	Ells
			{ Woollen Cloth	96	
	Urbino	1	7 as Bergamo		
	Winockxborough	1	8 as Bolduc		
	Wilmar	1	4	118	Ells
	Yfenghem		as Antwerp for Silks		
	Zara	1	13 as Sebenico		
	Zurich	1	4	116 $\frac{1}{2}$	Ells

Concave Mea-
sures.

Forrain Concave Measures have had the same fate as Long Measures to differ in Names and Quantities with most Nations; and by the aforeſaid Authour have like Contents and Correspondencies as follow.

Forreign Measures of Wine and Oyle.

Fother of Wine
the Content.

In Germany they call the Carriage of the drawing of 2 Horses, a Fother of Wine, and accompt 2 $\frac{1}{2}$ Rods for a Fother.

Rod of Wine
how much.

At Dort in Holland, they call a great Vessel 10 Feet Square, and one Foot deep: A Rod of Wine, every such Foot containing 7 $\frac{1}{2}$ Gallons Antwerp, every Gallon called there Stoop, weighing 6 lb.

Ame of Dort
the Content.

An Hoghead of Wine Dort Measure, called an Ame, contains 100 Gallons or Stoops; and every Gallon 10 Schreaves.

Some German
Measures.

Correnius in his *Janua Aurca*, mentions among the German Measures besides the Fother, Half Eymer, and Eymer, 3 sorts of Maasz, 1 of 24 Kannes, 1 of 12, and another of 3, together with the Noefel, Half Noefel, and Drittertheil, &c. but gives no accompt of the particular Contents thereof.

Malines affirms at Meyſen in Saxony 20 $\frac{3}{4}$ Ponderal, make 24 Mensural. And at Lipsich 32 $\frac{3}{4}$ Mensural 26 $\frac{1}{2}$ Ponderal, but at the rate of 6 Mensural for 5 Ponderal, it should be 26 $\frac{1}{2}$.

Hoghead the
Weight and
Content.

Pag. 30. he writes that an Hoghead of Wine weighs 500 l. the Cask 50 l. Wine Netto 450 lb. An Hoghead of Corn 400 lb. Cask 50 lb. Corn Netto 350 lb. so shall the Ton of Wine Netto be 1800 lb. with the Cask 2000 lb. of Corn but 1600 lb. with the Cask. Four Hogheads going to a Ton, and 2 Tons to a Last.

Milliar of Oil.
Ame of
Antwerp
how much.

Pag. 31. A Milliar of Oyle at Antwerp 1100 lb. a Butt 152 Stoops.

One Ame of Antwerp contains 300 Stoops, every Stoop weighing 6 lb. called a Stone. And 6 of these Ames of Wine make in

6 Ames of
Antwerp
compared with
the Measure of
other Places.

1	10	Algarve	34	Starre
		Anſoy, or Baſtard Spain	2	Pipes 16 Stoops
1	8	Artois	4	Hogheads
1	3	Auxere	3	Puncheon
1	3	Ay, as Artois		
1	7	Bologna	13	Corbes
1	3	Bordeaux	4	Hogheads

1 7 Calabria

- 1 7 *Calabria* ————— 8, *Salmes*
 3 3 *Canaries* ————— 2 Pipes of 150 Stoope, or 1 9.
 Butt. Every Butt at *Antwerp* 158 Stoope. They measure by
 the Roove of 30 lb, which at *Antwerp* is 5 Stoope. Every
 Butt contains 30 Rooves. And the Pipe 30 Rooves of 28 lb
 weight.
- 1 5 *Candia* ————— 80 Mostaches
Canado, or Condado Spain ————— 2 Butts
 1 3 *Coniac* ————— 2 Pipes or 4 Hogheads
 1 5 *Constantinople* ————— 180 Almes
 96 1/2 Almes of Oyle there is at *Venice* a Milliar.
- 1 5 *Corfu* ————— 37 Zare, or Sare.
 1 7 *Ferrara* ————— 12 Nastelli, of 8 Seccheio
 1 7 *Florence* ————— 16 1/2 Barrels of 20 Fiaschi
 or 18 Stoope *Antwerpe*, 3 Barrels is 1 Starre, and 1 Starre is
 54 Stoops *Antwerp*.
- 1 7 *Istria* ————— 15 Venas
 1 3 *Liborne* ————— 5 1/2 Hogheads
 1 10 *Lisbon* ————— 37 1/2 Almudas.
 1 Almuda is 1 1/2 Roove of Sevil, accompting 8 Sevil Somers or
 Covados to 1 Roove, that is 12 Cavados to 1 Almuda. Every
 Covado 4 Quartils, or Quarts.
 Oyl Measure is by *Alqueri* or *Canter*, 1 *Alqueri* is 6 Covados,
 1 *Cantar* 4 *Antwerp* Stoope.
- 1 2 *London* ————— 252 Gallons.
 So is 1 Ame of *Antwerp* 42 Gallons at *London*.
- 3 3 *Madera* ————— 2 Pipes, lacking 16 Stoops.
 1 3 *Orleans* ————— 4 Hogheads, lacking 10
 Stoops, or 60 lb of *Antwerp*.
- 1 7 *Padua* ————— 1 1/3 Cara.
 Oyl is by the Milliar of 1185 lb.
- 1 3 *Paris, as at Orleans.*
 1 Hoghead 36 Sextiers, 1 Sextier 4 Quarts, 1 Quart 2 Pints,
 1 Pint 2 Choppins or Obles, 1 Choppin 24 Poulceons. *Cotgrave*
 calls that a Muid of Wine which *Malines* calls an Hoghead,
 and saies 3 of them go to a Ton by the Customs of *Clermont*.
 And between the Muid and Sextier in quantity placeth a Barril
 to contain 9 Septiers or 72 Pints. The Septier he counts all one
 with the Sextier or Sextary, and when taken for a Wine Mea-
 sure is 8 *Paris* Pints, and in all these Vessels the Cask shall hold
 so much clear Wine besides the Lees. The Pint is the 288
 part of the Muid, almost as big as our Quart, weighing 27
 Ounces. The Chopine in *Latine Ciopina*, or *Ciopinta*, is called
Oble quasi Obolus being half the *Paris* Pint, But in *St. Denis* and
 some other places 3 go to a Pint. The Poulceon, a small measure
 of little use, save to try the Gage of the Small Sextier of 3 1/3,
 and Semisextier of 1 1/2; measures used by the *Apothecaries*. The
 Poulceon 1 3. *Alsted* mentions a *French* Measure called *Arroba*,
 or Roove to contain 2 Sextaries. But *Cotgrave* makes it contain
 as much as will weigh 25 lb. Others write of a *Coupe* con-
 taining about 4 1/2 Pints of *Paris*. Some an *Amphora* of about
 36 Quarts, but both these seem *Roman* Measures, and perhaps
 may be in use in those parts of *France* that border on *Italy*.
- 1 3 *Poitou* ————— 2 1/2 Pipes, or 5 Hogheads.
 1 7 *Piran* ————— 12 Urna.
 1 7 *Puglia, as Calabria.*
 Oyle also 8 Salmes, 1 Salme 10 Star, 1 Star 32 Pignatoli.
- 1 7 *Rome* ————— 7, Brenten.
 1 Brent 96 Pockal, (wrote also Bocal), or 13 1/2 Rubes or Stones
 of 10 lb, of 30 1/3, or 42 Stoops of *Antwerp*.
 For Honey the Pound is 44 1/3. The *Spaniards* call Bocal,
Azumbre. 1 Barrillis 32 Bocals. 1 Bocal 4 Foglietta's, that is
 128 in the Barrillis.

Almes of Ant-
 werp compared
 with the Mea-
 sures of other
 Places.

6 Ames of Antwerp compared with the Measure of other Places.

- 1 14 Seres or Sherry, as Canaries.
1 14 Seville

56 $\frac{1}{2}$ Rooves.

1 Roove, or Arroba, (in *Latine*) 8 Somers or Azumbres, 1 Sommer 4 Quartiles or Sextaries. 1 Quartil $\frac{1}{2}$ of a Stoope of *Antwerp*. They deliver 27 and 28 Rooves in a Pipe, but Oyle by 40 and 41 Rooves in a Pipe.

Hunt saith that 32 *Spanish* Sextaries are equal to 24 *Roman*, and that the *Spanish* Sextaries contain 3 $\frac{1}{4}$ Parillas, which he makes of Oyle 3 $\frac{3}{4}$ of Water 4 $\frac{3}{4}$ ferè, of Syrrup 6 $\frac{3}{4}$ ferè accompting 17 $\frac{1}{4}$ of Water = 23 $\frac{1}{2}$ of Syrrup. And mentions the *Modius* or *Moyo* to contain 16 Arroba's or Amphoras. *Heylin* p. 1044 affirms the Arroba of *Spain* to contain 25 Bushels.

- 1 7 Treviso 11 Confi. the 10. one Càra.
3 2 Tripoli 45 Metares, of 42 Rotules.
3 2 Tunis 60 Matali of 32 Rotules.
1 7 Venice 80 Mostati.

38 make 1 Butt, and 76 an Amphora, 16 $\frac{4}{5}$ Quarti Befonts Measure. The 4 one Bigontz. Bigonts is a *French* Hogthead 1 Quart 18 Stoope of *Antwerp*, 15 $\frac{2}{3}$ Quarti Measure Secchio or small Measure of 4 Tischauffer.

Amphora is 4 Bigonts or Bigontines, 16 Quarti Bigots Measure, 18 $\frac{1}{2}$ Quarti Secchio. Lagel is a Puncheon, Amphora is 2 Ames. Oyle and Honey some measure by Amphora, but most by the Milliar of 1210 lb.

Hunt reports the Follieta at *Venice* equal to the *Spanish* Sextarie or Quartil, and to contain in Water or Wine 16 $\frac{3}{4}$.

1 Congitella 4 Bocals, 1 Bocal 2 Medios.

1 Medius 2 Follieta's, that is 16 in the Congitella.

- 1 7 Verona 1 $\frac{1}{2}$ Cara, or 14 Brents.

1 Brent, 16 Baffes. Oyle by the Milliar of 1738 lb, which is 8 Brenten and 11 Baffes.

- 1 7 Vincenza 1 $\frac{1}{2}$ Cara.

Oyle by the Milliar of *Venice*.

- 1 5 Zant, as Corfu.

Foreign Beer-Measures.

Beer Measures.

English Barrel the Gallon whereof how many Stoope.
Holland Barrel how much.
Lubeck Barrel how much.
Dantlick Fat how much.
Corn Measures.
The Last.
The Muid.

The Barrel of Beer in *England* 36 Gallons, is 48 Gallons Wine-Measure. Every Beer-Gallon 2 Stoope in *Flanders*, and at *Amsterdam* 1 $\frac{2}{3}$ Stoope.

The Barrel of Beer in *Holland* containeth 54 Stoope, at *Amsterdam* 56 $\frac{1}{2}$ Stoope, accompting 60 Stoope there for 64 *Flemish*.

The Barrel of Beer of *Lubeck* is just 50 Stoope of *Antwerp*.

Foreign Corn-Measures.

The Last is differently reckoned, but with the *English* just 2 Tons or 4000 lb in dead Weight, reckoning Barley 5 Score to the Hundred.

In *France* several parts of the *Netherlands* and other places, they use a Measure called a Muid, Mudde, Moyo, &c. differently according to the Language of the Countrey where used; derived as conceived from *Modius*. This Measure in *France* as used for Land and Wine is spoken to before. The more proper use thereof is for Corn, Coals, Salt, and dry Commodities.

A Table of Corn Measure.

Ordinary French Corn-Measure according to Cotgrave.

Muid.	Septiers.	Mines.	Minots.	Boisseaux.	Quarts.	Pints
	12	24	48	144	576	1152
	Septier.	2	4	12	48	96
		Mine.	2	6	24	48
			Minot.	3	12	24
				Boisseau.	4	8
					Quart.	2

A Muid

A Muid of Coales is 16 Mines.

Muid of Coales

Hunt, on whole Authority I cannot say, counts the Common Measure of *France* to be 1 Muid, 24 Boisseaux, and that the same is about 18 Bushels of our Water Measure, and 32 of these Muids go to the Hundred: But by another sort of Measure he calls *Oldron*, 1 Hundred is 20 Tons, or 36 Muids.

A Septier of Coales and Oates 21 Boisseaux, though of Wheat but 12, as above, and is said to weigh 220 lb by *Nicot*. Nevertheless *Vigenere* will have the Septier in the Table above to be Rye Measure, and that of Wheat to weigh 240 lb. The Septier of *Moulin's* is 16 Boisseaux. The Small Septier is spoken of before as a proper Liquid Measure.

Septier of Corn
The Sorts.

The Mine above is ordinary. The Mine of *Clermont* double, viz. as much as the Mine of *Corn* Septier in the Table.

The Boisseaux is about 3 of our Gallons weighing 20 lb by *Cotgrave*, and our 3 Gallons 21 lb *Avoirdupois*.

Boisseaux
how much.

Some report a Bichot to be a Corn Measure used in *Burgundy*, and to be 2 Mettres, and 1 Mettre to contain 2 of that Countrey Boisseau.

Bichot and
Mettre.

The *Russia* Cherfrid is about 3 English Bushels, as *Fletcher* affirms.

Cherfrid of
Russia.

The Laste, or Last of *Amsterdam* is 27 Moyes or Mudden. 1 Mudde, 4 Scheppels. Or 1 Last is 29 Sacks, 1 Sack 3 Achtelings, or Archtelings.

Laste of Am-
sterdam the
content how
much in several
other places.

This Last of *Amsterdam* maketh in the following places, as at

1	8	<i>Antwerp</i> -----	37½	Vertules.
1	3	<i>Bordeaux</i> -----	38	Boisseaux, whereof 33 to the Last.
1	8	<i>Bridges</i> -----	17½	Hoot.
1	8	<i>Brussels</i> -----	10½	Mudden, by <i>Malines</i> the Mudde or Vertule there is One.
1	3	<i>Calais</i> -----	18	Rasiers, agree with <i>England</i> .
1	9	<i>Conningsbergh</i> ---	6	of a Last, 6 Last there being 7 at <i>Amsterdam</i> .
1	1	<i>Copenhagen</i> -----	23	Small Barrels, whereof 42 make a Last.
2	1	<i>Cyprus</i> -----	40	Medimnos, 1 Medimnus 2 Cipros.
1	9	<i>Dantick</i> -----	56	Scheppels, whereof 60 make a Last.
There 4				Scheppels are 1 Mudde, which is the Skippound of 340 lb.
1	8	<i>Delfe</i> -----	87	Achtelings.
1	8	<i>Dort</i> -----	28	Sacks.
1	8	<i>Dunkirk</i> -----	18	Rasiers, Water-Measure
1	1	<i>Ebbelrorff</i>	23	<i>Danic</i> Barrels of 36 to the Last.
				Some allow 42 to a Last.
1	4	<i>Emden</i> -----	55	Werps, whereof 61 make a Last, or 15½ of 4 Werps, <i>Malines</i> 15½, but then should the Last be 62.
1	8	<i>Enckhuysen</i> -----	42	Sacks.
1	1	<i>Fameren</i> -----	78	Scheppels, whereof 96 to the Last.
1	8	<i>Gaunt</i> -----	4	Muddes and 7 Halsters, or 55 Halsters.
				One Mudde there is 12 Halsters.
1	7	<i>Genoa</i> -----	23½	Mina.
2	6	<i>Goa</i> -----		Their Bharo for Pepper is 3½ Quintals of <i>Portugal</i> Weight every Quintal 100 lb. For Wheat, Rice, and other dry things 1 Candil is 20 Mao's or Hands, 24 Medida's 1 Mao, and 1 Medida 9½.
1	8	<i>Groninghen</i> -----	33	Muddes.
1	1	<i>Hamborough</i> -----	83	Scheppels, 90 whereof make a Last.
1	1	<i>Heyleger-haven</i> -----	80	Scheppels, of 96 to the Last.
1	1	<i>Horne</i> , as <i>Enckhuysen</i> .		
1	10	<i>Lisbon</i> -----	225	Alquiers, of which 240 to a Last, or 4 Moyo's of 60 Alquiers to the Moyo, and so in the <i>Portugal</i> Islands.
1	2	<i>London</i> -----	10½	Quarters, or 82 Bushels.
1	1	<i>Lubeck</i> -----	85	Scheppels, whereof 96 to the Last.
1	4	<i>Atechulborough</i>		
1	8	<i>Aledenblick</i> , as <i>Enckhuysen</i> .		
1	9	<i>Alcryn</i> -----	17	of a Last.
1	8	<i>Middleburgh</i> -----	40	Sacks 41½ to the Last in all <i>Zeland</i> .
1	1	<i>Nelleboghe</i> -----	23	Barrels, whereof 42 to the Last.
1	7	<i>Paglia</i> -----	32	Cara, of 36 Timani.
1	9	<i>Riga</i> -----	42	Loops.
1	3	<i>Rochell</i> -----	128	Bushels, 4 to every Sestier.
1	4	<i>Roslock</i> , as <i>Lubeck</i>		

Laft of Am-
sterdam how
much in feveral
other Places.

1	8	Rotterdam, as Delfe.	
1	3	Rouen—	20
1	8	Schonehaven—	88
1	14	Sevill—	54
1	7	Sicilia—	38
1	4	Stetin, as Coningsbergh.	
1	15	Sweden—	23
1	8	Texel—	58
1	7	Venice—	32

until 30 Mines. 1 Mine 4 Boiffeaux.

Achtelings.

Hanegas. A Laft there is 4 Cahis. 1 Cahis is 12 Hanegas.

Medimno's, of 6 Moyo's.

Barrells.

Loops.

Starr.

Salt Measures.

Foreign Salt-Measures.

Sack of Ar-
muyden, how
much in fe-
veral other
Places.

At *Armuyden* in *Zeland* they reckon $8\frac{2}{3}$ weighes for a Hundred, 1 Weigh $11\frac{1}{2}$ Sacks, 1 Sack 4 Measures. And 15 Weighes of *Brumage* Salt make the great Hundred.

The Sack of Salt of *Armuyden*, being 122 small Barrells for the 100, make in the other places, as at

1	8	Amsterdam—	102	Scheppels
1	8	Antwerp—	144	Vertels of 24 to the Laft, and 6 Laft to the 100.
But White Salt is measured by a Less Measure of 12 on the 100.				
1	8	Axells —	102	Measures.
1	8	Bruges—	104	Measures.
1	3	Brumage—	$\frac{2}{7}$	parts of 100 of 28 Moyo's, 1 Moyo, 12 Sacks, and by the Load, 10 Load in the 100, and 48 Moyos or Muys to the laft, or 21 Barrells.
1	14	Cadiz—	22	Cays, or Caies.
1	3	Calais —	130	Barrells 19 to the Laft, but 20 by Freighting.
1	8	Damme, as Axells.		
1	1	Denmark—	$6\frac{2}{3}$	Laft.
1	8	Deventer, as Amsterdam.		
1	8	Dunkirk—	92	Water-Measures, but 104 Land-Measures.
1	4	Emden —	100	Barrells of 14 to our Laft.
1	8	Gannt —	108	Sacks, or Barrells.
1	1	Hamburgh—	7	Laft, whereof 80 Barrells make the 100.
1	8	Ipre —	144	Measures.
1	10	Lisbon —	25	Moyo's.
1	2	London—	$7\frac{1}{2}$	Laft of 18 Herring Barrells, but by Weys, $11\frac{1}{2}$.
1	1	Lubeck—	7	Laft of 18 Barrells.
		Mary Port—	28	Moyo's.
1	8	Ostend—	98	Measures.
1	7	Piram —	70	Mofe.
1	8	Rotterdam—	100	Measures, whereof 6 make 1 Mudde of 18 to the 100.
1	3	Rouen —	$16\frac{1}{2}$	Muys, and fo almost all France.
1	14	St. Lucar—	21	} Cays, or Caies.
1	10	St. Tubal—	20	
1	15	Sweden—	112	Tunnes, or Barrells of 16 to the Laft.
1	7	Venice, as Piran.		
1	8	Utrecht, as Amsterdam.		

Sea-coal Measures.

Foreign Sea-Coal-Measures.

Laft of New-
castle how
much at feve-
ral other
Places.

The Laft of *Newcastle* Sea-coales is $7\frac{1}{2}$ Chalders, which *Malines* faith at *London* and *Yarmouth* make 10 Chalders. Some say more, besides the allowance at *London* of 21 for 20.

The same Laft is at

1	8	Alft—	200	Muddes.
1	8	Amsterdam—	$13\frac{1}{2}$	Hoot of 38 Measures to an Hoot, or Hoet.
1	8	Antwerp—	175	Vertels.
1	8	Bruges—	100	Measures for Oates.
1	3	Condet —	44	Muys, 80 Muys make 1 Cherke
1	8	Dort —	12	Hoot, also by Weights of 144 lb. one Weigh 24 Stone, 1 Stone 6 lb.

1 8 Gannt

1 8 Gaunt————144 Sacks, or 14 Muys.

1 8 Middleburgh, by Weighs of 180 lb to a Weigh.

1 8 Ostend, as Bruges.

1 3 Rouen————100 Barrels, allowing 104 for the 100.

1 8 Zeland————68 Herring Barrels.

Laste of New-

castle how

much at several

other Places.

Foreign Weights.

Generally 3 sorts of Weights are used for Merchandise.

1. Weights of great Content, as Hundreds, Kintalls, Centeners, Talents, Thou-
- sands, Weighs, Skippounds, Charges, Lispounds, Rooves, &c.
2. Weights of lesser Content, as Pounds, Mina's, Manehs, Rotuli, &c.
3. Small Weights, as Ounces of 12, 14, 16, 18, 20, 30, &c. to the Pound, and the
- Subdivisions of the Ounce.

Foreign

Weights of

3 sorts.

Talents, of the Hebrews, Greeks, and Latines are seen before.

Cantars, Centeners, or Kintals sometime wrote Quintals, accompted by Merchants

as Hundreds; are of 100, 112, 120, 125, 128, 132, and 140 Pounds.

Talents,

Cantars, &c.

Weighs, or Weys, are commonly 165 lb, or 180 lb, or 200 ½ lb for a Charge.

Skippounds, used in many places quasi Shippound, or Shippond, for as in Italy and other

Countreys the Carga, Cargo, or Charge is the Loading of an Horse of 300, or 400 lb.

So is the Skippound taken for the Divident of a Last of Corn Laden in a Ship. Skip-

pounds are of 300, 320, 340, and 400 lb to the Skippound. Cargo is often taken for

the whole Lading or Burden of a Ship.

Weighs, &c.

Skippounds,

Carga's.

Lispounds, of 15, 16, and sometimes 20 lb to the Lispound.

Rooves, or Arrobas of 10, 20, 25, 30, and 40 lb to the Roove.

Stones, of 6, 8, 10, 14, 16, 20, 21, 24, 32, and 40 lb. to some Stones.

Poade, of Russia by Heylin 140 lb.

Lispounds,

Rooves,

Stones.

Poade.

Mixias, is commonly understood to be 10000 Drams, of 8 to 1 3, and 12 3 to

Mixias.

1 lb Sestertio's of Cleopatra in Egypt and other places in Africa, were 2 ½ lb, for 50

Sestertio's made 125 lb, but in Thracia it was but 2 ½ lb.

Sestertio of

Africa, &c.

Pounds.

Pound is divided into more or less Ounces.

Mark Weight, commonly 8 3.

Markes.

Mark Pound 16 3, that is 2 Markes.

Mark Pound.

Mina Ptolomaica, 1 ½ Rotuli, or 18 Ounces, or 144 Drams, and in lesser Divi-

sions thus.

	Rotuli.	Ounces.	Drams.	Scruples.	Oboli.	Lupines.	Siliquas, or Carrats.	Aercoli.
Mina.	1 ½	18	144	432	864	1296	2592	6912
	Rotulus.	12	96	288	576	864	1728	4608
		Ounce.	8	24	48	72	144	384
			Dram.	3	6	9	18	48
				Scruple.	2	3	6	16
					Obolus.	1 ½	3	8
						Lupine.	2	5 ½
							Siliqua, or Carrat.	2 ½

A Table of the

Mina Ptolomaica.

Mane, or Maneh, in Arabia, double 1 of 16 3, and 1 of 20 3.

That called Alialica, Basaria, Alanthalica, and Aegyptia.

Maneh of

Arabia the

Sorts and

Names.

Rotulus.

This Romana, and is indeed of Alexandria, the Pound there being 20 Ounces.

Rotulus, in Arabia, Syria, Asia minor, Egypt and Venice, reckoned for a Pound

thus divided.

Rotulus,

A Table of the Arabian Rotulus.

Of Geodeticals.

Rotulus, or Pound.	Sachosi, or Ounces.	Sextaries, or Cicles.	Deniers, or Aureos.	Darchiny, or Drams.	Scruples, or Garma.	Obolos, or Orloffs.	Danigs, or Lupines.	Kirats, or Siliquas.	Aereola's, or Kestuffs.
	12	24	84	96	288	576	864	1728	3456
Sacros Sachos or Ounce.		2	7	8	24	48	72	144	288
		Sextarie or Cicle	3½	4	12	24	36	72	144
			Denier or Aureus Aunius Audanakus	1½	3½	6½	10½	20½	41½
				Dram or Darchiny Alky Oliginat	3	6	9	18	36
					Scruple Garmic or Kenmer	2	3	6	12
						Obolus Orloff or Onolaffat Onolum	1½	3	6
							Daning or Danic Lupine	2	4
								Carrat Kirat or Siliqua	2

Phylick Pound at Venice.
Lupines there.
Kestuff, how much.
Pound of Alexandria.
Italian Pound.

Some mention the Phylick Pound at Venice to have but 7 Drams in the Ounce.
The Lupines at Venice called Sextula's, because 1 ⅓ hath 72, which is 6 times 12.
Every Kestuff, or Aereolum (or Areolum) is the Weight of 2 Barley-Cornes, so is there in the Rotulus 6912 Graines.
The Alexandrian Pound 20 ⅓, the Ounce 8 ⅓, &c.
The Italian Pound generally is divided into 12 ⅓, 1 ⅓ into 2 Staters, and 1 Stater into 4 Drams; so hath 1 lb 24 Staters, 96 Drams.
But in Phylick there, and in other Places thus.

A Table of the Italian Phylick Pound.

	Ounces.	Loots.	Sizaynes, or Siliqua's.	Drams.	Scruples.	Obolos.	Siliqua's.	Graines.
Pound.	12	24	48	96	288	576	1728	5760
	Ounce.	2	4	8	24	48	144	480
		Loot.	2	4	12	24	72	240
			Sizayne or Siliqua	2	6	12	36	120
				Dram	3	6	18	60
					Scruple	2	6	20
						Obolus	3	10
							Siliqua	3½

Spain, some say hath a *Mina Romana*, which contains 20 ℥ . A Common Pound of Spanish 16 ℥ . and a Phyick Pound of 12 ℥ . each Ounce divided into 8 Drams. The Ounce of the *Toletan* Phyick Pound excepted, which hath 9 ℥ , as some affirm.

Pound Weights of Spain.

A Table of the Mina Romana of Spain.

	Libra.	℥.	Duels.	Quarterns.	Sixths.	℥.	Syrian Beans.	℥.	Obolos.	Carats.	Chalcos.	Graines
Mina Romana	1 $\frac{2}{3}$	20	60	80	120	160	240	480	900	2880	5760	11520
	Libra	12	36	48	72	96	144	288	576	1728	3456	6912
		$\frac{2}{3}$	3	4	6	8	12	24	48	144	288	576
			Duel	1 $\frac{1}{3}$	2	2 $\frac{2}{3}$	4	8	16	48	96	192
				Quartern	1 $\frac{1}{2}$	2	3	6	12	36	72	144
					Sixth	1 $\frac{1}{3}$	2	4	8	24	48	96
						3	1 $\frac{1}{2}$	3	6	18	36	72
							Syrian Beane	2	4	12	24	48
								℥	2	6	12	24
									Obolus	3	6	12
										Carat	2	4
											Chalcus	2

The Common Pound of Spain.

The Physick Pound of Toledo.

Tables of the Pounds of Spain.

	Marks.	Ounces.	Drams.	Adarmes or Adarames
Pound	2	16	128	256
Mark		8	64	128
		Ounce	8	16
			Dram	2

Pound	Ounces.	Drams.	Scruples.	Graines.
	12	108	324	6480
Ounce		9	27	540
		Dram	3	60
			Scruple	20

Pound Weights of France.

Pounds of France.

The Weight used by the Merchants for the most part is of 16 ℥ , called *Liure d'Anvers*, though in some places but 14, others 18 ℥ . *Cotgrave* writes the *Liure* or Pound of *Lyon* to be 15 ℥ . that of *Spaigne* but 14 ℥ . And divides the Pound of 16 ℥ . into 32 Halves, 64 Sezaines, 128 Treseaux, 256 Gros, 512 Demigros. And the Pound used by the *Farriers* consisting of 12 ℥ into 90 Drams, 270 Scruples, 540 Obolos. After *Malines* the Ordinary, or Pound commonly used for *Merchants* is parted thus.

The Pound Weight of Paris.

The Physick Pound of Lyons.

Tables of the Pounds of Paris and Lyons.

Pound	Ounces.	Grosse.	Scruples.	Graines.
	15	128	384	9216
Ounce		8	24	576
		Grosse	3	72
			Scruple	24

Pound	Ounces.	Drams.	Scruples.	Graines.
	12	96	288	5760
Ounce		8	24	480
		Dram	3	60
			Scruple	20

Cotgrave mentions a Weight called *Sextule* of 4 Scruples, or the Sixth part of 1 ℥ . *Sextule*, the Weight.

Pounds of Germany.

Tables of the Pound of Vienna and Physick Pound.

Pound Weights of Germany.

The Pound Weight of Vienna in Austria.

	Ounces.	Loots.	Quints.	Pennings.	Grains.
Pound	16	32	128	512	12800
Ounce	2	8	32	128	800
		Loot	4	16	400
			Quint	4	100
				Penning	25

The German Physick Pound, by Alsted.

	Ounces.	Drams.	Scruples.	Graines.
Pound	12	96	288	5760
Ounce	8	24	480	
		Dram	3	60
			Scruple	20

Pounds of the Low Countries.
Pounds of Bridges.

In the Low Countries they use Pounds of 12, 14, 15, &c. Ounces. At Bridges in Flanders they have 1 lb of 14 3/4, and 1 lb of 16 3/4. The 100 lb of 16 3/4 make 108 lb of 14 3/4, but the Ounces of 14 to the lb are the heaviest for 100, of these are 105 1/2 Ounces of 16 to the Pound, This lb is thus divided.

Table of the Bridges Pound.

	Ounces	Loots	Sizaines	Drams.
Pound	16	32	64	128
Ounce	2	4	8	
		Loot	2	4
			Sizaine	2

Dram, or Quint.

Weights at Antwerp.

At Antwerp they use to weigh by the Hundred Pounds even Weight, called Suttle, for which commonly at the Weigh House is allowed 101 lb. A Stone is 8 lb. The Skippound 300 lb. The Weigh 165 lb. The Carga or Charge 400 lb. which is two Bales of 200 lb each for an Horse to carry. The Pound there is 16 3/4. This 100 lb of Antwerp weigheth in the Places following,

The 100 lb at Antwerp how much at divers other Places.

- 1 3 Abbervile ————— 94 1/4 lb.
- 2 11 Achri ————— 17 1/4 Rotuli. The 100 A Cantar Tambaran.
- Adler ————— { 138 lb Ordinary Weight.
- { 91 lb To weigh Steel, Tinne and Copper.
- 1 8 Ailft ————— 108 lb.
- 3 1 Alcario ————— { 164 lb.
- { 78 Minas of 16 3/4 to the Mina.
- { 27 Rotuli of 6 lb to the Rotuli.
- 1 Peso is 1 1/2 Metallicum, or a Dram.
- 50 Metallici 1 Mark. Our Mark 42 Metallici.
- Musk and Amber sold by this Weight in Egypt.
- 2 11 Aleppo ————— 22 Rotuli of 100 to a Cantar.
- 1 Rotulus is 60 3/4, or 480 Metecalos or 3.
- 1 3/4 is 8 Metecalos or Dragmes.
- 1 3/4 or Metecalo is 1 1/2 Peso.
- 10 Peso's are 1 Onga, or Ongia, to weigh Civer.
- 2 1 Alexandria ————— { 108 Rotuli, of 100 to a Cantar.
- { 78 Minas of 20 Ounces.
- America Malica — { 90 lb, of 12 Ounces to the lb.
- { 36 Minas Sestertias of 30 3/4.
- 2 11 Aman, as Aleppo.
- 1 8 Amsterdam ————— 94 1/4 lb. And for Silkes they use the Weight of Antwerp.
- 1 7 Aquila ————— 147 lb.
- 1 3 Aquisnort ————— 102 lb.
- 2 2 Arabia ————— { 78 Rotuli.
- { 104 Maires, or Minas.
- { 148 Pounds.
- 936 Ounces, or Sachosi 12 to 1 Rotulus.

3 1 Arcadia

The 100 lb. at
Antwerp how
much at divers
other Places.

- 3 1 *Arcadia*— 92 lb. and 83 lb for Mavigetto.
1 5 *Archipelago*— 120 lb.
 Armara bona— { 105 lb. of 16 3 to the lb.
 { 93 lb. of 18 3 used for Silk and Copper.
 { 54 lb. of 32 3 Flesh Weight.
- 2 3 *Armenia*— 130 lb.
1 14 *Arragon*— { 106 lb.
 { 96 lb. Great Weight for Wooll.
1 8 *Arfchot*— 100 lb. all one with *Antwerp*
1 8 *Audinarde*— 110 lb.
1 3 *Avignon*— 111 lb. A Centener is 2 Frailes of 56 lb.
1 4 *Ausburgh*— 95 lb.
1 8 *Barrow Op Zome*— 98 lb.
1 14 *Barcelona*— { 96 lb. Wooll Weight.
 { 106 lb. Common Weight.
 { 131 lb. Saffron Weight.
- 1 4 *Bafil*— 96 lb. They use Centeners of 100 lb, 120 lb, and 132 lb.
1 7 *Bergamo*— 137 lb. and 108 lb. by the 2 Quintals.
1 1 *Bergen*— 90 lb. but uncertain weighing with a Sling.
1 4 *Bibrach*— 92 lb. of 16 3 to 1 lb as Constance.
2 11 *Barutti*— 21 Rotuli.
1 7 *Bologna*— 53 lb. of 30 3 to weigh Wax and Wooll by Rooves of 1 lb.
1 3 *Borgoingne*, as *Abbeville*.
1 4 *Botfen*— { 138 lb. Ordinary Weight.
 { 91 lb. To weigh Steel, Tinne, and Copper.
1 3 *Bourdeaux*, as *Abbeville*.
1 4 *Breslau*— 120 lb. by the Centener of 24 lb to 1 Stone, and 5 Stone to
 1 Centener. And 5 1/2 Stone to the Centener of 132 lb
 there also used.
- 1 7 *Brescia*— 184 lb. and for *Venice* Gold 136 lb.
1 8 *Bridges*— { 100 lb.
 { 93 lb. for Butter and Cheefe, The Stone 6 lb. and 20 Stone
 1 Weigh, but Wooll Weight is 108 lb. weighed by
 Stones of 6 lb, called Nailes or Neiles. 18 Neiles to the
 Hundred, 45 Neiles to the Weigh, 2 Weighs to 1 Pocket
 of Wooll. *Hunt* saies 18 Neiles is 144 lb of our Wooll
 Weight.
- 1 8 *Brussels*, as *Arfchot*.
 Bucca— 44 Ocha's.
1 14 *Burgos*— 93 Rotuli.
2 1 *Bursa*— 88 Rotuli.
3 4 *Cabo verde*— 107 1/2 lb, or Rotuli, A Quintal is 128 lb of 4 Rooves of 32 lb.
1 7 *Calabria*— 147 lb.
1 3 *Calais*— { 111 lb. Ordinary Weight.
 { 92 lb. Merchants Weight.
 { 114 lb. English Wooll Weight.
- 2 6 *Calicut*— 80 Aracoles. *Malines* p. 18. mentioning the Baccar or
 Bahar at *Calicut* to be at *Lisbon* 4 great Quintals of 112 lb
 to the Quintal, and that the 4 Quintals are 480 Aracoles,
 that is 120 Aracoles for 1 Quintal. And again that the
 Bahar is 20 Faracoles, which is 5 Quintals at *Lisbon* of
 32 lb per Roove, is not well to be understood Seeing
 the great Quintal at *Lisbon* is 128 lb or 4 Rooves of 32 lb
 per Roove; whereas 4 Quintals of 112 lb is but 448 lb,
 and 5 Quintals of 128 lb is 640 lb, unless there be 2 sorts
 of Bahars at *Calicut*, 1 of 48 Aracoles, and another of
 20 Faracoles. Or that the Bahar be 5 great Quintals at
 129 lb the Quintal, that is 645 lb for so many Pounds
 or *Portuguese* Rotuli are in 480 Aracoles for 100 lb of
 Antwerp, which answer to 107 1/2 lb of *Portugal* Weight
 by his own Concession in the same Page a little before.
- 3 3 *Canary Islands*— 107 lb. as *Sevill*.
1 5 *Candia*— { 138 lb. for Gold Thread.
 { 89 Rotuli, whereof 100 is a Cantar or Quintal.

The 100 lb at
Antwerp how
much at divers
other Places.

- 1 7 Carpi, as *Aquila*.
1 14 Castile—102 lb.
Cataio—87 Rotuli 100 to a Cantar.
1 7 Cesena, as *Bergamo*.
1 4 Collen—93 $\frac{1}{3}$ lb.
1 7 Como, as *Aquila*.
1 9 Coningsbergh—125 lb, which is a Centener. A Last of Wheat there
5200 lb. a Stone 40 lb, a Skippound 10 Stone, that is
400 lb.
1 4 Constance—92 lb. of 16 $\frac{3}{4}$ or 32 Loot. Some by the Centener of
100 lb, and some of 120 lb.
1 5 Constantinople.—{ 87 $\frac{1}{2}$ Rotuli, 100 to a Cantar.
39 Ochaa, *Hunt* writes it *Cohaa*.
2 $\frac{1}{2}$ Metallici, which is their Dram, make 3 of ours.
1 1 Coppenhagen—96 lb. There the Centener is 112 lb. A Stone is 10 lb.
A Skippound 32 Stone, or 20 Lippound of 16 Mark
Pound which is a Skippound, or 320 lb.
1 5 Corfu—{ 97 lb. Great Weight.
115 lb. Small Weight.
1 8 Cortrycke, as *Audinarde*.
1 9 Cracon—124 lb. The Centener there is 136 lb.
1 7 Crema, as *Aquila*.
1 7 Cremona—{ 143 lb. of 12 $\frac{3}{4}$ most used.
132 lb of 12 $\frac{3}{4}$, being 13 $\frac{3}{4}$ of the other.
60 lb. of 28 $\frac{3}{4}$ to the lb. used for Flesh.
2 1 Cyprus—20 $\frac{1}{4}$ Rotuli, 100 to the Cantar.
2 11 Damascus—26 Rotuli. There 1 Cantar is 5 Zurli, or Stone, and
1 Stone 20 Rotuli, 1 Rivola is 225 lb. *Antwerp*.
1 9 Dantick—120 lb. There 1 Last of Wheat is 4528 lb. The Last of
Rye 4245 lb. 1 Skippound 340 lb. of 10 great Stone.
1 Skippound 320 lb. of 20 Lippound. 1 Centener 125 lb.
1 Stone for Spices 24 lb. 1 Great Stone for Grofs Wares
34 lb. 1 Lippound 16 Mark Pound.
1 3 Diepe, as *Abbeville*.
1 8 Dixmude, as *Ailft*.
1 8 Doway, as *Audinarde*.
1 6 Dublin, and in Ire- { 91 $\frac{1}{2}$ lb. by the Great Hundred.
land generally.—{ 104 lb. Subtle Weight.
1 12 Edenburgh and all } 96 lb. and 103 $\frac{1}{2}$ lb. for 112 lb.
Scotland.—}
1 4 Erdford—85 lb. as at *Vienna*.
1 7 Faenza—132 lb.
3 2 Fez, or Fesse—96 lb. by *Hunt* wrote *Feas*, and noted as in *Portugal*.
1 7 Ferrara, as *Bergamo*.
Fio—96 $\frac{3}{4}$ Rotuli, or Scrutarij.
1 7 Fiume, as *Venice*.
1 8 Flanders—110 lb. for the most part, the places herein excepted.
1 7 Florence—125 lb. of 12 $\frac{3}{4}$ to the lb.
3 1 Forfori—65 Rotuli.
1 7 Forli, as *Aquila*.
1 3 France generally—111 lb. except herein excepted.
1 4 Frankford } as *Basil*.
1 4 Friburg—}
1 8 Gaunt as *Ailft*.
1 8 Gelderland—99 lb. The Places herein excepted
1 7 Genes, by Rooves to a Quintal of 4 Rooves, and 4 lb. over.
110 lb. a Quintal of Pepper.
114 lb. a Quintal of Ginger.
1 7—Geneva { 102 lb. Weight for Spices. A Carga is 270 lb small Weight.
85 lb. Great Weight.
1 4 Germany, A Centener of the small Weights is 100 lb, of the great 120 lb. and
132 lb. The Centener of 120 lb. is 5 Stone, of 24 lb.
per Stone.

- 2 6 Goa, as Portugal by Quintals, Arrobes or Rooves, &c. They have also another Weight called *Mao*, which signifieth the Hand, and weigheth 12 lb. used for Butter, Honey, Sugar, &c. in the Portugal Dominions. *The 100 lb. at Antwerp how much at divers other Places.*
- 1 14 Granada, as Armaria bona.
- 3 + Guynea, as Cabo verde.
- 1 1 Hamburgh———— 96 lb. The Centener 120 lb. of 12 Stone, 1 Stone 10 lb. A Lifpound 15 lb, and 20 Lifpound 1 Skippound.
- 1 4 Heidelbergh, as Basil.
- 1 8 Hertogenbosh, as Arschot.
- 1 8 Holland, as Gelderland.
- 1 8 Hulst, as Ailst.
- 1 8 Ipre, as Ailst.
- 1 7 Istria, as Venice.
- 1 5 Laarta———— 87 Rotuli, 100 to a Cantar.
- 1 5 Laconia———— { 138 lb.
78½ Rotuli.
- 1 7 Lansan, as Bergamo
- 1 5 Lavalona ————— 131 lb.
- 1 14 Leon ————— 109 lb.
- 1 5 Lepanto———— { 156 lb.
26 Rotuli, 1 Rotulus 6 lb.
- 1 4 Lipsich, as Basil.
- 1 10 Lisbone, See Calicut.
- 1 8 Lisle, as Audinarde.
- 1 2 London, and all— { 91½ lb. Gros Weight of the Kintal Weight 112 lb.
England.———— { 104 lb. Subtle Weight.
189½ Markes of 8 ⅔ Troy.
- 1 8 Louvaine, as Arschot.
- 1 1 Lubeck, as Coppenhagen.
- 1 7 Luca, as Aquila.
- 1 3 Lyons———— { 111 lb. ordinary Weight. A Centener is 112 lb.
102 lb. Almerick, or Weight of Geneva for Spices, abating
8 lb. per Cent.
94½ lb. by the Kings Weight to pay Custom by.
A Quintal is 100 lb. A Charge 300 lb. A Somme 400 lb.
- 3 3 Madera, as Cabo Verde.
- 1 8 Malines, as Arschot.
- 1 7 Mantua, as Aquila.
- 1 3 Marseilles ————— 111 lb.
- 3 2 Maroco, or Moroco, as Capo Verde.
- 1 14 Medina del Camporas Castile.
- 1 9 Melvin———— 124 lb. The Last of Wheat 5200 lb. The Skippound, and Stone as Coningsbergh.
- 1 4 Meysen———— { 100 lb. of 16 ⅔ to the lb. which is the Princes Weight,
called Zigoftatica.
96 lb. Merchants Weight.
148 lb. of 12 ⅔ to the lb.
- 1 7 Millan, as Cremona.
- 1 3 Mirabel, as Aquismort.
- 1 7 Mirandula, as Aquila.
- 1 7 Modena, as Faenza.
- 2 7 Molucco———— 88 Rotuli, 112, a Cantar.
- 1 3 Montpellier, as Avignon.
- 1 4 Munchen, as Ausburgh.
- 1 7 Naples ————— 120 lb. and for Venice Gold 134 lb.
- 1 15 Nareca ————— 120 lb. A Lifpound or Stone is 20 lb. and 20 Lifpound a Skippound, that is 400 lb. used for Rye, but for Wheat but 350 lb. to a Skippound.
- 1 7 Nicosia, or Nichosia, as Archipelago.
- 1 5 Nigropont———— 119 lb.
- 1 4 Norenburgh, as Constance.
- 1 4 Norlingen, as Ausburgh.
- 1 4 Offen, as Basil.

The 100 lb. at
Antwerp, how
much at divers
other Places.

3	2	Oran	94 Rotuli, 1 Cantar 5 Rooves, 1 Roove 20 Rotuli. 138 lb. for Spices. 1 Cantar 4 Rooves. 50 Rotuli for Corne, 1 Cantar 6 Rotuli. 61 Rotuli, for Cotton Wooll, 1 Cantar 15 Rotuli.
1	7	Oiranto	} as Bergamo.
1	7	Padua	
1	3	Paris	93 lb. accompting 4 Quarters of 25 lb to the Hundred.
1	7	Parma, as Aquila.	
1	4	Passau	87 lb.
1	7	Pavia, as Cremona.	
1	7	Piran, as Venice.	
1	7	Piedmont	} as Aquila.
1	7	Plaissance	
1	4	Poofen, as Breslaw.	
1	8	Popering, as Ailst.	
1	10	Portugal	107½ Rotuli or Araters. The great Quintal is 128 lb. of 4 Rooves, 1 Roove 32 lb. The Small Quintal is 112 lb. of 4 Rooves, 1 Roove 28 lb. The Quintal of Wax 168 lb, which is 1½ Quintal of 112 lb. of 4 Rooves of 42 lb. the Roove.
1	4	Prague, as Passau.	
1	7	Puglia, as Calabria.	
1	7	{ Ragusa Raviano Ravenna }	as Faenza.
1	7	Rechanati	137 lb. but to Gold Thread but 112 lb.
1	4	Regensbourgh, as Passau.	
1	15	Revell	120 lb which is a Centener. The Skippound there is 400 lb.
2	1	Rhodes	19½ Rotuli, A Cantar is 100.
1	9	Riga	120 lb. A Lifpound is 20 lb. and 20 Lifpound a Skippound.
1	7	Rimano, as Faenza.	
1	3	Rochel	111 lb and 119 lb. by the small Weight.
1	7	Romagna, as Naples.	
1	7	Rome	132 lb.
1	3	Rouen	{ 91 lb by the Viconte, accounting as at Paris. 94¼ lb. by the ordinary weight, and 4 lb. per Cent over.
1	4	Saltsburgh	{ 111 lb. Small Weight. 83 lb. great Weight.
1	3	St. Antoine	127 lb.
1	8	St. Omar, as Audinarde.	
3	3	St. Thomas, as Cabo Verde.	
1	14	Saragossa	112 lb. And the small Quintal 131 lb.
1	7	Savoy	{ 137 lb. 195 lb. Small Weight.
1	4	Saxony, as Meyfen.	
3	1	Sciba, as Antwerp, 320 lb.	is there a Skippound.
2	1	Scio, as Fio.	
1	13	Sequia, as Venice.	
1	14	Sevil	107 lb. { The great Quintal is 144 lb of 4 Rooves of 36 lb. The lesser Quintal is 120 lb. of 4 Rooves of 30 lb. The small Quintal is 112 lb. of 4 Rooves of 28 lb.
1	7	Sicilia	152 lb. of 12 3 per lb. 61 Rotuli of 30 3 is a Cantar of 24 Sestertios. 54 Rotuli for Flesh by Talents of 12 Sestertio's is 30 Rotuli.
1	4	Silefia, as Breslaw.	
1	13	Spelato, as Venice.	
1	4	Spiers, as Bibrach.	
1	4	Stetin	96 lb. The small Stone 10 lb. The great Stone 21 lb. The Centener 112 lb.
1	15	Stockholme	120 lb. The Skippound 320 lb. and also 340 lb. The Centener 120 lb. The Stone 10 lb.
1	4	Straelfont	92 lb. The Stone 10 lb. and the Lifpound 16 lb.
3	2	Suus, or Sus, or Fez.	

The 100 lb at Antwerp how much at divers other Places.

2	11	Syria	156	Minos, 1 Mina 100 Drams.
1	8	Tergos	107	lb.
1	3	Tholouse, as Avignon.		
3	2	Thunes, or Tunis	63	Rotuli.
1	9	Thoren	120	lb. The Stone is 24 lb.
1	8	Tournay, as Ailft.		
1	7	Treviso, as Bergamo.		
1	7	Trieste, as Venice.		
3	2	Tripoli, as Thunes.		
2	11	Tripoli	26½	lb.
1	14	Valentia	106	lb. by Quintals of 4 Rooves of 30 lb for Spices.
			134	lb. by Quintals of 4 Rooves of 36 lb.
				The small Carga is 360 lb. that is, 3 Quintals of 120 lb.
				The great Carga is 432 lb. that is 3 Quintals of 144 lb.
1	7	Venice	98½	lb. Great Weight, called <i>Ala Grossa</i> , used for Flesh, Butter, Cheefe, Leather, Dates, Yarne, Copper, Thread, Iron, Oile, Brimstone and Wooll.
			156	lb. Small Weight of 12 3, called <i>Ala Sotile</i> , most used for all Merchandise.
				An Ounce is 6 Saffi, 1 Saffi 24 Carrats, 1 Carrat 4 Graines.
				They also accompt by Thoufands, &c. with allowance of 2 lb. per Cent. in the Custom-House.
				1 Thousand 40 Mixti, 1 Mixti 25 lb.
				1 Carga 400, lb 1 Starre 220 lb. The Starre is Mensural.
				Starres for Corn 130 lb. Ginger 180 lb. Raisins 260 lb.
				The Starre contains 54 Pottles of Wine at Antwerp.
1	7	Veronna	90	lb. And for Gold Thread 143 lb.
1	4	Vienna	85	lb. as at Erdford, where also a Summe of Quick-Silver is 275 lb.
1	14	Villaco, or Vellica	80	lb.
1	4	Ulme, as Basil.		
3	2	Una	65	Rotuli for Cotton.
			75	Rotuli for Spices.
			94	Rotuli for Corn.
1	7	Urbini, as Bergamo.		
		Wallons Countrey, as Ailft.		
1	8	Walfland, as Gelderland.		
1	9	Wilde, as Riga.		
1	4	Wifel, as Ausburgh.		
3	1	Zaidin	77	Rotuli.
1	8	Zeland, as Gelderland.		
3	1	Zeroi	50	Rotuli.
1	8	Zurich Sea	110	lb.

Foreign Weights for Money.

Weights for Money

In Florence they use a Weight for Gold and Silver, and at Geneva for Silver called a Pound of 12 3. 1 3 is 24 Deniers, and 1 Denier is 24 Graines. So is there 6912 Graines in the Pound.

Pounds of Florence and Geneva.

In Naples their Pound is like wise divided into 12 3, and every Ounce into 8 Octany, or Octavos.

Pound of Naples.

The Mark Weight is used in many other Places, and at Antwerp containeth 8 3, and is heavier than their ordinary lb. by 5 upon the Hundred, as Malines saith. This Mark is divided in a double manner.

Mark of Antwerp.

Ounces.	English	Graines.
1 Mark	8	160
	Ounce	20
	English	32

Ounces.	Penny-weights.	Graines.
(2) Mark	8	192
	Ounce	24
	Penny-weight	24

Tables of Antwerp Mark.

A Table of the French Mark.

The Mark Weights of some other Places subdivided.

France.

	Ounce.	Groffes.	Deniers.	Graines.	Primes, or Garobs.	Seconds.	Tercies, or Malloquen.
Mark	8	64	192	4608	110592	2654208	63700992
	Ounce	8	24	576	13824	331776	7962624
		Groff	3	72	1728	41472	995328
			Denier	24	576	13824	331776
				Graine	24	576	13824
					Garob, or Prime	24	576
						Second	24

How many Carrats and Graines in their Ounce.

In France the Ounce is also divided into 2 Carrats, and every Carrat into 12 Graines.

Dantfick in Poland.

Tables of Mark Weights of Dantfick and Geneva.

	Ounces.	Pence.	Hellers.
Mark	8	256	512
	Ounce	32	64
		Peny	2

Geneva for Gold.

	Ounces.	Deniers.	Graines.
Mark	8	192	4608
	Ounce	24	576
		Denier	24

Meyfen and Norenburgh.

Meyfen in Saxony.

	Ounces.	Deniers.	Graines, or Momenta.
Mark	8	192	4608
	Ounce	24	576
		Denier, or Peny	24

Norenburgh in Germany.

	Ounces.	Loots.	Quints.	Primes.	Sestertios.
Mark	8	16	64	256	1024
	Ounce	2	8	32	128
		Loot	4	16	64
			Quint	4	16
				Prime, Peny, or Numulus.	4

Portugal and Venice.

Portugal.

	Ounces	Octavos, or Oitavos.	Great Grains.
Mark	8	64	288
	Ounce	8	36
		Oitavo or Octavo	4½

Venice.

	Ounces.	Silicos, or Quarts.	Siliquas, or Carrats.	Graines.
Mark	8	32	1152	4608
	Ounce	4	144	576
		Quart, or Silico,	36	144
			Carrat, or Siliqua	144
				4

Spain.

Spain.

Gold.

Silver.

Ounces. Castellanos. Tomines. Graines.			
Mark	8	50	400
	Ounce	6 $\frac{1}{4}$	50
		Castellano	8
		Tomine	12

Ounces. Drams or Octavos. Graines.		
Mark	8	64
	Ounce	8
		Dram, or Octavo

Rome.

Rome.

Ounces. Drams. Scruples. Obolos. - Siliquas, Primi, or Graines.					
Mark	8	64	192	384	1152
	Ounce	8	24	48	144
		Dram.	3	6	18
			Scruple	2	6
				Obolus	3
					Silique

Romana Libra, by Malines.

A Table of Romana Libra

Libra.	12.	84	168	336	840	3320	5040.
	Ounces, or Guilders.	Denarios.	Victoriatas.	Sesterio's.	Afles.	Quadrantes.	Sexantes.

The Ton of Gold in *Latine*, *Tina*, *feu Tonna*, by some called *Roman*, but by *Alsted*, *Tonnie of Gold* *German* is thus divided.

Pounds. Marks. Ounces. Loots. Drams.					
Tonne of Gold.	781 $\frac{1}{4}$	1562 $\frac{1}{2}$	12500	25000	100000
	Pound	2	16	32	128
		Mark	8	16	64
			Ounce	2	8
				Loot	4

A Table of the Tonne of Gold.

Scotland, divides their Pound into 24 Deniers, 1 Denier 24 Primes, 1 Prime 24 Seconds, 1 Second 24 Thirds, 1 Third 24 Fourths, &c.

Pound of Scotland.

The Correspondency of 100 Markes of *Antwerp* to the places following.

The 100 Marks of Antwerpe, how much at some other Places.

		<i>Adler</i>	76 $\frac{1}{2}$	<i>Hb.</i>
3	1	<i>Aegypt</i>	94	<i>Belles</i>
3		<i>Africa</i>	87	<i>Markes</i>
1	7	<i>Ancona</i>	103 $\frac{1}{4}$	<i>Markes</i>
1	7	<i>Aquila</i>	71	<i>Hb.</i>
1	4	<i>Ausburgh</i>	105 $\frac{1}{2}$	<i>Markes</i>
1	4	<i>Bambergh</i>	103 $\frac{1}{4}$	<i>Markes</i>
1	4	<i>Bavaria</i>		
1	4	<i>Bohemia</i>	87	<i>Markes</i>
1	4	<i>Bresla</i>	121 $\frac{1}{4}$	<i>Markes</i>

M m

1 14 *Burgas*

A Table of the French Mark.

The Mark Weights of some other Places subdivided.

France.

	Ounce.	Groffes.	Deniers.	Graines.	Primes, or Garobs.	Seconds.	Tercies, or Malloquen.
Mark	8	64	192	4608	110592	2654208	63700992
	Ounce	8	24	576	13824	331776	7962624
		Groß	3	72	1728	41472	995328
			Denier	24	576	13824	331776
				Graine	24	576	13824
					Garob, or Prime	24	576
						Second	24

How many Carrats and Graines in their Ounce.

In France the Ounce is also divided into 2 Carrats, and every Carrat into 12 Graines.

Dantick in Poland.

	Ounces.	Pence.	Hellers.
Mark	8	256	512
	Ounce	32	64
		Peny	2

Tables of Mark Weights of Dantick and Geneva.

Geneva for Gold.

	Ounces.	Deniers.	Graines.
Mark	8	192	4608
	Ounce	24	576
		Denier	24

Meyfen and Norenburgh.

Meyfen in Saxony.

	Ounces.	Deniers.	Graines, or Momenta.
Mark	8	192	4608
	Ounce	24	576
		Denier, or Peny	24

Norenburgh in Germany.

	Ounces.	Loots.	Quints.	Primes.	Sestertios.
Mark	8	16	64	256	1024
	Ounce	2	8	32	128
		Loot	4	16	64
			Quint	4	16
				Prime, Peny, or Numulus.	4

Portugal and Venice.

Portugal.

	Ounces	Oitavos, or Oitavos.	Great Grains.
Mark	8	64	288
	Ounce	8	36
		Oitavo or Oitavo	4½

Venice.

	Ounces.	Silicos. or Quarts.	Siliquas. or Carrats.	Graines.
Mark	8	32	1152	4608
	Ounce	4	144	576
		Quart, or Silico,	36	144
			Carrat, or Siliqua	144
				4

Spain.

Spain.

Spain.

Gold.

Silver.

Ounces. Castellanos. Tomines. Graines.			
Mark	8	50	400
		400	4800
Ounce		6¼	50
			600
	Castellano	8	96
		Tomine	12

Ounces. Drams or Octavos. Graines.			
Mark	8	64	4800
Ounce		8	600
	Dram, or Octavo		75

Rome.

Rome.

Ounces. Drams. Scruples. Obolos. Siliquas, Primi, or Graines.					
Mark	8	64	192	384	1152
					4008
Ounce		8	24	48	144
					576
	Dram.		3	6	18
					72
		Scruple		2	6
					24
			Obolus		3
					12
				Siliqua	
					4

Romana Libra, by Malines.

A Table of Romana Libra

Libra.	12.	84	168	336	840	3320	5040.
	Ounces, or Guilders.	Denarios.	Victoriatas.	Sellerio's.	Alfes.	Quadrantes.	Sexantes.

The Ton of Gold in *Latine*, *Tina*, seu *Tonna*, by some called *Roman*, but by *Alsted*, *Tonne of Gold* how called. *German* is thus divided.

Pounds. Marks. Ounces. Loots. Drams.					
Tonne of Gold.	78 1¼	1562 ½	12500	25000	100000
	Pound	2	16	32	128
		Mark	8	16	64
			Ounce	2	8
				Loot	4

A Table of the Tonne of Gold.

Scotland, divides their Pound into 24 Deniers, 1 Denier 24 Primes, 1 Prime 24 Seconds, 1 Second 24 Thirds, 1 Third 24 Fourths, &c. *Pound of Scotland.*

The Correspondency of 100 Markes of *Antwerp* to the places following.

The 100 Markes of Antwerpe, how much at some other Places.

	Adler	76 ½	lb.
3	1 Aegypt	94	Besses
3	Africa	87	Markes
1	7 Ancona	103 ¼	Markes
1	7 Aquila	71	lb.
1	4 Ausburgh	105 ½	Markes
1	4 Bambergh	103 ¼	Markes
1	4 Bavaria		
1	4 Bohemia	87	Markes
1	4 Bresla	121 ¼	Markes

M m

1 14 Bargas

The 100 Marks
at Antwerp,
how much at
some other
Places.

1	14	Burgas	116 $\frac{2}{3}$	Markes
1	7	Calabria	76 $\frac{1}{2}$	lb.
1	14	Catalonia	100	Markes
1	4	Collen	105 $\frac{2}{9}$	Markes
1	5	Constantinople	87	Markes
1	7	Crema	103 $\frac{1}{4}$	Markes
1	9	Dantick	105 $\frac{2}{9}$	Markes
1	4	Erdford	105 $\frac{2}{9}$	Markes
1	7	Florence	72	lb.
1	4	Franconia	103 $\frac{1}{4}$	Markes
1	4	Frankford	105 $\frac{2}{9}$	Markes
1	4	Friburgh	103 $\frac{1}{4}$	Markes
1	7	Genes for	Gold—116	Markes
		Silver—	77	Markes, or lb.
1	7	Geneva, as Paris and Lyons.		
1	5	Gracia	105 $\frac{2}{9}$	Markes
1	4	Hungary	87	Markes
1	4	Lipsich	105 $\frac{2}{9}$	Markes
1	2	London	89 $\frac{8}{9}$	lb.
1	3	Lyons	112	Markes, Merchants Weight
			102 $\frac{1}{2}$	Markes, Merchants Weights
1	4	Ments		The Kings Weight
1	4	Meysen	105 $\frac{2}{9}$	Markes
1	7	Millan		
1	7	Naples	76 $\frac{1}{2}$	lb.
2	6	Narsinga	87	Markes
1	4	Norenburg	103 $\frac{1}{4}$	Markes
4	2	Nova Spagnia	87 $\frac{1}{2}$	Markes
1	3	Paris, as Lyons.		
2	9	Persia	87	Mina's
4	3	Pern	87 $\frac{1}{2}$	Markes
1	7	Piedmont	99	Markes
1	7	Puglia	76 $\frac{1}{2}$	lb.
1	7	Rome	103 $\frac{1}{4}$	Markes
1	4	Saxony	105 $\frac{2}{9}$	Markes
1	14	Spain	107	Markes
1	4	Trevers, or Triers	105 $\frac{2}{9}$	Markes
1	7	Treviso	103 $\frac{1}{4}$	Markes
1	7	Turin	99	Markes
2		Turky	87	Markes
3				
1	7	Venice	103 $\frac{1}{4}$	Markes
1	7	Verona	103 $\frac{1}{4}$	Markes
1	7	Vicenza	105 $\frac{2}{9}$	Markes
1	4	Vienna	87	Markes
1	4	Ulme	105 $\frac{2}{9}$	Markes
1	4	Wissilburgh	103 $\frac{1}{4}$	Markes

Foreign Mo-
nies.

To close up the Forain Geodæticks, Moneys take their turn, concerning which three things are to be observed.

- 1st. Their Divisions, or greater and lesser Denominations.
- 2^{ly}. The Accompts thereof, and Exchanges.
- 3^{ly}. The Weight and Worth of the several Coines.

Accompts and
Exchange at se-
veral Places.

Malines, p. 240, 241, 257, 258, 259, and other Authours inform us concerning the former two, as followeth, viz. at—

- 2 11 Aleppo, The Exchange is made by Sultanies of 120 Alpers, or Dollers of 80 Alpers, every Alper 10 Macherines.
- 3 1 Alexandria, They Accompt by Ducats, either Ducat de Pargo, of 120 Maids, Ducat of Venice of 40 Maids, or Italian Ducat of 35 Maids.
- 1 7 Ancona, Exchange is made on the Ducat of 21 Gross, (which is in Specie 23 Gross) which Ducat is also 14 Carlini, and every Carlini 6 Bollidini, So is the Ducat 84 Bollidini.

1 4 *Arragon*, The Rial, or Ryal of Plate is 23 Dinero's (*Hunt* faith 13) and the Ducat is 12 Ryals, whereon they make Exchange. And they Accompt by Pounds of 20 s. and 12 d. And the Ducat of 12 Ryals. Every Ryal of 1 s. or 12 d.

Accompts and Exchange at several Places.

1 8 *Artois*, And in several other Places they Accompt and Exchange by Pounds or Liures Toirnois of 20 Stivers, or 40 Pence *Flemish*, whereof 6 called Guilders or Florius makes the Pound *Flemish* in all the 17 Provinces of the *Netherlands*. Which Pound is divided into 20 s. and every Shilling into 12 d. &c

Some reckon by the Pound *Parasit*, which is but 20 Pence, whereof 12 make 1 Pound *Flemish*, but their Accompts, as also the Finances of the Princes are kept by Pounds Tournois, and both Pounds divided into 20 s. and every Shilling into 12 Pence, admitting also the Subdivisions of Obolo's, Maille, Heller, Hallinck, Corte, Mites, Point, Engevin, Poot, and such like Copper Monies.

Alstead mentions the Florin in Germany to be 15 Bartz, every Bartz 2 Albes, every Albe 8 Oboli, or Nummos. So shall the Florin be 30 Albes or 240 Oboli.

1 4 *Augusta*, or *Ausburgh*, Accompts on the Dollar coined at 65 Creutzers, risen since to 72. Exchange is made on the imaginary rate of 65 Creutzers.

A Creutzer, is sometime called a Schreikenborger, and in *Latine*, *Crucigerus* and *Cruciatus*, being pieces stamped with a Cross. Their Gros make 12 Creutzers or 3 Bartz, so is their Bartz 4 Creutzers. Their Lyon Piece half a Creutzer. They have their Snubourgh, Blaphart, or Bohemico's of 3 and 3½ Creutzers. The Rix or Rycks Dollar is 30 Albes of 8 d. every Albe, or 72 Creutzers. Every Dollar as before. See the following Table, and afterwards in *Germany*.

Creutzer, here called and stamped.

	Gros.	Batz.	Albes.	Creutzers.	Lyon.	Pence.	Black.pennys.
Dollar	6	18	30	72	144	240	288
	Gros	3	5	12	24	40	48
		Batz	1½	4	8	13½	16
			Albe	2½	4½	8	9½
				Creutzer	2	3½	4
					Lyons	1½	2
						Peny	1½

A Table of the German Dollar.

3 2 *Barbary*, Generally Accompts are kept, and Commodities sold by Ducats of 10 3, each Ounce divided into 8 parts, which eight part is in Value about 12 d. Sterling.

1 14 *Barcelona*, As at *Arragon*.

1 4 *Bavaria*, Accompts and Exchanges both are by Guilders of 7 s. and 30 Pence to Shilling.

1 4 *Bohemia*, As in *Germany*, generally by the Dollar of 24 Bohemico's, called also White Gros, each of 3 Creutzers, other Divisions see in the Table following.

	Marke.	Dollars.	Angster's.	Bohemico's.	Creutzers.	Pence.
Scoc.	1½	2½	30	60	180	600
	Marke	1½	20	40	120	400
		Dollar	12	24	72	240
			Angster	2	6	20
				Bohemico	3	10
					Creutzer	3½

A Table of the Bohemian Scoc.

1 7 *Bologna*,

- 1 7 *Bologna*, They Accompt by Piastra, or Pounds (called also Piaftri,) each containing 20 Bolognesi. And exchange on the Ducat of 4 Piaftri.
- 1 8 *Brabant*, And in most places of the *Low Countries*, Monies, are accompted by the Pound *Flemish*, containing 20 s. *Flemish*. And every Shilling 12d. or Deniers called Single Stivers, 2 of which make 1 double Stiver. See *Flanders*.
- 1 4 *Breslaw*, They reckon by Markes of 32 Grofs, of 12 Heller to the Grofs, And exchange by 30 Florens to have at *Norendergh* 32 Florins, and at *Vienna* 34 Florins.
- 1 7 *Calabria*, Exchanges are made by the *Naples* Ducat, of 10 Carlini.
- 1 14 *Castile*, Exchanges are made on the Ducat of 375 *Marvedies*, which they call in the Bill of Exchange *Ducadas d' oro*, or *de Peso* to be paid out of the Bank is better by 6, or 8 pro *Milliar*. See *Spain*.
- 1 14 *Catalonia*, as at *Arragon*.
- 1 4 { *Cleves*, } Both Accompts and Exchanges are made by Dollars of 72 Creutzers.
 { *Collen*, }

A Table of the
Guilder of
Cleves and
Collen.

	Markes.	Morkens.	White pennies.	Shillings, or Stivers.
Their Guilder is	4	12	24	48
	Marke	3	6	12
		Morken	2	4
			White Penny	2

- 1 5 *Constantinople*, as *Aleppo*.
- 1 9 *Dantzick*, They accompt by *Polish* Guilders, of 30 Grofs, every Grofs 18d. They buy with the Great Mark of 60 Grofs, or the Little Mark of 15 Grofs. Also by the Scoc of 3 Great Marks. And exchange upon the *Florin Polish*, or the Pound *Flemish*. They have Dollars of 35 Grofs of 3 Shillings. And new Dollars of 24, 26, or 30 Grofs. Their Gilden is 80 Grofs. So is

A Table of the
Scoc of
Dantzick.

	Gildens.	Great Markes.	Dollars.	Guilders.	Little Markes.	Grofs.	Pence.
Scoc.	2 $\frac{1}{4}$	3	5 $\frac{1}{2}$	6	12	180	3240
	Gilden	1 $\frac{1}{3}$	2 $\frac{2}{3}$	2 $\frac{1}{3}$	5 $\frac{1}{3}$	80	1440
		Great Marke	1 $\frac{1}{2}$	2	4	60	1080
			Dollar	1 $\frac{1}{2}$	2 $\frac{1}{3}$	35	630
				Guilder	2	30	540
					Little Mark	15	270
						Grofs	18

- 1 1 *Denmark*, They Accompt by Markes of 20 Shillings. And Exchange upon the Dollar.
- 1 6 *Dublin*. See *Ireland*.
- 1 12 *Edenbourgh*. See *Scotland*.
- 1 4 *Embsen*, They reckon by Guilders, and exchange on the Rycks Dollar, but from *London* hither and thither upon the Pound *Sterling*.
- 1 8 *Flanders*, As before in *Brabant*. See a more particular Division of the *Flemish* Money in the following Table.

	Gilders.	Shillings.	Double-Stivers.	Single-Stivers.	Groats.	Ortgens.	Negen-Manneken.	Copper-Pence.	Mites.
Flemish Pound.	0	20	60	120	240	480	900	1920	2880
	Gilder	3 $\frac{1}{4}$	10	20	40	80	160	320	480
		Shilling	3	6	12	24	48	96	144
			Double-Stiver	2	4	8	16	32	48
				Single-Silver	2	4	8	16	24
					Groat	2	4	8	12
						Ortgen, or Ortken	2	4	6
							Negen-Manneken	2	3
								Copper-Penny.	1 $\frac{1}{2}$

A Table of the Flemish Pound.

Five Single Stivers are Currant in several Places of the Low-Countries for Six pence Sterling. Ortken in some places are called Duyts. Mites in some places of Flanders are called Corres, Engcuni, Points, Pites, Pootes.

7 Florence, They accompt by Crownes of 20 s. and 12 d. to the Shilling. And exchange by a Ducat called Largo, or Scripto in Banco. A Florin there is 24 Quatrini.

3 France, Generally they use Liures Solx and Deniers, and commonly accompt by them, as the English by Pounds, Shillings and Pence, but by an Edict made 1577, their Accompts are to be kept in French Crownes of 60 Sols to the Crown; or 3 Liures, that is Pounds Tournois. And exchange is made thereupon unless for some places in Italy, where they exchange for Number to have so many Ducats for so many Crownes of the Sum, not in Specie but imaginary, yet respecting the Value, or Par. See further in the Table and Notes thereupon.

	Liures.	Sols.	Liarts.	Doubles.	Deniers.
French Crown A	3	60	240	360	720
	Liure B	20	80	120	240
		Sols C	4	6	12
			Liart D	1 $\frac{1}{2}$	3
				Double E	2

A Table of the French Crowns

There are also Petit Deniers, and Mailles, but not considerable.

A. This Crown here to make Exchange by is equivalent to the Silver Coines of Lewis 13th and 14th, called Lewisses and imaginary, and not to be accompted for the French Gold Crowne, which is a real Coine, and of greater Value now, being worth 8 s. Sterling, or thereabouts, but when currant, at 6 s. Sterling. The Accompt and Exchange agreed in reality, 10 Sols then and yet commonly reckoned for an English Shilling. Of this Gold Crown was the Cardecue a quarter, and so valued in Sterling Money at 18 d. and should be wrote Quartid'escue, Escue, being French for a Crown.

French Crown of Gold how much.

Cardecue what and how to be wrought.

B, and C. The Liures (or Pounds sometime called Franks) and Sols (wrote sometime Soulx, sometime Solx, derived from the Latine, Solidus, as Liure from Libra are different. Those commonly used are called Tournois, and valued with Sterling Money as above. Of the Sols Barrois 14 make 20 Sols Tournois. The Sols Mauvais, is 2 Sols Tournois. The Sols Paris is 1 $\frac{1}{4}$ Sols Tournois. The Sols Bourdelois is half the Sols Paris. And so accordingly is the Liure to be accompted.

Liures and Sals how called, whence the word. Sorts of Sols.

Parts, and
Doubles not
used in Ac-
compts.

D, and *E*, Neither the Liarts nor Doubles, though both Cop-
per Coines are used in Common Accompts.

- 1 4 *Frankjörd*, Their Guilder or Florin by which they reckon is 60 Creutzers
divided by 20 s. and every Shilling into 12 Hellers according
to the Pound. But they exchange by the Dollar of 65 Creut-
zers payable in the two Yearly Fairs or Marts, one the Week
before *Easter*, and the other all the Moneth of *September*.

- 1 7. *Genoa*, All Accompts, and Exchanges are made by Crowns of 60 s. divided
into 20 s. and every Shilling into 12 Pence.

- 1 4 *Germany*, Every Batz by which generally they keep Accompts, is 4 Creutzers.
They Exchange on the Dollar imaginary at 65 Creutzers, and
so coined as was noted before at *Augusta*, though since risen
to 72 in Value.

They have Pieces of 3, 6, and 12 Creutzers, and by them,
and their Batz they value their own and *Exotick* Coines as the
Hungarian Ducat is 27 Batz. The Gold Guilder is 18 Batz.
The *Polish* Guilder or Dollar is 15 Batz, Teston 5 Batz, &c.
A Guilder was the name the Antient *Romans* gave to an Ounce,
and 8 $\frac{3}{4}$ made a Mark, and 12 Ounces or Guilders a Pound.
And there were Coined Pieces called *Nummi Dragmi*, or *Groschen*
the 8th part of a Dollar. Anno 1520. was the Gold Guilder
Coined for a General Coine, and valued in *Holland* at 28
Stivers, but now in *Specie* at double the Price. Nevertheless
Corn brought from *Poland* and the *East Countries*, is bought
and sold by the same at the old value of 28 Stivers.

Angelicies was the Sixth part of a Dollar, making 3 Batz, or
12 Creutzers. These Angelici becoming Tribute Pennies were
allayed, and so being made worse, did obtain the Name of
Batz or Bates (sometime wrote Batfes) *quasi* Base. And in
Thuringia they are called Gulielmi, and in *Bohemia*, Bohemici,
whereof they have also 12 Peeces dividedly, for 12 Pence;
which Penny is 2 Hellers in Accompt all over *Germany*.

Guilder of the
Romans.

Nummi
Dragmi.

Angelicies,
what.

Tribute Penny,
abased,
whence the
Name of Batz.

Pardauue-
Xerafin the
Stamp and
Value.

Tangas good
and bad.

Larin of Persia
the Value.

Pagode what.

- 2 6 *Goa*, Their Common Accompt is by their Ordinary Silver Coine a *Pardauue-
Xerafin*, having the Image of St. *Sebastion* at the one side, and
3 or 4 Arrows bound together at the other, which is worth
3 Testons, or 300 Res of *Portugal*, but varieth as the Exchange
riseth or falleth: And accordingly their other Coines, and Ac-
compts of which some are imaginary, and some real. They
have also some good and some bad Monies; for 4 good Tanga's,
or 5 bad Tanga's are reckoned to value 1 Pardauue-Xerafin.
And 1 Tanga is 75 Basarves. Of these Basarves 375 make 1
Pardauue-Xerafin. And 15 good Basarves are valued with 18
bad, which are made of Bad Tinne. By these other Countrey
Coines are rated, as the *Larin* of *Persia* is worth 105, and 108
Basarves, as the Exchange goes. A Paradauue of *Larins* is 5
Larins. And the Crowns of *Venice* or *Turkey* are almost worth
2 Paradauue-Xerafins. They have also a *Pagode*, or Gold
Crown, on which is the Figure of their *Idol*, worth about 8
Tanga's, And Gold Crowns of St. *Thomas* with his Image on
them, esteemed at 7, or 8 Tangas.

- 1 1 *Hamborough*, Their Dollar was first Coined at 31 Shillings Lups, and many
Years currant for 33, is now inhaunced to 54 s. Lups, of 3
White Penny, and every Shilling is 12 d. and every Penny 2
Hellens. They Accompt by Markes of 16 s. Lubish, and 12 d.
to the Shilling: But Exchange for *London* upon the Pound *Ster-
ling*, and for other Places on the Rycks Dollar of 33 s. now by
them inhaunced to 54 s. Lubish, or so many Stivers *Flemish*.

- 1 8 *Henault*, As *Artois*.

- 1 4 *Hungaria*, They accompt by Guilders of 10 s. of 30 d. to the Shilling, And
by *Florins* of 20 s. and 12 d. to the Shilling: And exchange on
the Ducat and Rycks Dollar worth 8 Shillings formerly, but
7 s. 7 d.

- 1 6 Ireland, They as the English Accompt by Pounds of 20 s. Sterling and Pence of 12 to the Shilling. Only their Harper valued in England but 9 d. was with them counted 1 s. So as their Pound is but $\frac{3}{4}$ of ours, or 15 s. Sterling. And thereon Exchanges are made.
- 1 4 Lipsich, as Bressa.
- 1 10 Lisbon, See Portugal.
- 1 8 Low Countries, generally as before at Brabant.
- 1 2 London, Exchanges are generally made for Germany and the Low Countries, on the Pound Sterling. For France on the French Crown of 60 Sols Tournois. For Italy, Spain, and other Places on the Ducat Dollar or Florin according to the Custom of the Place
- 1 7 Luca, For divers Places in Italy and Lyons in France Exchanges are made on the Ducat.
- 1 3 Lyons, as before in France.
- 1 14 Madrid, See Spain.
- 1 7 Millan, Accompts are kept by Ducats Imperial, divided by 20 s. and 12 d. to the Shilling. And Exchanges made on the same, accompting 80 s. to the Ducat Imperial : But they buy by a Ducat currant of 120 s.
- 1 7 Naples, They Accompt by Ducats, Taries and Graines. The Ducat is 10 Carlini or 5 Taries, for the Tarie is 2 Carlini, or Royals. And hereupon Exchanges are made for most places of Italy ; but for Lyons, they Exchange by Number, as 125 Ducats for 100 Crowns.
- 1 4 Norenbourgh, The Exchange is made on the Dollar of 65 Creutzers, and many times on the Guilder of Florin, of 60 Creutzers, which they also divide into 20 s. and every Shilling into 12 d. to keep Accompts by ; and some say the Creutzer is 4 d. every Peny is 2 Hellers. And 5 d. is called a Fynfer, or 5 Pennick.
- 1 7 Palermo, The Ducat is 13 Taries, 1 Tarie 2 Carlini, 5 Ryals of Spain are 6 Taries. They accompt by Ounces of 30 Taries, to 20 Graines every Tarie, and every Graine 6 Piccolie. And their Exchanges are made upon Florins of 6 Taries, or Tarij.
- 1 3 Paris, as before in France.
- 1 9 Poland, They Accompt by Markes and Exchange on the Dollar, and also on the Florin of 48 s. The Marke is One Third part of it.
- 1 4 Pomerania, They divide their Money as in the next Table following, Accompt by Markes of 16 Snudens, and Exchange upon the Ryckx Dollar of 32 s. or 2 Markes Snudens, so called to distinguish them from Markes Lups, and Shillings Lups.

	Marks-Snudens.	Lups-Shillings.	Shillings-Snudens.	Pence.	Hellers.
Ryckx Dollar	2	16	32	384	708
Marke Lups	Marke-Snuden	8	16	192	384
		Shilling-Lups.	2	24	48
			Shilling-Snuden	12	24
				Peny	2

A Table of the Ryckx Dollar of Pomeran.

- 1 10 Portugal, They Accompt by Milrais, Ducats, or Crusado's, &c. as in the Table following, And Exchange by the same Ducat of 400 Raies.

	Ducats, or Crusado.	Testons.	Rials.	Vintaines, or Half Rial.	Raies.
Mille Raies	2 $\frac{1}{2}$	10	25	50	1000
	Ducat, or Crusado.	4	10	20	400
		Teston.	2 $\frac{1}{2}$	5	100
			Rial.	2	40
				Vintaine, or Half Rial.	20

A Table of the Mille Raies.

Of these Ducats, Rials, (or Royals) and Raies (wrote also Reas, Reyse, and Res) are most in use for Accompt. They have Testons also of 4 Vintaines, 40 Raies are commonly accounted for Six Pence *Sterling*, and so accordingly were the other Coines valued, till the late advance whereby the Teston of 100 Raies were stamped, and made currant for 120 Raies, and so rated at 1 s. 6 d. *Sterling*, when before but 1 s. 3 d.

- 1 7 *Puglia*, as *Calabria*.
 1 9 *Riga*, They buy by Dollars, or Florins *Polish* of 18 Farthings, whereof 11 make 10 Dollars, but they Exchange upon the Ryckx Dollar.
 1 3 *Roan*, as before in *France*.
 1 7 *Rome*, Accompts and Exchanges are performed by Ducats *di Camera* of 13 July, or *Guili*, every Ducat which they divide into 20 s. and every Shilling into 12 d.
 1 11 } *Russia*, They have small Coine of 11 3. 2 penny weights fine, called *Dengen*,
 2 10 } whereof 320 Pieces weigh but a Mark of 8 3. They Exchange upon the Dollar of *Germany*; but for *London* upon their Rubble which is valued as a Double Ducat formerly, accounted equal to a Marke *Sterling*, or 13 s. 4 d.
 1 14 *Saragossa*, as *Arragon*.
 1 12 *Scotland*, They Accompt by Pounds, Shillings, and Pence, as in *England*, but one *Pund Scotch* is but 20 d *English*. Their Marke is 13 $\frac{1}{3}$ s. *Scotch*, currant in *England* at 13 $\frac{1}{2}$ d. Their Noble, or half Marke with them 6 $\frac{2}{3}$ s. with us 6 $\frac{1}{4}$ d. Their half Noble, and third part of their Noble proportionally. They have also Turnoners, Pence, and Half-Pence, and base Money of Bodles, Achifons, Babees, Placks, &c. accompting 6 Bodles to 1 d. *Sterling*, or 12 d. *Scotch*, 4 Bodles to 1 Achifon, 3 to 1 Babee, and 2 to 1 Plack. But they Exchange upon their Marke
 1 14 *Spaine*, as in *Madrid*, *Sevil*, and other Places their Accompts are all kept by *Malvedies*, or *Marveides* (wrote also *Merveides* and *Maravides*) whereof 375 are esteemed to make a Ducat of 11 Rials, though really every Ryal is but 34 Marveides, and so maketh but 374, as in the following Table, and so others keep Accompts accordingly. Exchange is made on this Imaginary Ducat of 375 Marveides to be payd in Bank with 5 on the 1000, which is the Salary of the Banker, or without the Bank to be payd without the same.

Base Money of
Scotland.

A Table of the
Spanish Ducat.

	Pieces of Eight.	Rials.	Quartilios.	Marveides.	Carnado's.
Ducat	1 $\frac{3}{4}$	11	44	374	2244
	Piece of Eight	8	32	272	1632
		Rial	4	34	204
			Quartilio	8 $\frac{1}{2}$	51
				Marveide	6

A Rial is about 6 $\frac{1}{2}$ d. *Sterling*.

- 1 4 *Stratsborough*, or *Strausburgh*, They have Blapharts, Gros, Bohemico's, all currant for 3 Creutzers a piece, 1 Creutzer at 2 d. One Peny at 2 Hellers, and 1 Heller at 2 Orthings.
 1 15 *Sweden*, They reckon by Markes, whereof 8 make a Dollar, whereupon they Exchange. And 2 Markes make a Clipping of 9 $\frac{1}{2}$ rivers.
 1 4 *Tirol*, The Dollar is 72 Creutzers, and the Creutzer 5 Fynfers or Hellers.
 2 11 *Tripoli*, as *Aleppo*.
 1 14 *Valentia*, as *Arragon*.
 1 7 *Venice*, Thirty Batz make 1 Souldey, and 20 Souldeys 1 Liure of *Venice*. Their Gold Ducat is valued equal to 40 Maides of *Alexandria*. They have also Copper Money, 1 Sessini make 2 Quatrini, and 1 Quatrini 4 Bagatini, and so 3 Quatrini, or 12 Bagatini make an Half penny *Sterling*, or thereabouts.

They

They accompt by Pounds *Flemish* of 10 Ducats or 20 s. and divide the Ducat into 24 Gros, and the Shilling into 12 Pence. And also by the Ducat 124 s. called *Ducato di Banco*, or Currant, and thereon Exchanges are made.

1 7 *Verona*, Their Accompts are kept by 20 s. and 12 d. to the Shilling. And they Exchange on the Ducat of 93 s.

1 4 *Vienna*, Both Accompts and Exchanges are kept and made by Guilders or Florins of 8 s. a piece, 30 d. to the Shilling, and 2 Hellers to the Penny.

They esteem the Ricks Dollar at 8 s. and the Ducat at 12 s.

1 4 *Ulme*, They reckon by Pounds of 20 s. and 12 Heller to the Shilling. And Exchange on the Dollar of 60 Creutzers.

That which remains to finish this long and tedious Chapter of *Geodæticals* is only to consider the weight and worth of the Coines of other Countreys as valued by the *Sterling* Standart, wherein because of the New Coines which may dayly be added by the Laws of the present or succeeding Governours, and those of different Fineness; to obviate the difficulties as well occasioned thereby, as by the rise and fall of Exchange, and so consequently of particular Coines, practised by Merchants: Every one that would arrive at satisfaction besides what can be here wrote, must add his diligent observance.

It may be remembred that the *English* Pound *Troy* is divided for Weight into 12 Ounces, every Ounce into 20 Penny-weights, and every Penny-weight into 24 Graines. And to try the Fineness of Silver the same divisions are kept, but for the Fineness of Gold, every Ounce is divided into 24 Carrats, and every Carrat into 4 Graines. And the old *Sterling* Standart for Silver is 11 Ounces 2 Penny-weights fine, and for Gold 22 Carrats fine.

They beyond Sea for Weight and Fineness of Silver divide their Ounce into 20 *English*, and every *English* into 32 Azes? And for weight of their Gold the like; But for the Fineness thereof divide their Ounce into 24 Carrats, and every Carrat into 12 Graines.

In the following Coines, understand the Value according to the *English* Division, allowing for the Ounce of Silver 11 Ounces 2 Penny-weights fine, 5 s. but for the Ounce of Gold 22 Carrats fine 3 l. 10 s. that is 3 s. 6 d. for the Penny-weight, and so proportionally for Coines of greater or lesser fineness, which Valuation makes the particular Pieces to differ from that found in several Printed Books, as they one from another, according to the times they were Printrd or Wrote in. Some valuing the Ounce of Silver so fine as aforesaid, and others that of 11 Ounces fine, at 5 s. and the Ounce of Gold of the fineness aforesaid but at 55 s. others at 3 l. some at 3 l. 6 s. and generally not above 3 l. 6 s. 8 d.

Neither are all Coines though of one and the same fineness alwaies valued alike proportionally. For K. *James*, May 14. 1612. by Proclamation ordered the bringers in of Forein Coine might receive at the Mint as followeth.

Foreign Coine.

Weight to try the fineness of Gold and Silver by in England. *Sterling* Standart. Weight beyond Sea to try the Fineness by. How the following Coines are valued.

All Coines not valued according to their Fineness.

	l.	s.	d.
For the Ounce of <i>Spanish</i> Silver Money of <i>Sevil</i> —————	0	5	0
The Ounce of <i>Mexico</i> Money.—————	0	4	10
Ingots of Silver, being 11 3/4. 2 pwts. fine—————	0	5	0
And so rateably for Silver of other fineness.			

	Car.	Gr.			
For 1 3/4. of	<i>Spanish</i> Pistolets being————	21	3 1/2 fine————	3	6 0
	<i>French</i> Crowns being————	22	0 fine————	3	6 0
	<i>Milreys</i> , <i>Crusado</i> long and short Gros.			3	6 2
	<i>Barbary</i> Gold being————	23	0 1/2 fine————	3	9 0
	<i>Hungary</i> Ducats, being————	23	1 fine————	3	9 0
	<i>Spanish</i> Ducats and Sultaines being	23	Car. 1 Gr. fine —	3	8 8
	<i>Zechines</i> , or <i>Checkeene</i> of <i>Venice</i> being	23	1 fine————	3	10 0
And for the Ounce of all other Gold being 22.			0 fine————	3	6 0

And in like sort to this day by the Artifice of *Merchants*, *Goldsmiths*, *Bankers*, &c. Some Coines are valued and currant in Traffique at a value higher, than by a due proportion in respect of their fineness to the *Sterling* Standart, they ought to be: But in the following Tables, the New Value is equally apportionated, yet without allowance for Coinage, which I take to be about 2 s. for the Pound *Troy* of Silver, and for the Pound *Troy* of Gold about 15 s.

Foreign Gold.		Fine	Pieces	Weight by	Weight by	Old			New						
		Car.Gr.	to the	Malines.	others.	Value.			Value.						
			Troy.	pmts. gr.	pmts. gr. L.	s.	d.		s.	d.					
Albertines, See Ducats.															
Angels the forts.	Angels {	with the three Lions	22	0	76	3	3 $\frac{1}{2}$ $\frac{5}{9}$	3	3 $\frac{1}{4}$	0	8	6	0	11	0 $\frac{1}{2}$
		with O.	23	0	72	3	8	3	6	0	9	0	0	12	2 $\frac{1}{4}$
	Angels of {	Batenborgh	21	3	72	3	8	3	6	0	9	0	0	11	6 $\frac{1}{4}$
		Flanaers, or best Flemish Angel	23	0	72	3	8	3	6	0	9	0	0	12	2 $\frac{1}{4}$
		H. M.	17	0	72	3	8	3	6	0	6	9	0	9	0
		Horne	23	1 $\frac{1}{2}$	72	3	8	3	6	0	9	6	0	12	4 $\frac{1}{4}$
		Thoron	22	1 $\frac{1}{2}$	72	3	8	3	6	0	9	0	0	11	10 $\frac{1}{4}$
Vienna	18	3	72	3	8	3	6	0	7	6	0	9	11 $\frac{1}{4}$		
Chautilion		23	3	79 $\frac{3}{4}$	3	0 $\frac{2}{3}$ $\frac{2}{19}$	2	23	0	8	10	0	11	4 $\frac{1}{4}$	
Cross Daggers of Scotland		22	0	72	3	8	3	6	0	11	0	0	11	8	
The half thereof		22	0	144	1	16	1	15	0	5	6	0	5	10	
Cro. the forts.	Crowns	Flemish Crown	22	0	108	2	5 $\frac{1}{3}$	2	5	0	6	0	0	7	9 $\frac{1}{4}$
		Floret Crown of France	23	3	100 $\frac{1}{2}$	2	9 $\frac{2}{6}$ $\frac{1}{7}$	2	9	0	7	0	0	9	8 $\frac{1}{4}$
		Charles French Crown	23	3	100 $\frac{1}{2}$	2	9 $\frac{2}{6}$ $\frac{1}{7}$	2	9	0	7	0	0	9	0 $\frac{1}{4}$
		Old French Crown	22	0	108	2	5 $\frac{1}{3}$	2	5	0	6	0	0	7	9 $\frac{1}{4}$
		New French Crown { Some	22	0	107 $\frac{1}{2}$	2	5 $\frac{2}{4}$ $\frac{5}{3}$	2	5 $\frac{1}{2}$	0	6	0	0	7	9 $\frac{1}{4}$
		New French Crown { Others	22	0	108	2	5 $\frac{1}{3}$	2	5	0	6	0	0	7	9 $\frac{1}{4}$
		Half Imperial Crown	22	1 $\frac{1}{2}$	107 $\frac{1}{2}$	2	5 $\frac{2}{4}$ $\frac{5}{3}$	2	5 $\frac{1}{2}$	0	6	0	0	7	11 $\frac{1}{4}$
Italian Crown								0	6	0	0	7	6		
Four Crowns of Portugal								1	6	2	0	10	0		
K. Philip's Crown of Spain		22	1 $\frac{1}{2}$	107 $\frac{1}{2}$	2	5 $\frac{2}{4}$ $\frac{5}{3}$	2	5 $\frac{1}{2}$	0	6	0	0	7	11 $\frac{1}{4}$	
Scotch Crown		22	0	108	2	5 $\frac{1}{3}$	2	5	0	6	0	0	7	9 $\frac{1}{4}$	
Thistle Crown		22	0	186	1	6 $\frac{3}{3}$ $\frac{0}{1}$	1	6 $\frac{3}{4}$	0	4	4 $\frac{3}{4}$	0	4	6	
Cru- fadoes the forts.	Crufado's {	or Ducat with the † of Portugal	22	1	105	2	6 $\frac{6}{7}$	2	6	0	6	0	0	8	1
		or Ducat with the ‡ of Portugal	22	3	105	2	6 $\frac{6}{7}$	2	6 $\frac{1}{2}$	0	6	2	0	8	3 $\frac{1}{4}$
		Great Crufado, or the Portuguese of Emanuel of Portugal	23	3	10 $\frac{2}{2}$	22	20 $\frac{4}{7}$	22	16	3	8	0	4	6	4 $\frac{1}{4}$
		Joannes Great Crufado	22	3	10 $\frac{2}{2}$	22	20 $\frac{4}{7}$	22	16	3	5	0	4	2	8 $\frac{1}{4}$
Dublion of Spain								0	14	6	0	15	2		
Du- cats the forts.	Ducats of {	Albertus, or { Single	23	3 $\frac{1}{2}$	105	2	6 $\frac{6}{7}$	2	6 $\frac{1}{2}$	0	6	6	0	8	8
		Albertines { Double	23	3 $\frac{1}{2}$	52 $\frac{1}{2}$	4	13 $\frac{5}{7}$	4	13	0	13	0	0	17	4
		Albertus of Austria { Single	23	3	78 $\frac{3}{4}$	3	1 $\frac{1}{2}$	3	0	0	9	0	0	11	5 $\frac{1}{4}$
		Albertus of Austria { Double	23	3	52 $\frac{1}{2}$	4	13 $\frac{5}{7}$	4	13	0	13	6	0	17	3 $\frac{1}{4}$
		$\frac{2}{3}$ parts of the same double Duc.	23	3	70 $\frac{1}{2}$	3	9 $\frac{3}{4}$ $\frac{2}{7}$	3	9	0	10	0	0	12	10 $\frac{1}{4}$
		$\frac{1}{3}$ part of the same double Duc.	23	3	126	1	21 $\frac{5}{7}$	1	21 $\frac{1}{2}$	0	5	7	0	7	2
		Arragon	23	3	105	2	6 $\frac{6}{7}$	2	6 $\frac{1}{2}$	0	6	6	0	8	7 $\frac{1}{2}$
Barbary and elsewhere, Some		23	0 $\frac{1}{2}$	106 $\frac{1}{2}$	2	6 $\frac{6}{7}$ $\frac{1}{1}$	2	6	0	6	4	0	8	3 $\frac{1}{2}$	
Barbary and elsewhere, Others		23	3	106 $\frac{1}{2}$	2	6 $\frac{6}{7}$ $\frac{1}{1}$	2	6	0	6	6	0	8	6 $\frac{1}{4}$	
Batenborg with the †		19	0	105	2	6 $\frac{6}{7}$	2	6 $\frac{1}{2}$	0	5	2	0	6	11	
Bishops Ducat		23	0 $\frac{1}{2}$	105	2	6 $\frac{6}{7}$	2	6 $\frac{1}{2}$	0	6	3	0	8	5 $\frac{1}{2}$	
Casim		23	3	105	2	6 $\frac{6}{7}$	2	6 $\frac{1}{2}$	0	6	6	0	8	7 $\frac{1}{2}$	
Denmark		20	0	106 $\frac{1}{2}$	2	6 $\frac{6}{7}$ $\frac{1}{1}$	2	6	0	5	4	0	7	2	
Emanuel of Portugal		23	3	105	2	6 $\frac{6}{7}$	2	6 $\frac{1}{2}$	0	6	6	0	8	7 $\frac{1}{2}$	
Ferdinand of Batenborg		19	0 $\frac{1}{2}$	106 $\frac{1}{2}$	2	6 $\frac{6}{7}$ $\frac{1}{1}$	2	6	0	5	2	0	6	10 $\frac{1}{4}$	
Ferdinand and Carolus of Horne		18	0	106 $\frac{1}{2}$	2	6 $\frac{6}{7}$ $\frac{1}{1}$	2	6	0	4	10	0	6	5 $\frac{1}{4}$	
Florence		23	1	108	2	5 $\frac{1}{3}$	2	5	0	6	4	0	8	2 $\frac{1}{4}$	
Ducats of {	George Rechin	21	3	106 $\frac{1}{2}$	2	6 $\frac{6}{7}$ $\frac{1}{1}$	2	6	0	6	1	0	7	9 $\frac{1}{4}$	
	Guelders	23	1	106 $\frac{1}{2}$	2	6 $\frac{6}{7}$ $\frac{1}{1}$	2	6	0	6	3	0	8	4 $\frac{1}{4}$	
	Guelphus of Batenborg	21	3	52 $\frac{1}{2}$	4	13 $\frac{5}{7}$	4	13	0	12	4	0	15	9 $\frac{1}{4}$	
	Hamborough								0	7	2	0	8	9	
	Holland	23	2	105	2	6 $\frac{6}{7}$	2	6 $\frac{1}{2}$	0	6	5	0	8	6 $\frac{1}{4}$	
	Hungary, or Half Noble	23	3 $\frac{1}{6}$	113 $\frac{1}{2}$	2	2 $\frac{1}{2}$ $\frac{2}{7}$	2	2 $\frac{1}{2}$	0	6	4	0	8	0	
	Other Hungary Ducats	23	1	104 $\frac{1}{2}$	2	7 $\frac{2}{2}$ $\frac{6}{9}$	2	7	0	6	4	0	8	5 $\frac{1}{4}$	
Italy { Some as Venice															
Italy { others		23	1	106 $\frac{1}{2}$	2	6 $\frac{6}{7}$ $\frac{1}{1}$	2	6	0	6	3	0	8	4	
Majorca		23	1	52 $\frac{1}{2}$	4	13 $\frac{5}{7}$	4	13	0	13	0	0	16	10 $\frac{1}{4}$	
Marie of Batenborg		20	0 $\frac{1}{2}$	106 $\frac{1}{2}$	2	6 $\frac{6}{7}$ $\frac{1}{1}$	2	6	0	5	4	0	7	2 $\frac{1}{4}$	
Navarre, and some others, as Majorca															
Nimneghen with Stephen		21	1	52 $\frac{1}{2}$	4	13 $\frac{5}{7}$	4	13	0	12	0	0	15	5 $\frac{1}{4}$	
Nimneghen of 1565		18	2	108	2	5 $\frac{1}{3}$	2	5	0	4	10	0	6	6 $\frac{1}{4}$	

Foreign Gold.

Foreign Gold.		Fine Pieces to the		Weight by Malines.		Weight by others.		Old Value.			New Value.					
		Car.	Gr.	lb.	Troy.	pwt.	gr.	pwt.	gr.	l.	s.	d.	l.	s.	d.	
Ducats of	Oswald Ducat Cusa	19	0 $\frac{1}{2}$	106 $\frac{1}{2}$	2	6 $\frac{6}{7}$	1	2	6	0	5	2	0	6	10 $\frac{1}{4}$	
	Pancratius Alleb. H. as Oswald															
	Peter Rechem, as Geo. Rechem															
	Portugal, fee Crusados, Milreys															
	Rome { Single { Some	23	3	105	2	6 $\frac{6}{7}$	1	2	6 $\frac{1}{2}$	0	6	6	0	8	7 $\frac{1}{2}$	
	Rome { Double { Others	23	3	106 $\frac{1}{2}$	2	6 $\frac{6}{7}$	1	2	6	0	6	6	0	8	6 $\frac{1}{4}$	
	St. Victor Rancratius, as {	23	3	52 $\frac{1}{2}$	4	13 $\frac{1}{2}$	7	4	13	0	13	0	0	17	3	
	George Rechem															
	Spain { Single { Some	23	1	105	2	6 $\frac{6}{7}$	1	2	6 $\frac{1}{2}$	0	6	6	0	8	5 $\frac{1}{4}$	
	Spain { Double { Others	23	2	105	2	6 $\frac{6}{7}$	1	2	6 $\frac{1}{2}$	0	6	6	0	8	6 $\frac{1}{2}$	
	Great	23	2	52 $\frac{1}{2}$	4	13 $\frac{1}{2}$	7	4	13	0	13	0	0	17	1	
	States of the United Prov. with Letters	22	0	24	10	0	0	10	0	1	10	0	1	15	0	Du-
	The Half thereof	22	0	52 $\frac{1}{2}$	4	13 $\frac{1}{2}$	7	4	13	0	12	4	0	16	0	cats.
	Stephanus of Batenborg	19	0 $\frac{1}{2}$	52 $\frac{1}{2}$	4	13 $\frac{1}{2}$	7	4	13	0	10	5	0	13	10 $\frac{3}{4}$	
	Suevia	23	1	104 $\frac{1}{2}$	2	7 $\frac{2}{3}$	0 $\frac{5}{9}$	2	7	0	6	4	0	8	5 $\frac{1}{4}$	
Valence	23	3	105	2	6 $\frac{6}{7}$	1	2	6 $\frac{1}{2}$	0	6	6	0	8	7 $\frac{1}{2}$		
Venice	23	3	106 $\frac{1}{2}$	2	6 $\frac{6}{7}$	1	2	6	0	6	6	0	8	6 $\frac{1}{4}$		
Victor Batenborgh, as Geo. Rechem																
Victor H.B. as Marie of Batenborg																
W. B. Margaret Foren	21	3	106 $\frac{1}{2}$	2	6 $\frac{6}{7}$	1	2	6	0	6	1	0	7	9 $\frac{1}{2}$		
Water Ducats, as Marie of Batenborg																
Zeland { Single	23	0 $\frac{1}{2}$	105	2	6 $\frac{6}{7}$	1	2	6 $\frac{1}{2}$	0	6	3	0	8	5 $\frac{1}{2}$		
Double	23	0 $\frac{1}{2}$	52 $\frac{1}{2}$	4	13 $\frac{1}{2}$	7	4	13	0	12	6	0	16	11		
Ducats with the Checquer, as Denmark																
Floret of France	22	0	100 $\frac{1}{2}$	2	9 $\frac{2}{3}$	7	2	9	0	6	5	0	8	4 $\frac{1}{4}$		
The New Floret, See Gilden St. Andrew																
Golden Fleece, or Toyson d'or	23	3 $\frac{3}{4}$	81 $\frac{1}{6}$	2	22 $\frac{4}{5}$	0 $\frac{7}{8}$	2	22 $\frac{3}{4}$	0	9	2	0	11	3 $\frac{1}{4}$		
Gold Guilder, or Guildren	18	3	112 $\frac{1}{2}$	2	3 $\frac{1}{2}$	5	2	6	0	4	5	0	6	4 $\frac{1}{4}$		
St. Andrew { Old	18	1	108	2	5 $\frac{1}{3}$	3	2	3 $\frac{1}{4}$	0	4	10	0	6	5 $\frac{1}{4}$	Guil-	
New, or Floret	18	3	108	2	5 $\frac{1}{3}$	3	2	3	0	5	0	0	6	7 $\frac{1}{2}$	ders	
Arnoldus	12	0	138	1	17 $\frac{1}{2}$	2 $\frac{2}{3}$	1	17 $\frac{1}{2}$	0	2	7	0	3	3 $\frac{3}{4}$	the	
Carolus	14	0	126	1	21 $\frac{1}{2}$	7	1	21 $\frac{1}{8}$	0	3	6	0	4	2 $\frac{1}{4}$	sorts.	
Clemmer	13	0	114	2	2 $\frac{1}{9}$	0 $\frac{9}{9}$	2	2	0	3	6	0	4	4 $\frac{1}{4}$		
Collen	17	3	114	2	2 $\frac{1}{9}$	0 $\frac{9}{9}$	2	2 $\frac{1}{4}$	0	4	8	0	5	11 $\frac{1}{4}$		
David of { The Harp	15	0	114	2	2 $\frac{1}{9}$	0 $\frac{9}{9}$	2	2	0	4	0	0	5	0 $\frac{1}{4}$		
{ Triers	17	2	114	2	2 $\frac{1}{9}$	0 $\frac{9}{9}$	2	2	0	4	8	0	5	10 $\frac{1}{4}$		
{ Utrecht	16	0	114	2	2 $\frac{1}{9}$	0 $\frac{9}{9}$	2	2	0	4	3	0	5	4 $\frac{1}{4}$		
Frederick of Bavaria	14	0	117	2	1 $\frac{1}{3}$	3	2	1	0	3	8	0	4	6 $\frac{3}{4}$		
Gulielmus	18	1	108	2	5 $\frac{1}{3}$	3	2	5	0	5	10	0	6	5 $\frac{1}{4}$		
Horne			158 $\frac{3}{4}$	1	12 $\frac{1}{2}$	6 $\frac{7}{7}$	1	12 $\frac{1}{4}$	0	4	11	0	6	4		
Joannes	16	0	109 $\frac{1}{2}$	2	4 $\frac{1}{2}$	3 $\frac{1}{3}$	2	4 $\frac{1}{2}$	0	4	6	0	5	7		
Peter of Louvaine	17	0 $\frac{1}{2}$	114	2	2 $\frac{1}{9}$	0 $\frac{9}{9}$	2	2	0	4	5	0	5	8 $\frac{1}{4}$		
Philip	15	3	111	2	3 $\frac{1}{3}$	7	2	3 $\frac{1}{4}$	0	4	2	0	5	5		
The Half thereof	15	3	222	1	1 $\frac{1}{3}$	7	1	1 $\frac{1}{8}$	0	2	1	0	2	8 $\frac{1}{2}$		
Renish Guilder	22	0	102 $\frac{3}{4}$	2	8 $\frac{1}{4}$	1 $\frac{1}{4}$	2	8	0	6	5	0	8	2		
Saxon	17	3	113	2	2 $\frac{1}{3}$	1 $\frac{1}{3}$	2	2 $\frac{1}{4}$	0	4	8	0	5	11 $\frac{1}{4}$		
States of the United Provinces	20	0	120 $\frac{3}{4}$	1	23 $\frac{1}{2}$	1 $\frac{1}{2}$	1	23 $\frac{1}{2}$	0	4	8	0	6	3 $\frac{1}{4}$		
Lions, Golden Lion of Flanders	23	3	89 $\frac{1}{4}$	2	16 $\frac{1}{2}$	1 $\frac{1}{2}$	2	16 $\frac{1}{2}$	0	7	8	0	10	1 $\frac{1}{4}$		
parts thereof	23	3	133 $\frac{7}{8}$	1	19 $\frac{1}{2}$	3 $\frac{1}{2}$	1	19	0	4	11	0	6	9		
part thereof	23	3	267 $\frac{3}{4}$	0	21 $\frac{1}{2}$	1 $\frac{1}{2}$	0	21 $\frac{1}{2}$	0	2	5	0	3	4 $\frac{1}{2}$		
Louiffes, of Louis 13th and 14th of France	22	0	54	4	10 $\frac{1}{2}$	1	4	8	0	15	0	0	15	6 $\frac{1}{2}$		
The Half thereof	22	0	108	2	5 $\frac{1}{2}$	1	2	4	0	7	6	0	7	9 $\frac{1}{4}$		
Marke of Bohemia																
20 Markes of Scotland	22	0	36	6	16		6	10	1	2	0	1	3	4	Mark	
10 Markes of Scotland	22	0	72	3	8		3	5	0	11	0	1	8		and	
5 Markes of Scotland	22	0	144	1	16		1	14	0	5	0	5	10		Mil-	
6 Markes of Suevia															reys.	
Milreys or Ducat of Portugal	22	1	48	5	0		4	20	0	13	4	0	17	8 $\frac{1}{4}$		
Halfe Milreys	22	1	96	2	12		2	10	0	6	8	0	8	10		
Counterfeit Milreys	21	0	48	5	0		4	20	0	12	6	0	16	8 $\frac{1}{2}$		

		Foreign Gold.		Fine	Pieces	Weight by	Weight by	Old	New							
				Car.Gr.	to the	Malines.	others.	Value.	s.	d.						
					Troy.	pwt.	gr.	pwt.	gr.	l.						
Nobles the forts.	Nobles of	Bridges	23	0	88½	2	17½	2	17	0	7	4	0	9	11	
		Flanders, or Flemish Noble	23	0	54	4	10⅔	4	10	0	12	0	0	16	3	
		Half Flemish Noble	23	0	108	2	5⅓	2	5	0	6	0	0	8	1½	
		Gaunt	23	0	54	4	10⅔	4	10	0	12	0	0	16	3	
		Half Noble, as the Hungary Ducat														
		Half Noble with the Lyon, as Bridges														
		Henry Noble of France	22	0	51	4	16⅓	4	16	0	13	4	0	16	5½	
		The Half thereof	22	2½	108	2	5⅓	2	5	0	6	8	0	7	7¼	
		Hollan	23	3	48	5	0	4	20	0	14	3	0	18	10¼	
		Overysfel and } as Holland														
Utrecht																
		Zeland, as Gaunt														
		Pezzo of Peru, by Heylin, p. 1064														
Pisto- lets the forts.	Pistolets of	De Lege & Legion	18	0	108	2	5⅓	2	5	0	4	9	0	6	4¼	
		Italy { Some	21	2½	108	2	5⅓	2	5	0	5	9	0	6	7½	
		Italy { Others	22	0	108	2	5⅓	2	5	0	5	10	0	7	9¼	
		Scotland	19	2½	108	2	5⅓	2	5	0	5	2	0	6	11¼	
		Spaine	{ Single { Some	21	3½	108	2	5⅓	2	5	0	5	10	0	7	9¼
				{ Others	22	0	108	2	5⅓	2	5	0	5	10	0	7
		Double	{	22	0	54	4	10⅔	4	8	0	11	8	0	15	6½
				Of 26 Ryals	22	0	45	5	8	5	6	0	14	0	0	18
				Portuguese, See Great Cruzado of Portugal												
		Pof- tulates the forts.	Poftulates of	Bourbon	12	0½	136½	1	18⅓	1	18	0	2	7	0	3
Cleves	9			0½	156	1	12⅓	1	12¼	0	1	9	0	2	2¼	
Dog and Cat, as Bourbon																
Fran. Frier	9			0	156	1	12⅓	1	12¼	0	1	8	0	2	2½	
Horne	10			0½	156	1	12⅓	1	12¼	0	1	11	0	2	5¼	
Juliers, or Guliers	9			3	156	1	12⅓	1	12¼	0	1	10	0	2	4½	
6 Pound Scotch	22			0	80	3	0	3	0	0	10	0	0	10	6	
12 Pound Scotch	22			0	40	6	0	6	0	1	0	0	1	1	0	
Burgundy	23			0½	105	2	6⅔	2	6½	0	6	3	0	8	5½	
Campen and Swoll	12			3	114	2	2⅓	2	2	0	3	2	0	4	3¼	
Ri- ders the forts.	Riders of	Deventer, as Campen and Swoll														
		Flanders	23	3	105	2	6⅔	2	6½	0	6	6	0	8	7½	
		Friesland } of the Year 1583	21	0	108	2	5⅓	2	5	0	5	6	0	7	5	
		Guelthers }														
		Guelthers Ryder	14	0	114	2	2⅓	2	2	0	3	6	0	4	8¼	
		Guelthers new Ryder	23	3	105	2	6⅔	2	6½	0	6	9	0	8	7½	
		Philip Clincart	14	0	114	2	2⅓	2	2	0	3	6	0	4	8¼	
		Ryder with the Loaves	10	0½	114	2	6⅓	2	2	0	2	7	0	3	4	
		Scotland { Some others	19	2½												
		{ Others	22	0												
Roy- als the forts.	Ruble of	States of the United Provinces	22	0	36	6	16	6	12	1	0	0	1	3	4	
		The Half thereof	22	0	72	3	8	3	6	0	10	0	0	11	8	
		Muscovic (by Heylin 13 s. 4 d.)								0	10	0	0	13	4	
		Poland								0	13	4	0	13	4	
		Austria	{ Single	23	3½	49½	4	20⅓	4	20	0	14	4	0	18	5
				{ Double	23	3½	24¾	9	16⅓	9	16	1	8	8	1	16
		Campen and Swoll	23	0	40½	4	20⅓	4	20	0	14	2	0	17	9	
		The Half thereof	23	0	99	2	10⅓	2	10	0	7	1	0	8	10½	
		Flanders, or the Key	23	0	69¾	3	10⅓	3	10½	0	10	0	0	12	6¼	
		The Half thereof	23	0	139½	1	17⅓	1	17¼	0	5	0	0	6	3¼	
Royals or Reals of	Royals or Reals of	Imperial Loyal	23	3½	69	3	11⅓	3	11	0	11	0	0	13	2½	
		The Half thereof	18	0	105⅓	2	6⅔	2	6½	0	4	11	0	6	6¼	
		Philip with the Spread Eagle	18	2	106½	2	6⅔	2	6	0	5	0	0	6	7¼	
		Philip of Spain	23	0	69¾	3	10⅓	3	10½	0	10	0	0	12	6¼	
		Ship of Flanders, or Schuytken	22	1	109½	2	4⅓	2	4½	0	6	3	0	7	9	
		Shock of Bohemia, some say 8 s. some								0	9	0	0	9	0	
		Stiver Pieces, 9 Stiv. Pieces of { Batenborg	7	1	176	1	8⅓	1	8½	0	1	3		6	6¼	
		{ and Frize }														
		Sultanies of 120 Aspers	23	1	90	2	15	2	16	0	7	0	0	9	10¼	

Foreign Gold.			Fine	Pieces	Weight by	Weight by	Old			New					
			Car.Gr.	to the	Malines.	others.	Value.			Value.					
				lb Troy.	pwt.s.	gr.	pwt.s.	gr.	l.	s.	d.	l.	s.	d.	
Tabe of China									0	5	4½	0	7	2	
Toman of Persia, by Heylin, p. 839. valued at 20 Crowns, seemeth not to be any Coine, but a denomination used in Accompt									5	0	0	5	0	0	
Unicorn of Scotland			22	0	99¼	2	10	¾	0	6	0	0	8	½	
Xeriffe of Goa in India, by Heylin by Grimstone p. 197. worth 300 Res of Portugal									0	6	0	0	7	6	
Zechines, or Checkeenes of Venice			23	1	90	2	16		0	7	6	0	9	10¼	
Foreign Silver.			Fine	Pieces	Weight of	Weight of	Sterling			Value by			Foreign		
			3 pwt.s.	to the	the Pieces	the Pieces	Value.			some Authors.			Silver		
				lb Troy.	3. pwt.s.	gr.	s. d.			s. d.			Coines:		
Albi of	Bamberg		4	18	273	0	0	21	⅔	0	1	+	0	1	Albi, or Albes.
	Collen	Some	5	10	345	0	0	16	⅔	0	1	+	0	1	
		Others	5	10	342	0	0	16	⅔	0	1	+	0	1	
		Others	5	10	179	0	1	8	⅔	0	2	—	0	2	
	Frankford, or Bamberg														
	Mentz, as Collen														
	Norenbergh and													as Bamberg.	
	Palatine of the Rhine--														
	Trier, as Collen														
Aiteen, or Atten of Muscovia												0	4	+	
Altine of Poland													0	4½	
Angel of Scrikelborg			10	7½	78¼	0	3	1⅓	0	8½	+	0	8		
Asper of Turkie													0	1¼	
Babec of Scotland													0	0½	
Batz of + Creutzes.	Bavaria													Batz of several sorts.	
	Brandenburgh														
	Colmograve														
	Cost. of 1530.														
	Eriburgh		5	7	109½	0	2	4⅔	0	3	+	0	3		
	Ottinge														
	Raynsburgh														
	Roy														
Kemptor half Batz			4	12½	192½	0	1	5⅔	0	1½	+	0	1½		
Munchien half Batz			4	12½	186	0	1	6⅓	0	1½	+	0	1½		
3 Batz, See Snaphane															
Remesh, or Bemish of Switz												0	2½		
Bianco, or Biamco of Italy												0	8		
Blanks													0	0¼	
Half Ruyters Blank of Holland			3	0	144	0	1	16	0	1¼	+	0	1¼	Pieces of 3. Carolus.	
Bologne													0		0¼
Carlini of Italy													0		6
Carolus Guilder, as ⅔ of the Philips Dollar															
3 Carolus of	Carolus and Salsburgh														
	Campidona														
	Ernestus														
	Frankford		9	0	78¼	0	3	1⅓	0	7¼	+	0	7		
	Ottingus														
Patavia															
Reynsborg															
Causteto, or Caveto of Italy													0	3¼	

Foreign Silver.		Fine Pieces to the		Weight of the Pieces		Sterling Value,		Value by others.	
		3. pwt. 15. Troy.		3. pwt. gr.		s. d.		s. d.	
Creutz. of divers sorts.	Crenciat of John of Cleve	8	7	39 $\frac{3}{4}$	0	6	0 $\frac{4}{5}$ $\frac{8}{3}$	1	1 $\frac{1}{2}$ +
	of Ausburge and Ulme	5	5	384	0	0	15	0	0 $\frac{3}{4}$ +
	of Poland							0	0 $\frac{3}{4}$
	12 Creutzers of { Bavaria—	8	7 $\frac{1}{2}$	57	0	4	5 $\frac{1}{9}$	0	9 $\frac{1}{2}$ +
	Vienna—							0	9
	other 12 Creutzer Pieces	8	7 $\frac{1}{2}$	61 $\frac{1}{2}$	0	3	21 $\frac{2}{4}$ $\frac{2}{1}$	0	8 $\frac{3}{4}$ +
	And some	10	10	same			same	0	11+
	10 Creutzers of { Frise—	8	7	64 $\frac{1}{2}$	0	3	17 $\frac{1}{4}$ $\frac{3}{3}$	0	8 $\frac{1}{4}$ +
	Ravenburg—							0	8
	Salsburg—								
	Saxony—								
	6 Creutzers of { Insburgh—	10	10	124 $\frac{1}{2}$	0	1	22 $\frac{2}{8}$ $\frac{2}{3}$	0	5 $\frac{1}{2}$ —
	Vienna—	8	7 $\frac{1}{2}$	114	0	2	2 $\frac{1}{1}$ $\frac{0}{9}$	0	4 $\frac{3}{4}$ +
	other 6 Creutzer Pieces	10	10	123	0	1	22 $\frac{2}{4}$ $\frac{4}{1}$	0	5 $\frac{1}{2}$ +
	3 Creutzers of { Bavaria—	4	8	375	0	0	15 $\frac{2}{1}$ $\frac{2}{7}$	0	0 $\frac{3}{4}$ +
Dollars of divers sorts.	Vienna—	4	8 $\frac{1}{3}$	129	0	1	20 $\frac{2}{4}$ $\frac{8}{1}$	0	2 $\frac{1}{4}$ —
	other 3 Creutzer Pieces	5	10	136 $\frac{1}{2}$	0	1	18 $\frac{1}{9}$ $\frac{8}{1}$	0	2 $\frac{1}{2}$ +
	2 Crosses and Harpe	4	0	180	0	1	8	0	1 $\frac{1}{4}$ +
	(France, See Louis								
	Crowns of { Italy								5
	Turky								0
	Cupstoke								1
	Deghen, or { of { Muscovia & de Narde	11	13	545 $\frac{1}{3}$	0	0	10 $\frac{2}{4}$ $\frac{0}{9}$	0	1 $\frac{1}{4}$ +
	Denghen—								0
	Denier, Petit Denier of { Paris	1	10	270	0	0	21 $\frac{1}{3}$	0	0 $\frac{1}{4}$ +
	Tor	1	10	337 $\frac{1}{2}$	0	0	17 $\frac{1}{1}$ $\frac{1}{5}$	0	0 $\frac{1}{4}$ +
	Dicken of a Wing								1
	Albania, or the Cross Dollar								4
	Basil of 6 Creutzers	10	13 $\frac{1}{2}$	15	0	16	0	3	10+
	Batenburgh—								3
Dollars or Dollers of	Bohemia. Ne.Op. {	7	15	13 $\frac{1}{2}$	0	17	18 $\frac{2}{3}$	3	1 $\frac{1}{4}$
	Bommel—								3
	Brisgau—	10	15	15	0	16	0	3	10 $\frac{1}{2}$ —
	Cambray, The Sixteenth part	6	10	123	0	1	22 $\frac{3}{4}$ $\frac{4}{1}$	0	3 $\frac{1}{4}$ +
	Christopher, 45.	10	10	12 $\frac{1}{4}$	0	18	19 $\frac{1}{1}$ $\frac{3}{7}$	4	5 $\frac{1}{4}$ +
	Frisland of 1601.	9	0	13 $\frac{1}{2}$	0	17	18 $\frac{2}{3}$	3	7 $\frac{1}{4}$ —
	See Guelders								3
	Ryckx Dollar Ouncia—	11	5	12 $\frac{1}{2}$	0	19	4 $\frac{1}{5}$	1	10 $\frac{1}{4}$ +
	Ryckx Dollar of 1567	10	12	12 $\frac{1}{2}$	0	19	4 $\frac{4}{5}$	4	7+
	Others—	10	13	12 $\frac{1}{2}$	0	19	4 $\frac{4}{5}$	4	7 $\frac{1}{4}$ +
	Others—	10	14	12 $\frac{1}{2}$	0	19	4 $\frac{4}{5}$	4	7 $\frac{1}{2}$ +
	Others—	11	0	12 $\frac{1}{2}$	0	19	4 $\frac{4}{5}$	4	2+
	Others—	11	3	12 $\frac{1}{2}$	0	19	4 $\frac{4}{5}$	4	9 $\frac{1}{4}$ +
	Others—	11	3	12 $\frac{1}{2}$	0	19	4 $\frac{4}{5}$	4	10
	Guelders and Frisland { Some—	10	4	14 $\frac{1}{2}$	0	16	20 $\frac{1}{9}$	3	10 $\frac{1}{4}$ +
	Others—	9	0	12 $\frac{3}{4}$	0	19	22 $\frac{3}{2}$ $\frac{4}{5}$ $\frac{7}{7}$	4	0 $\frac{1}{4}$ +
	(Some—	9	0	15	0	16	0	3	2 $\frac{3}{4}$ +
	Guelders and Utrecht { Others—	10	10	13	0	18	11 $\frac{1}{3}$	4	4 $\frac{1}{4}$ +
	(Others—	10	12	13	0	11	11 $\frac{1}{3}$	4	4 $\frac{3}{4}$ +
	See Zutphen								4
	Gulheimus of Sweden	10	10	12 $\frac{1}{4}$	0	18	19 $\frac{1}{1}$ $\frac{1}{7}$	4	5 $\frac{1}{4}$ +
	Gustavus of Liege the 38th.	10	4	12 $\frac{1}{4}$	0	18	19 $\frac{1}{1}$ $\frac{1}{7}$	4	3 $\frac{3}{4}$ +
	Holland	9	0	13 $\frac{1}{1}$	0	18	0	3	7 $\frac{1}{4}$
	Holland with the Crown	8	0	13 $\frac{1}{1}$	0	18	0	3	2 $\frac{1}{4}$
	Ismensen, as Basil								3
	Luneburgh—	10	16 $\frac{1}{2}$	15	0	16	0	3	10 $\frac{3}{4}$
	Phillip—	10	0	10 $\frac{1}{2}$	1	2	9 $\frac{1}{1}$	5	0 $\frac{1}{4}$ +

Foreign Silver.		Fine 3 pmts.	Pieces to the Troy.	Weight of the Pieces 3 pmts. gr.	Sterling Value, s. d.	Value by others. s. d.	
Dollars, or Dollars of	Half of the <i>Phillips Dollar</i> —	10	0	21 $\frac{3}{7}$	0 11 4 $\frac{4}{5}$	2 6 $\frac{1}{4}$ +	2 6
	Fourth Part—	10	0	42 $\frac{6}{7}$	0 5 14 $\frac{2}{5}$	1 3 +	1 3
	Fifth Part—	10	0	53 $\frac{4}{7}$	0 4 11 $\frac{1}{2}$ $\frac{3}{5}$	1 0 +	1 0
	Tenth Part—	10	0	107 $\frac{1}{7}$	0 2 5 $\frac{1}{2}$ $\frac{2}{5}$	0 6 +	0 6
	Twentieth Part—	5	0	107 $\frac{1}{4}$	0 2 5 $\frac{1}{4}$ $\frac{1}{3}$	0 3 +	0 3
	Fortieth Part—	5	0	214 $\frac{1}{2}$	0 1 2 $\frac{1}{4}$ $\frac{2}{3}$	0 1 $\frac{1}{2}$ +	0 1 $\frac{1}{2}$
	Two third Parts of the same <i>Dollar</i> —	10	0	16 $\frac{1}{4}$	0 14 22 $\frac{2}{5}$	3 4 +	3 4
	<i>Poland</i> { Some as <i>Batenborgh</i> —						
	Others of 60 <i>Creutzers</i> —	11	3 $\frac{1}{2}$	15	0 16 0	4 0 $\frac{1}{4}$ +	4 0
	<i>Prince of Orange</i> or <i>Lyon Dollar</i> —	9	0	13	0 18 11 $\frac{1}{3}$	3 8 $\frac{3}{4}$ +	4 0
	<i>Reynsburgh</i> , as <i>Basil</i> —						
	<i>Riga</i> —	10	2 $\frac{1}{2}$	13 $\frac{1}{2}$	0 17 18 $\frac{2}{3}$	4 0 $\frac{1}{2}$ +	4 0
	<i>Scotland</i> with the <i>Cross Daggers</i> —	11	2	11 $\frac{3}{4}$	1 0 10 $\frac{1}{4}$ $\frac{0}{7}$	5 1 $\frac{1}{4}$ +	5 0
	<i>States General</i> of the <i>United Provinces</i> —	9	0	12 $\frac{3}{4}$	0 19 22 $\frac{3}{5}$ $\frac{4}{7}$	4 0 $\frac{1}{4}$ +	4 0
	<i>Suecia</i> , or { <i>Merchants Dollar</i> —						3 2
	{ <i>Ryckx</i> , or <i>Imperial Dollar</i> —						5 2
	<i>Tremone</i> , as <i>Brifgau</i> —						
	<i>Utrecht</i> , See <i>Guilders</i> —						
	<i>Zeland</i> , with the <i>Eagles</i> —	9	0	13 $\frac{1}{2}$	0 17 18 $\frac{2}{3}$	3 7 $\frac{1}{4}$ —	3 7
	<i>Zutphen</i> and <i>Guilders</i> of 1586—	10	4	13 $\frac{2}{3}$	0 17 10 $\frac{1}{4}$ $\frac{9}{1}$	4 0 $\frac{1}{4}$ +	4 0
Duyts	<i>Drier</i> —						0 0 $\frac{2}{4}$
	<i>Duplus</i> —	2	0	324	0 0 17 $\frac{7}{9}$	0 0 $\frac{1}{4}$ +	} not currant in England. nor this.
	<i>Dupli Simple</i> —	5	10	882	0 0 6 $\frac{2}{4}$ $\frac{6}{9}$	0 0 $\frac{1}{4}$ +	
	<i>Dupli Mavi</i> $\frac{1}{9}$ of <i>Guliel.</i> of <i>Turing</i> —	2	15	440	9 6 13 $\frac{1}{1}$ $\frac{1}{1}$	0 0 $\frac{1}{4}$ +	
	9 <i>Duyts</i> <i>Peny</i> of <i>Charles</i> and <i>Philip</i> —	4	14	129	0 1 20 $\frac{2}{4}$ $\frac{8}{3}$	0 2 $\frac{1}{4}$ +	0 2 $\frac{1}{4}$
	11 <i>Duyts</i> of { <i>Charles-Limburg</i> —	4	15	120	0 2 0	0 2 $\frac{1}{2}$ +	0 2 $\frac{1}{2}$
		6	0	144	0 1 16	0 2 $\frac{1}{2}$ +	0 2 $\frac{1}{2}$
		11	3 $\frac{1}{2}$	270	0 0 21 $\frac{1}{3}$	0 2 $\frac{1}{2}$ +	0 2 $\frac{1}{2}$
	17 <i>Duyts</i> —	10	10	147	0 1 15 $\frac{2}{4}$ $\frac{9}{9}$	0 4 $\frac{1}{2}$ +	0 4 $\frac{1}{2}$
	17 <i>Duyts</i> of { <i>Charles</i> —						
	17 <i>Duyts</i> of { <i>Guilthers</i> —						
17 <i>Duyts</i> of { <i>Liege</i> —							
17 <i>Duyts</i> of { <i>Limburgh</i> —							
17 <i>Duyts</i> of { <i>Lodowicke</i> —							
17 <i>Duyts</i> of { <i>Philip</i> —							
17 <i>Duyts</i> of { <i>Philip of Flanders</i> —							
17 <i>Duyts</i> of <i>Sluce</i> —	9	5	148	0 1 14 $\frac{3}{4}$ $\frac{4}{7}$	0 4 +	0 4	
Groots	<i>Flabes</i> in the <i>Low Countries</i> —						1 4
	<i>Finferkin</i> —						0 0 $\frac{1}{2}$
	<i>Fleece</i> , See <i>Stivers</i> .—						
	<i>Florins</i> , by <i>Heylin</i> —						3 0
	<i>Franks</i> of <i>Turky</i> —						2 0
	<i>Franks</i> of <i>France</i> , 3 to a <i>Crown</i> —	10	0	26 $\frac{1}{4}$	0 9 3 $\frac{3}{7}$	2 0 $\frac{1}{2}$ +	2 0
	<i>Gagatta</i> of <i>Italy</i> —						0 1
	<i>Gmbij</i> of <i>Rome</i> —						0 6
	<i>Grot</i> , or <i>Groot</i> —						0 1 $\frac{1}{4}$
	3 <i>Groots</i> or <i>Deniers</i> —						
		5	10	117 $\frac{1}{4}$	0 2 0 $\frac{1}{4}$ $\frac{4}{7}$	0 3 +	0 3
	5 <i>Groots</i> of { <i>Flanders</i> —						
		10	6 $\frac{1}{2}$	146 $\frac{1}{2}$	0 1 15 $\frac{2}{9}$ $\frac{1}{3}$	0 4 $\frac{1}{2}$ +	0 4 $\frac{1}{2}$
		5	13	145	0 1 15 $\frac{2}{9}$ $\frac{1}{9}$	0 2 $\frac{1}{2}$ +	0 2 $\frac{1}{2}$
		11	3	135	0 1 18 $\frac{3}{1}$	0 5 $\frac{1}{4}$ +	0 5 $\frac{1}{4}$
	5 $\frac{1}{2}$ <i>Groots</i> of { <i>Others</i> —						
		10	14	135	0 1 18 $\frac{2}{3}$	0 5 +	0 5
5 $\frac{1}{2}$ <i>Groots</i> of 1520—							
<i>Ala. Flanders</i> }	9	14	120	0 2 0	0 5 $\frac{1}{4}$ —	0 5 $\frac{1}{4}$	

Groth

Foreign Silver.

Fine Pieces Weight of Sterling Value
 3 pwt. 15 Troy. 3 pwt. gr. s. d. s. d.
 by others.

Gros of
divers
forts.

<i>Ambafs</i>	4	12½	94½	0	2	2½	0	3	+	0	3
<i>Ausburgh</i> { Some	5	7	108	0	2	5½	0	3	—	0	3
Others	6	4½	155	0	1	13½	0	2½	+	0	2½
Others of 3 Batz											9
<i>Bafil</i>	9	0	118½	0	2	0¾	0	3¼	+	0	3¼
<i>Bassaw</i>	5	7	116½	0	2	6¾	0	3¼	+	0	3¼
<i>Bohemia</i> , 1½ Silver Grosh	3	7½	87	0	2	18½	0	2½	+	0	2½
<i>Brisau</i> , as Bassaw											
<i>Brisgrave</i> } as Basil											
<i>Campido</i> }											
<i>Corinthia</i> }											
<i>Coningstein</i>	5	7	108	0	2	5½	0	3	+	0	3
<i>Curienfis</i> , as Bassaw											
<i>Duodena</i> , or the 12. par. of the Sil. Grosh	3	3½	874½	0	0	6½	0	0¼	—	0	not currant in England.
<i>Ferdinando</i> of Dantsicke	5	0	180	0	1	8	0	1¾	+	0	2
<i>George</i> and Wormeser, as Ambafs											
<i>Kempton</i> , as Bassaw											
<i>Markegrave</i> , as Ambafs											
<i>Mary</i>											1¼
<i>Melvin</i> 3 Grosh { of 1340	10	4	138	0	1	17½	0	4¼	+	0	5
Others	10	10	138	0	1	17½	0	4¼	+	0	5
<i>Meysen</i>											2¼ or 3d
<i>Noiling</i> , as Ambafs											
<i>Poland</i>											1½
<i>Poland</i> Six Grosh	6	0	13½	0	17	7¾	2	4	+	2	4
<i>Prague</i>	9	12½	180	0	1	8	0	3¼	+	0	3¼
<i>Prussia</i> , 3 Grosh alb.	10	10½	138	0	1	17½	0	4¼	+	0	5
<i>Reynsburgh</i>	6	4½	155	0	1	13½	0	2½	+	0	2½
<i>Salzburg</i> { Some	6	2½	118½	0	2	0¾	0	3¼	+	0	3¼
Others	4	12½	39	0	6	3¼	0	7½	+	0	7½
<i>Saxony</i> , as Coningstein											
<i>Scafhuyfen</i> , as Basil											
<i>Sigismund</i> of 1532, and 1535	10	4	69	0	3	11½	0	9½	+	0	9½
<i>Sigismund</i> of Prussia 1534	10	11	69	0	3	11½	0	9½	+	0	10
Others with the Armes of Dantsick	10	0½	69	0	3	11½	0	9½	+	0	9½
<i>Silver Grosh</i> Common											2
<i>Taven</i> , as Basil											
<i>Vienna</i>	6	4	132	0	1	19¾	0	3	+	0	3
1½ Silver Grosh	3	7½	87	0	2	18½	0	2½	+	0	2½
4 Grosh Penny	8	0	81	0	2	23¾	0	6¼	+	0	6
<i>Albertus</i> { Double	10	15	14½	0	16	10¾	4	0	—	4	0
Single	10	15	29½	0	8	5¾	2	0	—	2	0
Half	10	15	58½	0	4	2¾	1	0	—	1	0
Quarter	10	15	116½	0	2	1¾	0	6	—	0	6
<i>Carolus</i> , as 2 of the Philips Doller											
<i>Flanders Silver Guilder</i>											2 0
<i>Gulielmus</i> of Turing	6	15	129	0	1	20¾	0	3¼	+	0	3¼
<i>Harp</i> of Ireland, or Silver Harp	11	0	82	0	2	22½	0	8½	+	0	9
<i>Half Harp</i>	11	0	164	0	1	11¼	0	4¼	+	0	4½
<i>Base Irish Harp</i>	3	0	82	0	2	22½	0	2¼	+	0	2¼
<i>Old Harp</i>	9	6	102	0	2	8¾	0	5¼	+	0	6
<i>Junetine</i> , or <i>Justine</i> of Italy											1 6

Gu iders
the forts.Gulders,
or
Gulden

Foreign Silver.

Foreign Silver.			Fine	Pieces	Weight of	Sterling	Value	Value	
			to the	the Pieces	the Pieces	Value	by others.		
			3. pwt.	lb. Troy.	3. pwt.	gr.	s. d.	s. d.	
Lion of Gulielmi—	{ Some—	2 5	150	0 1	14 $\frac{2}{3}$	0 0 $\frac{3}{4}$ +	0 1		
	{ Others—	2 5	179	0 1	8 $\frac{2}{3}$	0 0 $\frac{3}{4}$ +	0 1		
Lieure of France, See Quar. Cro. by Heylin							2 0		
Louis of France—		11 2					4 6		
Half, Quart. and Eight part accordingly									
Lyarts of France, H.—		3 0						not currant in England.	
Lyre of { Geneva—							1 4		
	{ Venice—						0 9		
Maille, Old Petit Maille—		1 0	450	0 0	12 $\frac{4}{5}$	0 0 $\frac{1}{8}$ +		not cur. in Eng.	
Magenburgh, 3 Armes—		5 8 $\frac{1}{3}$	27	0 8	21 $\frac{1}{3}$	1 1 +	1 1		
Other Piece—		11 3 $\frac{1}{2}$	51	0 4	16 $\frac{1}{7}$	1 2 +	1 2		
Mark of Denmark—							2 2		
Mark of Scotland—		11 2	54	0 4	10 $\frac{2}{3}$	1 1 $\frac{1}{4}$ +	1 1 $\frac{1}{2}$		
Half and Quarter accordingly									
Markesticke of { Lady Mary—		10 16 $\frac{2}{3}$	27	0 8	21 $\frac{1}{3}$	2 2 +	2 2		
	{ Lubeck—								
Medine of Cairo—							0 2 $\frac{1}{4}$		
Murjenigo—							0 11		
Nummi Dragme { Some—		6 0	140	0 1	17 $\frac{1}{7}$	0 2 $\frac{3}{4}$ +	0 3		
	{ Others—	6 2 $\frac{1}{2}$	118 $\frac{1}{2}$	0 2	0 $\frac{4}{9}$	0 3 $\frac{1}{4}$ +	0 3		
Peny of { Bohemia { White—		5 7	924	0 0	6 $\frac{1}{7}$	0 0 $\frac{1}{4}$ +	not currant in England.		
	{ Black—	2 13 $\frac{1}{2}$	990	0 0	5 $\frac{2}{11}$	0 0 $\frac{2}{5}$ +			
Holland—		0 19	518	0 0	11 $\frac{2}{5}$	0 0 $\frac{1}{9}$ +			
Peny, called the Brats Peny—		4 10	120	0 2	0	0 2 $\frac{1}{4}$ +	0 2		
Half thereof—		4 10	240	0 1	0	0 1 +	0 1		
Half Ruyters Black Peny—		4 14	256	0 0	22 $\frac{1}{2}$	0 1 +	0 1		
Pfound, or Pfound—							0 4 $\frac{3}{4}$		
Plappot—							0 2 $\frac{1}{2}$ or 2 $\frac{3}{4}$		
Poali of Italy—							0 6		
Pound, 3 Pound of Scotland—		11 2					5 0		
Polpate, or Baldpate of Scotland—		11 2					0 10 $\frac{1}{2}$		
Half thereof—		11 2					0 5 $\frac{1}{4}$		
Quart. of { France—		10 6 $\frac{2}{3}$	39	0 6	31 $\frac{2}{3}$	1 5 +	1 6		
	Lorraine—	9 8 $\frac{1}{2}$	39	0 6	31 $\frac{2}{3}$	1 3 $\frac{1}{2}$ +	1 4		
		Philip—							
		Savoy—	10 16 $\frac{1}{2}$	39	0 6	31 $\frac{2}{3}$	1 6 +	1 6	
Rappen Muntz—							0 2 $\frac{1}{2}$		
Rouflick—							0 1 $\frac{1}{2}$		
Albertus of Austria—		10 15	120	0 2	0	0 5 $\frac{1}{4}$ +	0 6		
Half and Quarter accordingly									
Pieces of his of 3 Ryals—		10 15	40	0 6	0	0 5 $\frac{1}{4}$ +	1 6		
Italy { Some—		9 17	108	0 2	5 $\frac{1}{3}$	0 5 $\frac{1}{4}$ +	0 6		
	{ Others—	9 14	108	0 2	5 $\frac{1}{3}$	0 5 $\frac{1}{4}$ +	0 6		
	{ Others—	9 11	108	0 2	5 $\frac{1}{3}$	0 5 $\frac{1}{4}$ +	0 6		
Mexico, 8 Ryals—		11 0	13 $\frac{2}{3}$	0 17	13 $\frac{1}{3}$	4 4 +	4 4		
Rome, Courle Ryals—		7 0	108	0 2	5 $\frac{1}{3}$	0 4 +	0 4		
Spain—		11 3 $\frac{1}{2}$	108	0 2	5 $\frac{1}{3}$	0 6 $\frac{1}{2}$ +	0 6		
Spanish 8 Ryals called Pieces of 8—		11 4	13 $\frac{1}{2}$	0 17	18 $\frac{2}{3}$	4 5 $\frac{1}{4}$ +	4 4		
States General of the United Provinces		10 0	10 $\frac{2}{3}$	1 2	9 $\frac{2}{3}$	5 0 $\frac{1}{2}$ +	5 0		
The 20th. part of the same, with the Arrows accordingly									
Venice—		11 10	96	0 2	12	0 7 $\frac{3}{4}$ +	0 8		
Ryals of Gelders and Friesland—		9 0	12 $\frac{3}{4}$	0 19	22 $\frac{3}{4}$	4 0 $\frac{1}{4}$ +	4 0		
Salvator of Venice—		11 10	96	0 2	12	0 7 $\frac{3}{4}$ +	0 8		

Carde-
cues the
forts.

Ryals of
several
forts.

Foreign Silver.		Fine 3 pmts.	Pieces to the 16 Troy.	Weight of the Pieces 3. pmts. gr.	Sterling Value, s. d.	Value by others. s. d.
Shillings of several forts.	<i>Saffenars double</i>	10	6 $\frac{1}{2}$	146 $\frac{1}{2}$	0 1 15 $\frac{2}{3}$ $\frac{2}{3}$	0 4 $\frac{1}{2}$ + 0 4 $\frac{1}{2}$
	<i>Scaby of Turkey</i>					0 6
	<i>Schaneberger</i>					0 1 $\frac{3}{4}$
	<i>Scya of Turkey</i>					0 6 $\frac{1}{4}$
	<i>Senube, or S'nube of Bohemia</i>	5	7	129	0 1 20 $\frac{2}{4}$ $\frac{8}{3}$	0 2 $\frac{1}{2}$ + 0 2 $\frac{1}{4}$
	Half thereof.	5	7	258	0 0 22 $\frac{1}{1}$ $\frac{4}{3}$	0 1 $\frac{1}{4}$ + 0 1 $\frac{1}{4}$
	<i>Sestling</i>					0 0 $\frac{3}{4}$
	<i>Bridges of 1582</i>	5	0	57	0 4 5 $\frac{1}{9}$	0 5 $\frac{1}{2}$ + 0 5
	<i>Dantfieke</i>					0 0 $\frac{3}{4}$
	8 Shilling of <i>Dantfieke</i> of 1541	10	12	156	0 1 12 $\frac{1}{1}$ $\frac{2}{3}$	0 4 $\frac{1}{4}$ + 0 4
	<i>Flanders</i>					0 7 $\frac{1}{2}$
	<i>Frisland of 1586</i>	6	0	57	0 4 5 $\frac{1}{9}$	0 6 $\frac{3}{4}$ + 0 6
	<i>Gaunt of 1583</i>	7	7	54	0 4 10 $\frac{2}{3}$	0 8 $\frac{3}{4}$ + 0 9
	<i>Germany</i>					0 5 $\frac{1}{4}$
	<i>Guelthers, as Frisland.</i>					0 9 $\frac{1}{4}$
French Sols of divers forts.	<i>Hamborough</i>					0 1 $\frac{1}{4}$
	<i>Lubeck</i>					0 1 $\frac{1}{4}$
	<i>M. E. and Philip of Flanaers</i>	11	3	135	0 1 18 $\frac{2}{3}$	0 5 $\frac{1}{4}$ + 0 5
	<i>Scotland</i>					0 1
	<i>Switz, or Helvetia</i>					0 1 $\frac{1}{4}$
	<i>Utrecht</i>					0 1 $\frac{1}{4}$
	<i>Zeland</i> } as <i>Frisland</i>					0 1 $\frac{1}{4}$
	<i>Sicherling</i>					0 1 $\frac{1}{4}$
	<i>Snapbanen, Coined for 3 Batz.</i>	7	7 $\frac{1}{2}$	39 $\frac{1}{4}$	0 6 0 $\frac{4}{5}$ $\frac{8}{3}$	1 0 + 1 0
	<i>Snapbanen of</i> { <i>Cleve</i> } { <i>Deventer</i> } { <i>Nimmeghen</i> }	7	11	48	0 5 0	0 10 + 0 10
	<i>Soldi of Genoa</i>					0 0 $\frac{3}{4}$
	<i>Soli of Wersburgh, Dantfick and Prussia</i>	5	6 $\frac{1}{4}$	157 $\frac{1}{2}$	0 1 12 $\frac{4}{7}$	0 2 + 0 2
	<i>Soulx, or Solx of France</i>					0 1
	<i>Soulx stamped, called Soulx Marque</i>					0 1 $\frac{1}{4}$
Styvers of divers forts.	<i>The Old Soulx with</i> $\frac{1}{4}$	4	5	175	0 1 8 $\frac{2}{3}$ $\frac{2}{3}$	0 1 $\frac{1}{2}$ + 0 2 $\frac{1}{4}$
	<i>Ordinary French Soulx</i>	3	10	147	0 1 15 $\frac{2}{4}$ $\frac{9}{9}$	0 1 $\frac{1}{2}$ + 0 1 $\frac{1}{4}$
	<i>Late French Soulx</i>	3	6 $\frac{1}{2}$	147	0 1 15 $\frac{2}{4}$ $\frac{9}{9}$	0 1 $\frac{1}{4}$ + 0 1
	<i>Double Hand of one Soulx</i>	3	15	132	0 1 19 $\frac{2}{1}$ $\frac{1}{1}$	0 1 $\frac{3}{4}$ + 0 1 $\frac{1}{4}$
	<i>Two Soulx Pieces, or Doubles</i>	6	6 $\frac{2}{3}$	117	0 2 1 $\frac{1}{1}$ $\frac{3}{3}$	0 3 $\frac{1}{2}$ — 0 3
	<i>Four Soulx Pieces accordingly.</i>					
	<i>Cambray</i>	3	5	135	0 1 18 $\frac{2}{3}$	0 1 $\frac{1}{2}$ + 0 1 $\frac{1}{4}$
	<i>Embsen</i>					0 1 $\frac{1}{4}$
	<i>Gaunt of 1583</i>	3	0	175 $\frac{1}{2}$	0 1 8 $\frac{1}{3}$ $\frac{2}{9}$	0 1 — 0 1
	<i>Groeninghen</i> } as <i>Cambray</i>					
	<i>Liege</i>					
	<i>States General of the United Provinces</i>	4	0	168	0 1 10 $\frac{2}{7}$	0 1 $\frac{1}{2}$ + 0 1 $\frac{1}{4}$
	<i>Utrecht</i>	3	0	167	0 1 10 $\frac{8}{1}$ $\frac{2}{67}$	0 1 + 0 1
	<i>Old Styver</i>	3	14 $\frac{1}{6}$	120	0 2 0	0 2 + 0 2
	<i>New Styver</i>	3	13 $\frac{1}{3}$	120	0 2 0	0 2 — 0 2
Stryers of	<i>Half Styver</i>	3	10	201	0 1 4 $\frac{4}{8}$ $\frac{7}{7}$	0 1 + 0 1
	<i>Quarter Styver Oort</i>	1	17 $\frac{1}{2}$	158	0 1 12 $\frac{1}{7}$ $\frac{6}{9}$	0 0 $\frac{3}{4}$ + not currant in England.
	<i>Eight part Stryver Duyt</i>	1	14	474	0 0 12 $\frac{1}{7}$ $\frac{2}{9}$	0 0 $\frac{1}{4}$
	<i>Old Double Styver</i>	7	7 $\frac{1}{2}$	120	0 2 0	0 4 — 0 4
	<i>Old Three Stryvers</i>	11	3 $\frac{3}{4}$	120	0 2 0	0 6 + 0 6
	<i>Old Four Stryvers</i> { <i>with the Eagle</i> } { <i>Charles and</i> } { <i>Philip</i> }	7	7 $\frac{1}{2}$	60	0 4 0	0 8 — 0 8

Foreign Silver.			Fine 3 pmts.	Pieces to the lb. Troy.	Weight of the Pieces 3 pmts. gr.	Sterling Value. s. d.	Value by others: s. d.			
Styvers	Three Styvers, or Fleece		10	10	108	0 2 5 $\frac{1}{3}$	0 6 $\frac{1}{4}$ +	0 6	Styvers of several sorts.	
	Flemish Six Styvers		10	0	54	0 4 10 $\frac{2}{3}$	1 0 +	1 0		
	The Bre 1499		10	4	156	0 1 12 $\frac{1}{3}$	0 4 +	0 4		
	The Key and Joane		10	4	156	0 1 12 $\frac{1}{3}$	0 4 +	0 4		
	Five Styvers of	{	Cambray	6	6 $\frac{1}{2}$	48	0 5 0	0 8 $\frac{1}{2}$ +		0 8
			Some	6	6 $\frac{1}{2}$	51	0 4 16 $\frac{1}{7}$	0 8 +		0 8
			Others	8	1 $\frac{1}{2}$	48	0 5 0	0 10 $\frac{3}{4}$ +		0 10
			Guelthers	8	1 $\frac{1}{2}$	48	0 5 0	0 10 $\frac{3}{4}$ +		0 10
			Horne, as Cambray	7	11	48	0 5 0	0 10 +		0 10
	{	Some	6	6 $\frac{1}{2}$	48	0 5 0	0 8 $\frac{1}{2}$ +	0 8		
		Others	6	6 $\frac{1}{2}$	51	0 4 16 $\frac{1}{7}$	0 8 +	0 8		
Others		6	6 $\frac{1}{2}$	51	0 4 16 $\frac{1}{7}$	0 8 +	0 8			
Baden, Chrysofome		10	10 $\frac{1}{2}$	39	0 6 3 $\frac{2}{3}$	1 5 $\frac{1}{2}$ +	1 5	Testons of divers sorts.		
Berne	{ Ottoman	11	5 $\frac{1}{6}$	45	0 5 8	1 4 +	1 4			
	{ Vincent	11	5 $\frac{1}{6}$	45	0 5 8	1 4 +	1 4			
Castile, as Berne.		11	5 $\frac{1}{6}$	45	0 5 8	1 4 +	1 4			
Ferrara, Hercules, and Alphonsus		10	7	42	0 5 17 $\frac{1}{7}$	1 4 +	1 4			
France, Francisus		10	7	42	0 5 17 $\frac{1}{7}$	1 4 +	1 4			
Friburg, Nicolas, as Berne		10	4 $\frac{1}{2}$	42	0 5 17 $\frac{1}{7}$	1 3 $\frac{3}{4}$ +	1 4			
Geneva		10	7	42	0 5 17 $\frac{1}{7}$	1 4 +	1 4			
Lorrain of 1524, and 1529		10	7	42	0 5 17 $\frac{1}{7}$	1 4 +	1 4			
{	as Berne	Lucerne, Episcopus	11	5 $\frac{1}{6}$	45	0 5 8	1 4 +		1 4	
		Mantua, Francis	10	4 $\frac{1}{2}$	42	0 5 17 $\frac{1}{7}$	1 3 $\frac{3}{4}$ +		1 4	
Millan	{	Galleacius, and	11	5 $\frac{1}{6}$	45	0 5 8	1 4 +	1 4		
		Lodovicus	11	5 $\frac{1}{6}$	45	0 5 8	1 4 +	1 4		
Montferat, George and Guill.		10	4 $\frac{1}{2}$	42	0 5 17 $\frac{1}{7}$	1 3 $\frac{3}{4}$ +	1 4			
Navarre	{	Henricus	10	7	42	0 5 17 $\frac{1}{7}$	1 4 +	1 4		
		Anna	11	5 $\frac{1}{6}$	45	0 5 8	1 4 +	1 4		
as Baden		11	5 $\frac{1}{6}$	45	0 5 8	1 4 +	1 4			
Portugal, Io. V. L.		10	7	42	0 5 17 $\frac{1}{7}$	1 4 +	1 4			
Savoy, Carolus	{	Some	11	5 $\frac{1}{6}$	45	0 5 8	1 4 +	1 4		
		Others	10	10 $\frac{1}{2}$	39	0 6 3 $\frac{2}{3}$	1 5 $\frac{1}{2}$ +	1 5		
Sedun, Nicol, dan, Adrian		11	5 $\frac{1}{6}$	45	0 5 8	1 4 +	1 4			
Solod, Ursus, as Berne		10	18	26 $\frac{1}{4}$	0 9 3 $\frac{2}{7}$	2 2 $\frac{3}{4}$ +	2 2			
Turones of France		4	10	138	0 1 17 $\frac{1}{2}$ $\frac{2}{3}$	0 2 +	0 2			
Vieryfers, Double		4	10	138	0 1 17 $\frac{1}{2}$ $\frac{2}{3}$	0 2 +	0 2			
Single accordingly										

The Coines that are all Brads used in Foreign Countries are many, and admit of several Subdivisions : But (as the Lawyers say, *de minimus non currat Lex*) they being so finall and inconsiderable, and few of them being currant in any other place than respectively where Coined, are not worth the remembrance here.

Nothing is further needful to finish this Chapter, then to shew how to set down or express any Geodetical Number, which to do, place the highest Denominate Number to the left hand, and all the rest in a straight line in order to the right hand, with a little line, or prick or two, or some such note of distinction between them; and over the head of every Number, or near the same, set the Character, Symbole, or Note, whereby it may be known of what nature or kind of Geodetical the same is, as to express Four Pounds Twelve Shillings and Three Pence, set them as at A. Ten Ounces Three Drains, Two Scruples, and Fifteen Graines, as at B.

l. s. d.			l. s. d.			l. s. d.			Examples.
A.	4	: 12 : 3	or thus	4—12—3	or thus	4 + 12 + 3			
3 3 3 gr.			3 3 3 gr.			3 3 3 gr.			
B.	10 .	3 . 2 . 15	or thus	10—3—2—15	or thus	10 + 3 + 2 + 15			

And if the Geodeticals be fracted in like manner after the Number is set down, place the Denomination at or near the head thereof, as 1/4 d. 2/3 s. 1/2 l. 3 1/2 Ton, &c which Numbers, though Without those Denominations should have been as Abstract Fractions; yet now are restrained by those Denominations. The first to be one Quarter or Fourth part of a Penny, which is a Farthing. The Second to be Two Thirds of a Shilling, which is 8 Pence; for one Shilling broken into Three parts or Groats, two of

of them will amount to so much. The third Fraction likewise now is Four Fifths of a Pound, which is in Value 16 s. for if one Pound or 20 s. be divided into 5 parts, and 4 of them parts be put together, it maketh 16 s. The Fourth Number is 3 Tons, and 17 Twentieth parts of one Ton, that is 17 Hundred, whence also is to be observed, That when an Integer and a Fraction is mixt, the Fraction is alwaies set to the right hand of the Integer, and is a part of parts of one of those Integers, let the *Geodætical* Denomination be what it will. So $4\frac{1}{4}l.$ the $\frac{1}{4}$ shall understand a quarter of 1 l. not a quarter of 4 l. and the like of all others.

English Geodæticals where to be understood.

Further also may be observed, to save the often repeating the words *Sterling*, or *English* in the following Examples; let the same be understood to those *Geodæticals* of Coin Weight, &c. though not expressed, where the Denominations or Notes do not express them to be Foreign *Geodæticals*.

CHAP. II.

Reduction of Geodæticals.

Simple Elements of Geodæticals. How agree and differ to Integers and Fractions.

THE Nature of *Geodæticals* with their Notes and Denominations declared in the præcedent Chapter, the rest of their Simple Elements are next to be spoken to. *Geodæticals*, as they partake of the Nature of Abstract and Contract Numbers, and arise from others; so their Numeration is both Original and Ortive; that in *Addition*, *Subtraction*, *Multiplication* and *Division*; and this in *Reduction*. And (as Fractions) have properly their Ortive Numeration, though but accidental, and for conveniency, fall first under consideration, before the more Essential, and Original Numeration. Yet different herein, that *Reduction* of *Fractions* declareth the proportion of one Number to another, or of broken parts to broken parts; but *Reduction* of *Geodæticals*, the Denominations of one Number, lesser or greater contained in another.

Reduction of what use and how called.

Reduction of *Geodæticals* bringeth Numbers of one Denomination to another, and to sheweth how to express one and the same Number in Value under different Names or Denominations. As 8 s. or $\frac{2}{5}$ of a Pound, which is alike valuable. For which reason *Reduction* is sometime called *Equation*. And sometime is useful to avoid Fractions, sometime to facilitate those Operations which without *Reduction* are tedious. The Number given to be reduced is called The *Reducend*. The several Denominations, are *Reducers*. And the Number obtained by *Reduction* is the *Result*.

Reducend Reducers Result what

The Sorts of Reduction.

Reduction of *Geodæticals* is either General, or Special.

General Reduction of *Geodæticals* is { Proper { Synthetical.
or
Analytical.
or
Proportional { Analytical.
and
Synthetical.

Proper of the First Sort included under 3 Cases.

Proper Synthetical Reduction serveth to reduce Subtiller or Smaller Denominations into Groffer or Greater, as Pence into Shillings or Pounds, &c. and is performed by *Division* under one of these 3 Cases.

I. Data Single. Quæsitæ Single Rule.

1. Case. When the Number to be reduced is Single-Integral, and the Denomination desired is single.

Then divide the Number to be reduced if it may be, by so many as one of the Greater do contain of the Lesser Denomination; If it cannot be divided, abbreviate it as a Fraction.

1. Example.

1. Example. To know how many Pounds *Sterling* are in 81600 Pence. I divide 81600 by 240 the Pence in one Pound, and the Quotient 340 l. is the Result.

Reducend	81600	(340 l. Result.
Reducer	240	0
	2	

2. Example,

2. *Example*, To know what parts of a Pound 8 s. or 8 d. are, because 8 will not be divided by 20 the Shillings in a Pound, nor yet by 240 the Pence in a Pound; they are both abbreviated as Fractions, and 8 s. at A. is seen to be $\frac{2}{5} l.$ and 8 d. at B. $\frac{1}{30} l.$

$$A. \frac{8}{20} \left| \frac{4}{10} \right| \frac{2}{5} l. \quad B. \frac{8}{240} \left| \frac{4}{120} \right| \frac{2}{60} \left| \frac{1}{30} l.$$

2. *Case*, When the *Reducend* is Integral and single, and the Denominations desired are plural.

Then divide the *Reducend*, by so many of the Denomination given, as make one of the next Greater desired, and then successively continue the Division of the Quotients resulting by so many as make one of the next Greater Denomination desired.

1. *Example*, To know how many Shillings and Pounds there are in the former Sum of 81600 d. First I divide 81600. by 12 the Pence in a Shilling, and the resulting Quotient is 6800 s. which divided by 20, the Shillings in 1 l. the Result is 340 l. as before.

$$\begin{array}{r} 81600 \text{ (6800 s. (340 l.} \\ 1222 \quad 2 \quad 0 \\ \hline \end{array}$$

2. *Example*, To know how far 285120 Barley-Cornes laid end to end will reach, dividing by 3 the Barley-Cornes that make 1 Inch, and by 12 the Inches in 1 Foot, and by 5 the Feet in 1 Pace, and by 1056 the Paces in 1 English Mile: The Result is $1 \frac{1}{2}$ Mile. And the several intermediate Quotients declare the respective Inches, Feet, and Paces therein.

$$\begin{array}{r} 285120 \text{ (95040 (7920 (528 (1056 or } 1 \frac{1}{2} \text{ Mile.} \\ 3333 \quad 1222 \quad 5555 \quad 1056 \\ \hline \end{array}$$

3. *Case*, When the *Reducend* hath some Fraction annexed thereto.

Then reduce the *Reducend* into an Improper Fraction, and divide the same after the manner of a Fraction, by so many of the given Denomination, as make one of the desired Denomination, Or else divide the Whole Number first, and add the Fraction to the Quotient.

Example. To know how many Shillings are in $196 \frac{1}{2} d.$ First I reduce $196 \frac{1}{2}$ into the Improper Fraction $\frac{393}{2}$, and then divide by 12, the Result is $16 \frac{3}{8} s.$ or dividing 196 by 12, the Result is 16 s. 4 d. to which the half-penny added makes it 16 s. $4 \frac{1}{2} d.$ all one with $16 \frac{3}{8} s.$

$$\text{Thus } \frac{393}{196 \frac{1}{2}} d. \quad \frac{12}{1} \left) \frac{393}{2} \left(\frac{131}{8} = 16 \frac{3}{8} s. \text{ or thus } \frac{7(4}{1222} (16 \cdot 4 \frac{1}{2} \right.$$

Proper Analytical Reduction, serveth to reduce Groffer or Greater Denominations into Subtiller, or Smaller, as Pounds into Shillings, or Pence, &c. and is performed by Multiplication under one of these 3 Cases.

1. *Case*, When the Number given to be reduced is of one Denomination and Integral, and the desired Denomination single

Then multiply the *Reducend* by so many as one of the Greater do contain of the Lesser Denomination.

Example, To know how many Pence are in 340 l. Sterling, I multiply 340 by 240, because so many Pence are contained in one Pound, and the Product 81600 is the Result.

$$\begin{array}{r} \text{Operation} \quad 340 l. \\ \quad \quad 240 d. \\ \hline \end{array}$$

$$\begin{array}{r} 13600 \\ 680 \\ \hline \end{array}$$

In 340 l. Sterl. 81600 pence

$$6 \overline{) 81600} 7$$

or thus

$$\begin{array}{r} 340 \text{ Reducend} \\ 240 \text{ Reducer} \\ \hline \end{array}$$

$$\begin{array}{r} 136 \\ 68 \\ \hline \end{array}$$

81600 Result.

2.
Data Plural.
Quæſita Plural
Rule.

2. *Caſe*, When the *Reducend* is Integral and of divers Denominations, or it is deſired to know how many of the ſeveral intermediate Denominations (if any ſuch be) are between the Denomination given, and that into which it is to be reduced.

Then multiply the Number to be reduced by ſo many as one of the next Leſſer Denominations to the *Reducend* containeth, and ſucceſſively the Product reſulting by ſo many as one of the next Leſſer Denomination to the Denomination of the Product doth contain. And if any odd Numbers belong to the reſpective Denominations add them to the reſpective Products after the manner of Integers.

1. *Example*.

1. *Example*, In the former Sum of 340 *l.* if it were deſired to know how many Shillings as well as Pence were contained therein. I multiply 340 *l.* by 20, the Shilling in 1 Pound, and the Product 6800 are the Shillings therein, which multiplied by 12, the Pence in 1 Shilling, produce 81600 *d.* as before. See C.

2. *Example*.

2. *Example*, If it be deſired to know how many Shillings, Pence, and Farthings there are in 355 *l.* 15 *s.* 4 *d.* 3 *q.* *Sterling*. After *Multiplication* by 20, the 15 *s.* are added, and after multiplication by 12, the 4 *d.* are added, and after multiplication by 4 the 3 *q.* are added as at D, and the Total Reſult is 341539 *q.*

$$\begin{array}{r}
 340 \text{ l.} \\
 20 \\
 \hline
 6800 \text{ s.} \\
 12 \\
 \hline
 13600 \\
 6800 \\
 \hline
 \text{C. } 81600 \text{ d.} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 355 \text{ l. } 15 \text{ s. } 4 \text{ d. } 3 \text{ q.} \\
 20 \\
 \hline
 7100 \\
 15 \text{ added} \\
 \hline
 7115 \text{ s.} \\
 12 \\
 \hline
 14230 \\
 71154 \text{ added} \\
 \hline
 85384 \text{ d.} \\
 4 \\
 \hline
 341536 \\
 3 \text{ added} \\
 \hline
 \text{D. } 341539 \text{ q.} \\
 \hline
 \end{array}$$

Examples in
ſeveral Deno-
minations.

In like manner any *Geodetical* of like Nature *English*, or Foreign may be reduced to a Leſſer Denomination, obſerving to multiply by the Number of Leſſer Denominations contained in the Greater, and adding the odd Numbers if any be.

Examples in Long Meaſure.

Long Meaſure.

How many Barley Cornes being laid end to end will reach from *Rye* to *London* being 60 Miles?

$$\begin{array}{r}
 \text{Thus } 60 \text{ Miles} \\
 8 \\
 \hline
 480 \text{ Furlongs} \\
 40 \\
 \hline
 19200 \text{ Perches.} \\
 16\frac{1}{2} \\
 \hline
 115200 \\
 19200 \\
 9600 \\
 \hline
 316800 \text{ Feet} \\
 12 \\
 \hline
 633600 \\
 316800 \\
 \hline
 321600 \text{ Inches.} \\
 3 \\
 \hline
 11404800 \text{ Barly Corns} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{or Thus, } 60 \text{ Miles} \\
 1056 \\
 \hline
 360 \\
 300 \\
 600 \\
 \hline
 63360 \text{ Paces} \\
 5 \\
 \hline
 316800 \text{ Feet.} \\
 12 \\
 \hline
 633600 \\
 316800 \\
 \hline
 3801600 \text{ Inches.} \\
 3 \\
 \hline
 11404800 \text{ Early Corns.} \\
 \hline
 \end{array}$$

Example

Example in Square Measure.

In 1423 Acres, 2 Roods, 30 Perches; How many Perches?

Square Measure

Thus 1423 Acres.

$$\begin{array}{r}
 4 \\
 \hline
 5693 \\
 2 \text{ added} \\
 \hline
 5694 \text{ Roods} \\
 40 \\
 \hline
 227760 \\
 30 \text{ added} \\
 \hline
 227790 \text{ Perches}
 \end{array}$$

or thus,

1423 Acres.

160 Perches in 1 Acre.

$$\begin{array}{r}
 85380 \\
 1423 \\
 110 \text{ added } 110 \text{ Perches,} \\
 \hline
 227790 \text{ Perches resulting.}
 \end{array}$$

Example in Weight.

C. qr. lb.

In 19—2—20 Avoirdupois Weight. How many Grains?

Avoirdupois Weight.

Thus

19 C.

4

76

2 added

78 qr.

28

624

156

20 added

2204 lb.

16

13224

2204

35264 3

8

282112 5

3

846336 9

20

16926720 gr.

or Thus

19 C.

112 Pounds in 1 C.

38

196 added 76 lb.

197

2204 lb.

7680 Graines in 1 lb.

176320

13224

15428

16926720 Graines resulting.

Example in Time.

In 40 Years, 12 Dayes, 10 Hours, How many Minutes?

Time.

40 Years.

305

200

240

12022

14522 Dayes

24

58488

29244

10

359938 Hours

60

21050280 Minutes.

Besides the 12 Daies in the *Reducend*, are added 10 more, (which is 22 in all) because every 4th. year being Leap-Year, and that year having 366 Daies in 40 Years is 10 Daies.

3. Case.

3.
Data mixt.
Rule.

3. Case, When the Reducend hath some Fraction annexed thereto.
Then Reduce the Reducend into an Improper Fraction, and multiply the same of
ter the manner of Fractions, by so many of the given Denominations as make one of
that desired.

Example.

Example, To know how many Pence there are in $16 \frac{1}{8} s$ I reduce them into the
Improper Fraction $\frac{131}{8}$, and the multiply by 12, and the Result is $196 \frac{1}{2} d$.

$$\frac{131}{16 \frac{1}{8} s.} \quad \frac{131}{8} \times \frac{12}{1} = \frac{393}{2} = 196 \frac{1}{2} d. \quad \frac{393}{2} \left(196 \frac{1}{2} \right)$$

Proof of Pro-
per Reduction.
Proportional
Reduction
belongs to
Proportions.

Proper Reduction of both sorts, Case by Case is alternately proved by the other; as
by the former Examples is evident without further instance.

Proportional Reduction, is both Analytical and Synthetical, using both Multiplication
and Division, and helpeth to reduce Denominations of Measure, Weight, Coine, &c.
of one Kind or Countrey to another, English to Foreign, and Foreign to English, and is
called Proportional, because it properly belongeth to the Doctrine of Proportions and
Rule of Three handled in the 4th. Book, where more fully thereof may be seen. but
because often in that Rule an Unit falls to be one of the Three Numbers, which
neither multiplying nor dividing becomes useles, and so such kind of Reductive
Questions become transient when the given Geodeticals contain in them some Part
Denomination or Value common to both, and may be translated hither under 3 Cases.

If an Unit, one
of the Data
fall under
3. Cases.

I.
Data, Single.
Rule.

1. Case, When the Geodetical given is Integral and Single.
Multiply the given Number by such common Parts or Value, as make one of the
Denominate Integers, and divide the Product by the common Parts or Value in the
other.

1. Example.

1. Example, In 300 Ells English, How many Yards?
Here because Ells contain 45 such Inches as Yards 36, or 5 Quarters, as Yards 4
I multiply the 300 by one, and divide the Product by the other, and find 375 Yards.

$$\begin{array}{r} \text{Thus} \quad 300 \\ \quad 45 \\ \hline 1500 \\ 1200 \\ \hline 13500 \end{array} \quad \begin{array}{r} \times \\ 278 \text{ yards} \\ 13500 (375 \\ 3666 \\ 33 \end{array} \quad \begin{array}{r} \text{or thus} \quad 300 \\ \quad 5 \\ \hline 1500 \end{array} \quad \begin{array}{r} 32 \\ 1500 \\ 4 \end{array} (375$$

1. Example.

2. Example, In 108 Pieces (or Ryals) of Eight, at 4 s. 4 d. per piece : How many
Crowns English at 5 s. per Piece ?

Here Pence being the Denomination common to both, the Product of 108 multiplied
by 4 s. 4 d. or 52 d. and divided by 5 s. or 60 d. giveth $93 \frac{1}{3}$ Crowns, the
Resolution, or 93 Crowns and 3 s. Sterling.

$$\begin{array}{r} \text{Thus,} \quad 108 \\ \quad 4 \frac{1}{3} \\ \hline 432 \\ 36 \\ \hline 468 \end{array} \quad \begin{array}{r} \times 3 \\ 468 \Delta \\ 5 \end{array} (93 \quad \begin{array}{r} \text{or thus} \quad 108 \\ \quad 52 \\ \hline 216 \\ 540 \\ \hline 5616 \end{array} \quad \begin{array}{r} 2(3 \\ 5616 \\ 60 \end{array} (93$$

2.
Data Plural.
Rule.

2. Case. When the given Geodetical is Integral and Plural.
Reduce all the Denominations into the lowest, and then divide or abbreviate
before.

1. Example

1. Example, To know what parts of a Pound 7 s. 6 d. or 12 s. 3 d. 2 q. are , 1. Example. These Numbers reduced, the first into Pence, and the second into Farthings, and abbreviated because they will not be divided: Declare the former $\frac{3}{8}$ l. at E. and the other $\frac{5}{9}$ l. at F.

7 s. 6 d.

12

E

14

76

90

90

124

9

3

3

8

l.

F.

12 s. 3 d. 2 q.

12

24

123

147

4

590

590

960

59

60

l.

2. Example. Suppose a Ropemaker marry his Daughter to a Sope-maker , and for her Portion give her Twenty Ropes, and on every Rope 20 Knots, and on every Knot 20 Purles, and in every Purle 20 Three Half-pence : How much Money had she for her Portion?

To resolve this Question, the number of Three Half Pence is first known by Multiplication, and then dividing that Product by 8 the Three Half Pence in one Shilling, and after by 20 the Shillings in one Pound ; or else by 160 the Three Half-Pence in one Pound : The Portion is found to be 1000 l.

20 Ropes

20

400 Knots

20

8000 Purles.

20

160000

3 Half-Pence

160000

8

2000

1000

10

l.

160000

16

10000

10

1000

l.

3. Case, When the Geodatical Reducend is a mixt Number, or a Fraction. 3. Data mixt, or Fractions. If a mixt Number reduce it into an Improper Fraction, and let the Numerator of the Fraction Proper or Improper be multiplied by the Parts that make one of the Denominate Integers, and divide that Product by the Denominator with the common parts desired if any be. And thus the value of any Fraction or Remainer upon a Division may be known.

1. Example, To know the value of $\frac{7}{10}$ l. The Numerator 7 being multiplied by the parts of a Pound, which are either 20 Shillings, 240 Pence, 960 Farthings, &c. according to that part multiplying, dividing the Product by 10, the Denominator; shall the Quotient of that Division be denominate.

7 l.

20 s.

10

7

140

140

10

14

s.

240 d.

7

1680

1680

12

168

12

14

s.

960 q.

7

6720

6720

48

672

48

14

s.

2. Example. To know how many Liures Tournois at 20 d. per Liure , are in 100 $\frac{1}{2}$ l. 2. Example. Sterling. The 100 $\frac{1}{2}$ l. reduced is $\frac{201}{2}$ l. which multiplied by 240 d. the parts of one Integer produce 48240, this divided by 2 the Denominator multiplied into 20 the common parts of the Liure desired, resolve the Question into 1206 Liures.

Thus $\frac{201}{100}$ l.

201

240

8040

402

48240

48240

2

48240

12

40

12

40

12

1206

12

l.

48240

20

48240

1

48240

1

48240

1

1206

1

l.

55

Proportional

Proof of Proportional Reduction.

Proportional Reduction of each sort is proved, by reversing the Question and Work, by making the *Divisors* and *Multipliers* in the one Question, the contrary in the other. As in the former Instance, If 300 Ells contain 375 yards, and they were given to know how many Ells were therein, Then multiply 375 yards by 4 (the former Divisor) and that Product 1500 divided by (the former Multiplier) 5, will return 300 Ells, and shew the Works right.

Special Reduction what and how done.

Special Reduction consists in some Select Rules more brief and commodious than the common way. As—

1.
Pence brought into Pounds, &c. at once.

1. To bring Pence into Pounds, Shillings, and Pence at one Work, divide the Sum to be reduced by 24, and from the Quotient cut off the right hand Figure, which is Primes, every Unite in Value 2s. and the Remainder of the Division is Pence.

Example.

Example, If 85390 Pence be divided by 24, I take 12d. for 1s. out of the 22 Pence remaining on the Division, and add to the 14s. or 7 Primes cut off from the Quotient, and with the 10d. left of the 22, I obtain the Total 355l. 15s. 10d.

$$\begin{array}{r}
 \times \text{ pence} \\
 \times 37(22 \text{ l. primes} \\
 85390(355 \text{ 7} \\
 24444 \text{ —————} \\
 222 \text{ 355:14} \\
 \hline
 1:10 \\
 355:15:10
 \end{array}$$

Common Way.

$$\begin{array}{r}
 (1 \\
 117 \\
 8539(0 \text{ d. } 71(15 \text{ s. } (355 \text{ l.} \\
 22222 \quad 2220 \\
 \times 111
 \end{array}$$

2.
Farthings brought into Pounds, &c. at once.
Example.

2. To bring Farthings into Pounds, Shillings and Pence, at one work, divide the Sum by 96, and from the Quotient cut off the right hand Figure as before, and for every 48 remaining add a Shilling, the rest are Farthings.

Example, If 417231 Farthings be divided by 96, the Quotient right hand Figure 6 is 12s. the 15 left on the Division is Farthings, or 3d. 3q. added make the Total 434l. 12s. 3d. 3q.

$$\begin{array}{r}
 45(1 \text{ q.} \\
 3349(5 \text{ l. primes} \\
 417231(+34'6 \\
 96666 \text{ —————} \\
 999 \text{ 334:12:} \\
 \hline
 3\frac{3}{4} \\
 434:12:3\frac{3}{4}
 \end{array}$$

Common Way.

$$\begin{array}{r}
 \times \\
 \times (3 \text{ q. } 212(3 \text{ d. } 1 \text{ s. } 1 \text{ l.} \\
 417231 \quad 104207(269(2(434 \\
 444444 \quad 12222 \quad 2220 \\
 111
 \end{array}$$

3.
Pounds brought into Shillings.
Example.

3. To reduce Pounds into Shillings, double the Number of Pounds, and to the right hand adjoyne a Cypher.

Example, If 30l. be brought into Shillings, 30 doubled is 60, and a Cypher adjoyned makes it 600s.

The Common way.

$$\begin{array}{r}
 30 \text{ l.} \\
 2 \\
 \hline
 600 \text{ s.}
 \end{array}$$

$$\begin{array}{r}
 30 \text{ l.} \\
 20 \\
 \hline
 600 \text{ s.}
 \end{array}$$

4.
Shillings brought into Pounds.
Example.

4. To reduce Shillings into Pounds, cut off the right hand figure, and take half the residue, as in the 13th. Section of *Division of Integers* was taught to divide by 20.

Example, If 8692s. be brought into Pounds, 2 cut off the half of 869, the rest is 434l. and 1 remaining makes 2 cut off to be 12s.

$$\begin{array}{r}
 (1 \text{ s.} \\
 869'2 \\
 \hline
 434 \text{ l.}
 \end{array}$$

Common way.

$$\begin{array}{r}
 (1 \\
 869(2(+34 \text{ l.} \\
 2220
 \end{array}$$

5. To

5. To reduce Shillings into Pence, double the Number given, and placing it one place nearer to the right hand, add it to the given Number.
Example, To bring 16 s. into Pence, the double of 16 is 32, placed and added accordingly, make 192 Pence.

5.
Shillings
brought into
Pence.
Example.

16 s.	Common way.
32	16 s.
<hr/>	12
192	<hr/>
	32
	16
	<hr/>
	192 d.
	<hr/>

6. To reduce Pounds into Primes or 2 s. adjoyn a Cypher to the right hand of the given Number.
Example, 45 l. brought to Primes is 450 Primes.

6.
Pounds into
Primes.
Example.

450 Primes.	Common way.
	45
	10
	<hr/>
	450 Primes.
	<hr/>

7. To reduce Shillings into Primes of Pounds, Take half the given Number.
Example, 346 Shillings reduced make 173 Primes.

7.
Shillings into
Primes.
Example.

$\frac{1}{2}$ 346 173 Primes	Common way.
	$\frac{346}{222}$ (173 Primes

8. To reduce any Number to another Denomination, that hath more Common Parts than one, any of those Common Parts may be taken and those are to be chosen, that make the work shortest.
Example, If one Dollar be worth 4 s. 8 d. How many Dollars are in 348 l. ? Here because Groats as well as Pence are common parts to 4 s. 8 d. and also to Pounds; I accept Groats before Pence, and working thereby find 1491 Dollars and 6 Groats over.

8.
Reduction by
any of the
Common Parts.
Example.

Common way.		
4 s. 8 d. 3 Groats <hr/>	348 l. 60 <hr/>	x 6228 Groats. 20880 (1491 Dollars. 14444 x11
12 2 <hr/>	20880 <hr/>	4 s. 8 d. 12 <hr/>
14 Groats. <hr/>		48 8 <hr/>
		56 <hr/>
		13920 6200 <hr/>
		83520 <hr/>

Special Reduction may be proved by the General, as in all the last 8 Examples is apparent, the Result by the Common Way is equal to the other.
Johnson adviseth to prove the Fractionary Operations in Geodæticals by finding the value of each Fraction, and compare them with their value after Reduction, which will be alike if the work be right.
1. Example, If $\frac{1}{2}$ l. and $\frac{3}{4}$ l. be reduced to one Denominator as Vulgar Abstract Fractions, they make $\frac{2}{4}$ and $\frac{3}{4}$, and because $\frac{1}{2}$ l. is 10 s. and so is $\frac{2}{4}$ l. and $\frac{3}{4}$ l. is 6 s. 8 d. and so is $\frac{3}{4}$ l. their Reduction appears right.

Proof of Special Reduction
Johnson's
Proof of Reduction of
Fractions.

$$\frac{1}{2} l. \frac{20}{2} (10 s. \quad \frac{3}{4} l. \frac{30}{4} (10 s. \quad \left| \quad \frac{1}{2} l. \frac{60}{2} (6 s. \quad \frac{2}{4} l. \frac{20}{4} (6 s. \right.$$

2. Example,

2. *Example*, If $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{5}{7}$ of $\frac{7}{9}$ of an old Harper, or Nine-pence be reduced, it will make $\frac{1}{3}$ or Three-pence; and so by finding the Value of the Right hand Fraction $\frac{1}{3}$ it is 7 Pence, of which $\frac{5}{7}$ taken is 6 d. and $\frac{2}{3}$ of that is 4 d. of which $\frac{1}{4}$ is 3 d. also.

C H A P. III.

Addition of Geodæticals.

Geodæticals
added.

1.
Integers of
one Denomina-
tion.
Rule.

1. *Example*.

2. *Example*.

Addition of Geodæticals may be considered according to the different nature of the Geodæticals proposed to be added.

1. *Case*. If the Numbers to be added be *Integral Geodæticals* alone, and of the same Denomination,

Then add the Numbers as *Abstract Integers* in Book 1. Part 1. Chap. 5. and to the Total adjoine or understand the Denomination given with the Numbers to be added.

1. *Example*, Two Flocks of Sheep, one of 910, and the other of 563 are to be added, the Total shall be 1473 Sheep.

2. *Example*, Three Men are indebted to another, one 340 l. the second 520 l. and the third 600 l. The Total of those Debts shall be 1460 l.

First Example.

$$\begin{array}{r} 910 \\ 563 \\ \hline \text{Total } 1473 \text{ Sheep.} \end{array}$$

Second Example.

$$\begin{array}{r} 340 \\ 520 \\ 600 \\ \hline \text{Total } 1460 \text{ l.} \end{array}$$

2.
Integers of se-
veral Denomi-
nations.
To place the
Data.

To add them.

Example.

2. *Case*. If the Addends be *Integral Geodæticals* greater and smaller of different Denominations,

Then place the greater Denomination alwaies to the Left hand, and in order to the Right hand the next Denominations, and also Units under Units, and Tens under Tens, &c. of every Denomination respectively, and Numbers of like Quality or Denomination under Numbers of the same Quality or Denomination. And when there are wanting some Denominations to fellow with others, some supply the places of such wanting Denominations with Cyphers.

Then begin with the right hand file, and smallest Denomination first, and adding all those smaller Numbers together, mark how many of the next Greater Denomination, you can take out of the Total of those smaller Numbers so reckoned up, and so many Units reserve in your mind, and the Overplus, if there be any, subscribe under the line and file where you are reckoning. And as in *Integers* and the former sort of Greater Geodæticals you reckoned the Article before, so reckon them you carry away now into the next Left hand Denomination. And this do if there be 2. 3. or more Denominations in the Numbers given to be added.

Example, If 4 Men were indebted to me, viz. A. 22 l. 13 s. B. 15 l. 10 s. 8 d. C. 10 l. 5 s. 2 d. and D. 3 l. 4 s. 7 d. or any other Sums, and I would know the Total that is owing by them all. Then I set them as at E. and beginning with the Pence, I say 7 and 2 is 9, and 8 is 17, which is 1 Shilling and 5 Pence over, wherefore I set down the remaining 5 Pence, and the one Shilling I carry to the Denomination of Shillings, and the Work will stand as at F. Then the one Shilling I carry and 4 is 5, and 5 is 10, and 3 is 13, and 10 is 23, and 10 is 33 Shillings, out of which I can take but one of the next greater Denomination, which is Pounds, and there resteth 13 s. which I set down as at G. Now the one Pound reserved I add to the next Denomination thus, saying 1 and 3 is 4, and 5 is 9, and 2 is 11, where this being the Greater Geodætical and highest Denomination of that kind, I reserve the Articles to the next file, and subscribe the digits as before in *Integers* and going forward find the Total to be 51 l. 13 s. 4 d.

Debits of

pricked.

Example.

and Foreign in this Case: The like Order may be observed as in the *Examples* ensuing.

Examples of

Weight.

Weight.

Measure.

Measure.

Time and Motion.

Time and Motion.

3.
Fractions or
Mixt.
Rule.

*Misc.
Rule.*

Examples,

Example. To add $\frac{1}{2} l.$ with $\frac{3}{5} l.$ the Total as at *I*, is $\frac{4}{5} l.$ by *Book 1. Part 2. Chap. 3.*
Case 1. And to add $\frac{1}{2} l.$ and $\frac{1}{3} l.$ together, the Total as at *K*. is $\frac{5}{6} l.$ by *Case 2.* there.
 And to add $40 \frac{3}{4} l.$ with $18 \frac{1}{2} l.$ the Total as at *L*. is $59 \frac{1}{4} l.$ by carrying an Unite from the Fractions to the Integers, and adding them after the manner of Integers in *Book 1. Part 1. Chap. 5.*

<i>I.</i>	<i>K.</i>	<i>L.</i>
<i>l.</i>		<i>l.</i>
$\frac{1}{5} \times \frac{3}{5} = \frac{4}{5}$	$\begin{array}{r} 5 \\ \hline 3 \quad 2 \\ \hline 1 \quad 1 \\ 2 \quad \times \quad 2 \\ \hline 6 \end{array} = \frac{5}{6}$	$\begin{array}{r} 40 \frac{3}{4} \\ 18 \frac{1}{2} \\ \hline 59 \frac{1}{4} \end{array}$

Addition of Geodeticals falling under the first Case, admits of like Proof with Addition of Integers. And those under the Third Case are to be proved, as the Addition of Integers and Fractions, according to the respective Operations made use of in the Addition : For if Integers be added, the Work is proved as Addition of Integers, if Fractions, as Addition of Fractions. *Addition of Geodeticals* under the second Case is to be proved by *Subtraction*, as is shewed in the next Chapter.

Addition of *English* Pounds, Shillings and Pence, in the Second Case in this Chapter above shewed : Some Schoolmasters teach to prove thus. Cast away all the Nines from the Pounds to be added, and what remains double, and bring to the Shillings, and cast away 9 also thence, and what remains treble, and bring to the Pence, and all the Nines being cast away there, note the last Remain. Then reject Nines in like manner from the Total, and if the last Remain here be like the former, they approve the Work.

As in the former Example where the Total was 51 l. 13 s. 5 d. casting away 9 from the Pounds to be added, there remains 5, which doubled is 10, and 9 cast therefrom, 1 remains, which reckoned to the Shillings, and 9 cast away as oft as can be there remains 6, this trebled is 18, from which twice Nine cast, there rests 0 to be brought to the Pence, and from thence all the Nines rejected there remains at least 8. then in the Total the Pounds make 6 Units, which doubled are 12, and 9 rejected leaves 3 to be reckoned to 4 in the Shillings, which 7 tripled make 21, from which 18, which is twice 9 rejected, there rests 3 to be added to 5 in the Pence, which all make 8 parallel to the former Remain.

But for the Reason above remembered in Addition of Integers, all the Proofs by casting away 9 is uncertain, and the true Proof of *Addition* is by *Subtraction*, as before taught in *Subtraction of Integers*, and needs no further repetition here.

Addition of Geodetical Fractions besides the Work above-mentioned may be proved by finding the Value of each Fraction before Addition, and adding them as *Geodetical Integers*, and comparing the Total with the Value of the Total of the added Fractions.

Example, If $\frac{1}{2} l.$ be added to $\frac{1}{3} l.$ the Total will be $\frac{5}{6} l.$ The Value of $\frac{1}{2} l.$ is 10s. and $\frac{1}{3} l.$ is 6s. 8d. which added together make the Total to be 16s. 8d. and so much is $\frac{5}{6} l.$

$$\frac{1}{2} l. \frac{20}{2} \left(10 s. \right) \quad \left| \quad \frac{1}{3} \frac{240}{3} \left(\frac{80}{12} \right) \left(6 s. 8 d. \right) \quad \begin{array}{r} s. \quad d. \\ 10 \quad 0 \\ 6 \quad 8 \\ \hline 16 \quad 8 \end{array} \quad \left| \quad \frac{5}{6} l. \frac{240}{5} \quad \frac{1200}{6} \left(\frac{200}{12} \right) \left(16 s. 8 d. \right) \quad \begin{array}{r} 5 \\ \hline 1200 \end{array}$$

Subtraction of Geodæticals.

IN *Subtraction of Geodeticals*, the different Nature of the proposed Numbers is to be considered.

1. Case.

1. *Case.* If the Numbers given be Integral *Geodeticals* only, and of the same kind or denomination,
Then subtract the Lesser Number from the Greater, as was taught in Abstract Integers, *Book 1. Part 1. Chap. 6.* and understand the remain to be of the same Denomination with the given Numbers.

1. *Integers of one Denomination. Rule.*

1. *Example,* If there were delivered to one 546 French Crowns to buy some Commodities with, and he did disburse but 354 of them; then will remain in his hands 192.
2. *Example,* If one deliver me to keep for him 1006 *Liures Tournois*, and receive again at several times 821, then will 185 rest in my hands.

1. *Ex. Fr. Δ.*

Delivered	546
Disbursed	354
Remaineth	192

2. *Ex. l. Tour.*

Delivered	1006
Received	821
Remaineth	185

2. *Case.* If the given Number be Integral *Geodeticals* of different Denominations, Greater or Smaller, English or Foreign,
Then place the Greater or highest Denomination of the Number from which Subtraction is to be made alwaies to the left hand, and the other Numbers in order to the right hand, and under the same the Subtrahend, so as Pounds may stand under Pounds, Shillings under Shillings, &c. likewise the Arithmetical Places of Units under Units, Tens under Tens, &c. are to be kept. And where any *Geodetical* Denomination is wanting, the same may be supplied with Cyphers to keep place.

2. *Integers of several Denominations. To place the Data.*

Then begin at the right hand and deduct the Lower Numbers or Figures of the Subtrahend and least *Geodetical*, out of the upper standing over them particularly, subscribing the Remain respectively under the same Files.

To Subtract them.

If the neather Figure happen to be greater than the upper, then in imagination borrow one of the Denomination next to the left hand, and add to the upper Number, which is too little, and make Subtraction from both, and subscribe the Remain as before; and for that borrowed accompt one back in the next File, reckoning the next Figure to be subtracted 1 more than it is, or the next Figure to be subtracted from 1 less than it is.

When borrowing needful.

Example, If *A.* lend to *B.* 344*l.* 10*s.* 6*d.* and *B.* paid *A.* 124*l.* 6*s.* 9*d.* How much is yet owing? I place the Numbers as at *C.* and because 9*d.* is greater than 6*d.* I borrow 1 Shilling, which is 12*d.* and put to 6*d.* and from the Total 18*d.* I take 9*d.* and there resteth 9*d.* then coming to the Shillings, I reckon that 1 that I borrowed and 16 is 17, which because I cannot take from 10, the Number over them I borrow 1 Pound, which is 20*s.* and put to the 10 makes 30*s.* from which 17 taken there remaineth 13*s.* to be subscribed, and that 1*l.* borrowed reckoned to the 4*l.* next maketh 5, which I proceed with as in *Subtraction of Integers*, and finish the rest of the work accordingly, and by the last Remain find yet due to *A.* 219*l.* 13*s.* 9*d.* as at *D.*

Example.

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Lent by <i>A.</i>	344	10	06
Paid by <i>B.</i>	124	16	09

C.

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Lent by <i>A.</i>	344	10	06
Paid by <i>B.</i>	124	16	09
Rest due to <i>A.</i>	219	13	09

D.

When many Numbers are given from which Subtraction is to be made, or many Numbers to be subtracted from one, or many: Let all the Plurals of each sort be severally added into one, as is taught in *Subtraction of Integers*, and then proceed in Subtraction as above.

When the Data are many.

In Subtraction of Weights, Measures, Time, Motion, and other *Geodeticals*, English and Foreign in this Case: Let the like order be observed, as in the ensuing Examples.

Examples of Weight, Measure, Time, and Motion.

Example,

Weight.					Measure.			Time.			Motion.			
Ton.	C.	qrs.	lb.	3.	Acres	Roods	Rods.	Years	Daies.	Ho.	Sig.	Deg.	Min.	Sec.
40	18	3	24	12	340	2	39	1648	330	19	11	20	40	31
29	19	3	12	14	121	3	36	345	360	20	3	19	50	33
18	19	0	11	14	218	3	03	1302	334	23	8	00	49	58

Fractions or
mixt.

3. Case. If the given Numbers be Fracted Geodeticals, Greater or Lesser, or Integral mixed with Fracted,

Then proceed in the Substraction with the Fractions as Fractions, and Integers as Integers, according to their respective Rules.

Examples.

Examples, To subtract $\frac{1}{5} l.$ from $\frac{7}{15} l.$ the Remain is $\frac{6}{15} l.$ as at E. by Book 1. Part 2. Chap. 4. Case 1. And to Subtract $\frac{1}{4} l.$ from $\frac{2}{5} l.$ the Remain will be $\frac{3}{20} l.$ as at F. by Case 2. there. And if $3\frac{1}{3} l.$ be taken from $13 l.$ there will be left $9\frac{2}{3} l.$ by borrowing an Unite which is 3 Thirds, to take the $\frac{1}{3}$ from, and for the same paying 1 to the 3, which is 4, and subtracting it from 13, after the manner of Integers above-mentioned, as at G.

$$\begin{array}{rcl}
 \text{E.} & & \text{F.} \\
 \frac{7}{15} l. - \frac{1}{15} l. = \frac{6}{15} \text{ or } \frac{2}{5} & \text{or} & \frac{2}{5} l. - \frac{1}{4} l. = \frac{3}{20} l. \\
 & & \underbrace{\frac{2}{5} l. - \frac{1}{4} l.}_{12} = \frac{3}{20} l.
 \end{array}$$

$$\begin{array}{rcl}
 \text{G.} & & \\
 \text{Lent} & \text{---} & 13 l. \\
 \text{Paid} & \text{---} & 3\frac{1}{3} \\
 \text{Rest} & \text{---} & 9\frac{2}{3}
 \end{array}$$

Proof of Geodetical Substraction of the First and Third Cases.

Substraction of Geodeticals falling under the First Case is proved like Substraction of Integers and Fractions, And the Substractions under the third Case, as the Substraction of Integers, according to the respective Operations used in the Substraction: For if Integers be taken from Integers, the Work is proved as Substraction of Integers, if Fractions from Fractions, as Substraction of Fractions, and generally Addition by Substraction, and Substraction by Addition.

Of the Second Case and reciprocally Addition thereby.

The Substraction of Geodeticals falling under the Second Case, admits of Geodetical Addition for Proof. For if the remain be added to the Number subtracted, the Total will be parallel to the Number from which Substraction is made when the Work is right. And consequently when from the Total of any Geodetical Addition of this sort, one or more of the Numbers added, be subtracted; the Remain of that Substraction will be equal to the Residue of those Numbers added into that Total; and thereby prove that Addition right.

Addition of this sort may also be proved by beginning at the left hand to subtract the respective Files from the Figure of the Total under them, and if any thing remain to underwrite, or remember to make allowance of the same, in the next right hand Figure of the Total, as was shewed before in the Proof of Substraction of Integers, Book 1. Part 1. Chap. 6. For further Evidence view the Examples of Addition at H. and Substraction at I.

Addition				Proof by Substraction.			
l.	s.	d.	Thus	or	Thus		
22	13	00	22—13—00	22—13—00	51—13—05		
15	10	08	15—10—08	15—10—08	29—00—05		
10	05	02	10—05—02	10—05—02	22—13—00		
03	04	07	03—04—07	03—04—07	29—00—05		
			51—13—05	51—13—05			
			xx—x—0	xx—x—0			

Substraction

Subtraction		
<i>l.</i>	<i>s.</i>	<i>d.</i>
51	13	05
Lent by <i>H.</i> to <i>I.</i> at several times.		
15	10	08
<i>I.</i> 10	05	02
03	04	07
Paid at times to <i>H.</i>		
29	00	05
Total of all the Payments to be subtracted.		
22	13	00
Remaineth due to <i>H.</i>		
51	13	05
Proof of the Subtraction.		

Subtraction of Geodætical Fractions; besides the Proof above mentioned may be proved by finding the Value of each Fraction before Subtraction, and subtract them as Integers, and compare the Remain with the Value of the Remain of the Subtracted Fraction.

Example, To take $\frac{1}{4} l.$ from $\frac{1}{3} l.$ leaves $\frac{1}{12} l.$ the Value of $\frac{1}{4} l.$ is 5 *s.* and $\frac{1}{3} l.$ is 6*s.* 8*d.* from which 5 *s.* subtracted leaves 1 *s.* 8 *d.* and so much is $\frac{1}{12} l.$

<i>l.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>
$\frac{1}{4} l. = \frac{20}{4} (5 s.)$	$\frac{1}{3} l. = \frac{240}{3} (\frac{80}{12} (6 s. 8 d.))$	6	8
		5	0
		1	8

C H A P. V.

Multiplication of Geodæticals.

TO multiply *Geodæticals*, consider what Case the proposed Numbers fall under and accordingly proceed in their *Multiplication*.

1. *Case*. If the Numbers given to be multiplied be Single *Geodæticals* both Integral, both Fracted, mixt Numbers, or the one an Integer, and the other a Fraction.

Then multiply the Integers as Integers, and the Fractions as Fractions, and the Mixt Numbers as mixt Numbers; of the first sort is to be seen, *Book 1. Part 1. Chap. 7.*

1. *Example*, If 30 Men stood in a straight line, every one 3 Feet before the other; and it were desired to know how many Feet there were between the Front and the Rear. The Answer is the Product at *A.* 90 Feet.

2. *Example*, If 40 Planks each of 10½ Feet long were laid end to end; and the Question were, how far they would reach, The Answer is the Product at *B.* 420 Feet.

3. *Example*, If a piece of Land contain in length 10¼ Yards, and in breadth 5⅓ Yards: And the Question were to know the Area, or Content of the same, or how many Square Yards the whole did contain. The Answer is the Product at *C.* 54⅔ Square Yards.

4. *Example*, If ½ *l.* be multiplied by ⅓ *l.* The Product is ⅙ *l.* as at *D.*

<i>A.</i>	<i>B.</i>	<i>C.</i>	<i>D.</i>
30 Men.	40 Planks.		
3 Feet.	10½ Feet.	Yards.	
90	400		
	20		
	420 Feet.		
		12	

$$\frac{41}{10\frac{1}{4}} \times \frac{16}{5\frac{1}{3}} = \frac{656}{12} \text{ or } 54\frac{2}{3}$$

Square.

$$\frac{1}{2} l. \times \frac{1}{6} l. = \frac{1}{12} l.$$

Weight.					Measure.			Time.			Motion.			
Ton.	C.	qrs.	lb.	3.	Acres	Roods	Rods.	Years	Daies.	Ho.	Sig.	Deg.	Min.	Sec.
40	18	3	24	12	340	2	39	1648	330	19	11	20	40	31
29	19	3	12	14	121	3	36	345	360	20	3	19	50	33
18	19	0	11	14	218	3	03	1302	334	23	8	00	49	58

Fractions or mixt.

3. Case. If the given Numbers be Fracted Geodeticals, Greater or Lesser, or Integral mixed with Fracted,

Then proceed in the Substraction with the Fractions as Fractions, and Integers as Integers, according to their respective Rules.

Examples.

Examples, To subtract $\frac{1}{15} l.$ from $\frac{7}{15} l.$ the Remain is $\frac{6}{15} l.$ as at E. by Book I. Part 2. Chap. 4. Case 1. And to Subtract $\frac{1}{4} l.$ from $\frac{2}{5} l.$ the Remain will be $\frac{1}{12} l.$ as at F. by Case 2. there. And if $3\frac{1}{3} l.$ be taken from $13 l.$ there will be left $9\frac{2}{3} l.$ by borrowing an Unite which is 3 Thirds, to take the $\frac{1}{3}$ from, and for the same paying 1 to the 3, which is 4, and subtracting it from 13, after the manner of Integers above-mentioned, as at G.

$$\begin{array}{c}
 \text{E.} \qquad \qquad \qquad \text{F.} \qquad \qquad \qquad \text{G.} \\
 \begin{array}{l}
 l. \quad l. \\
 \frac{7}{15} - \frac{1}{15} = \frac{6}{15} \text{ or } \frac{2}{5}
 \end{array}
 \begin{array}{l}
 l. \quad \frac{2}{5} \\
 \frac{2}{5} - \frac{1}{4} = \frac{1}{12}
 \end{array}
 \begin{array}{l}
 \text{Lent} - 13 l. \\
 \text{Paid} - 3\frac{1}{3} \\
 \text{Rest} - 9\frac{2}{3}
 \end{array}
 \end{array}$$

Proof of Geodetical Substraction of the First and Third Cases.

Substraction of Geodeticals falling under the First Case is proved like Substraction of Integers and Fractions, And the Substractions under the third Case, as the Substraction of Integers, according to the respective Operations used in the Substraction: For if Integers be taken from Integers, the Work is proved as Substraction of Integers, if Fractions from Fractions, as Substraction of Fractions, and generally Addition by Substraction, and Substraction by Addition.

Of the Second Case and reciprocally Addition thereby.

The Substraction of Geodeticals falling under the Second Case, admits of Geodetical Addition for Proof. For if the remain be added to the Number subtracted, the Total will be parallel to the Number from which Substraction is made when the Work is right. And consequently when from the Total of any Geodetical Addition of this sort, one or more of the Numbers added, be subtracted; the Remain of that Substraction will be equal to the Residue of those Numbers added into that Total; and thereby prove that Addition right.

Addition of this sort may also be proved by beginning at the left hand to subtract the respective Files from the Figure of the Total under them, and if any thing remain to underwrite, or remember to make allowance of the same, in the next right hand Figure of the Total, as was shewed before in the Proof of Substraction of Integers, Book I. Part 1. Chap. 6. For further Evidence view the Examples of Addition at H. and Substraction at I.

<i>Addition</i>				<i>Proof by Substraction.</i>			
<i>H.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>	Thus	or	Thus	
	22	13	00	22—13—00	22—13—00	51	13—05
	15	10	08	15—10—08	<hr/> 15—10—08	29	00—05
	10	05	02	10—05—02	10—05—02	<hr/> 22	13—00
	03	04	07	03—04—07	03—04—07	<hr/> 29	00—05
	<hr/> 51	13	05	<hr/> 51—13—05	<hr/> 29—00—05	<hr/>	
				<hr/> xx—x—0	<hr/>		

Substraction

Subtraction

<i>l. s. d.</i>			
51	13	05	Lent by <i>H.</i> to <i>I.</i> at several times.
<hr/>			
15	10	08	} Paid at times to <i>H.</i>
<i>I.</i> 10	05	02	
03	04	07	
<hr/>			
29	00	05	Total of all the Payments to be subtracted.
<hr/>			
22	13	00	Remaineth due to <i>H.</i>
<hr/>			
51	13	05	Proof of the <i>Subtraction</i> .
<hr/>			

Subtraction of Geodætical Fractions; besides the Proof above mentioned may be proved by finding the Value of each Fraction before Subtraction, and subtract them as Integers, and compare the Remain with the Value of the Remain of the Subtracted Fraction.

Example, To take $\frac{1}{4} l.$ from $\frac{1}{3} l.$ leaves $\frac{1}{12} l.$ the Value of $\frac{1}{4} l.$ is 5 s. and $\frac{1}{3} l.$ is 6 s. 8 d. from which 5 s. subtracted leaves 1 s. 8 d. and so much is $\frac{1}{12} l.$

<i>l.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>
$\frac{1}{4} \frac{20}{4} (5 s.$	$\frac{1}{3} \frac{240}{3} (80$	6	8
		5	0
		1	8

C H A P. V.

Multiplication of Geodæticals.

TO multiply *Geodæticals*, consider what Case the proposed Numbers fall under and accordingly proceed in their *Multiplication*.

1. *Case*. If the Numbers given to be multiplied be Single *Geodæticals* both Integral, both Fracted, mixt Numbers, or the one an Integer, and the other a Fraction.

Then multiply the Integers as Integers, and the Fractions as Fractions, and the Mixt Numbers as mixt Numbers; of the first sort is to be seen, *Book 1. Part 1. Chap. 7.*

1. *Example*, If 30 Men stood in a straight line, every one 3 Feet before the other; and it were desired to know how many Feet there were between the Front and the Rear. The Answer is the Product at *A.* 90 Feet.

2. *Example*, If 40 Planks each of $10\frac{1}{2}$ Feet long were laid end to end; and the Question were, how far they would reach, The Answer is the Product at *B.* 420 Feet.

3. *Example*, If a piece of Land contain in length $10\frac{1}{4}$ Yards, and in breadth $5\frac{1}{3}$ Yards: And the Question were to know the Area, or Content of the same, or how many Square Yards the whole did contain. The Answer is the Product at *C.* $54\frac{2}{3}$ Square Yards.

4. *Example*, If $\frac{1}{2} l.$ be multiplied by $\frac{1}{6} l.$ The Product is $\frac{1}{12} l.$ as at *D.*

<i>A.</i>	<i>B.</i>	<i>C.</i>	<i>D.</i>
30 Men.	40 Planks.	656	1
3 Feet.	$10\frac{1}{2}$ Feet.	Yards.	
<hr/>	<hr/>		<hr/>
90	400	$\frac{656}{12}$ or $54\frac{2}{3}$	$\frac{1}{2} l. \times \frac{1}{6} l. = \frac{1}{12} l.$
<hr/>	20	Square.	<hr/>
	420 Feet.		12

2.
One of the
Data Single,
and the other
Plural.

2. *Case.* If one of the given Numbers be a Plural *Geodetical*, and the other a Single. Then reduce the Plural Number into the lowest Denomination, and multiply the Result by the other, or if the Single *Geodetical* be a Digit or other small Number; you may easily multiply every of the Denominate Numbers severally by that Number, and carry in imagination or otherwise to the next greater Denomination, so many as the respective Products will afford, subscribing the Remaines.

Example.

Example. If a Merchant pay to 20 Porters 1 l. 6 s. 8 d. a piece, and it were desired to know how much Money in all was paid. First 1 l. 6 s. 8 d. reduced into Pence is 320, which multiplied by 20, produceth 6400, and reduced back into Shillings and Pounds giveth 26 l. 13 s. 4 d. as at E, or otherwise 20 Eight-pences which is 40 Groats, or 13 s. 4 d. and the 13 s. carried to the place of Shillings leaveth the 4 d. to be subscribed, then 20 times 6 s. is 6 times 20 s. or 6 l. to be carried to the Denomination of Pounds, and the 13 s. left to be subscribed. So at last the 6 l. added to the 20 times 1 l. or 20 l. makes 26 l. 13 s. 4 d. as at F.

1 l. 6 s. 8.	E.	Thus, or Thus, or Thus,	
20		l. s. d. l. F. l. s. d.	
20		1—6—8 1—6—8	
6		20 20	
	4(4 d. 6400 xz (53 3 s.		
26		26—13—4 20	13—4 Product of 20 by 8d.
12	320 pence	6—0 6—0	Product of 20 by 6s.
20 Porters		20	Product of 20 by 1 l.
52		26—13—4	
268	6400		
320			

3.
Data Plural,
and the Pro-
duct Simple.
Rule.
Example.

3. *Case.* If both the given Numbers be Plural *Geodeticals*, and the Product required Simple, and of the lowest Denomination, Then reduce the Numbers into their lowest Denominations, and multiply the Results as Integers. *Example.* If 20 l. 10 s. 3 d. be multiplied by 5 l. 4 s. 2 d. both being reduced make 4923 d. and 1250 d. which multiplied produce 6153750 d. and they if occasion be may be reduced into Shillings and Pounds, as at G.

l. s. d.	l. s. d.	
20—10—3	5—4—2	4923
20	20	1250
400	100	246150
10	4	9846
410	104	4923
12	12	6153750
820	208	
4108	1042	
4923	1250	

4.
Data Plural
and Product
Compound.
Variety 1.
Example.

4. *Case.* If both the Given Numbers be Plural *Geodeticals*, and the Product required Compound, Then first after Reduction of the Numbers and Multiplication of the Results as before, divide the Product by the Product of every Lesser Denomination contained in one Greater multiplied together. *Example.* In the Numbers last Exemplary, because 240 Pence are contained in 1 Pound, and Pence are the Lesser Denominations in both the given Numbers, the Product of 240 multiplied by 240, which is 57600 dividing the 6153750 the Product above obtained, the Quotient will be 106 l. and if the Value of the Fraction remaining be gotten and added thereto; the whole *Geodetical* will be 106 l. 16 s. 8 $\frac{1}{2}$ d. as at H.

l. 240	(4	4815	(41	414		360	180	90	5
240	38(81	20	3874	12	360	180	90	5	
9500	51537(50(106L.		3874	828	4388(8 d.	576	288	144	8
480	57666 0/0	96300	57666 0/0	414	576				
	577		57						
57600	5			4968					

Or Secondly, Under the Reduced Numbers, Place the Number of Lesser Denominations contained in one Greater; and multiply as in Fractions.

Example, Under the before-reduced Numbers 4923 and 1250, placing 240, and multiplying them as Fractions, the alike Product will arise as followeth at A.

205125	1	(13	(120
1641 125	12/60	119/8	1656(8 d.
4923 1250	20512/5(106L.	3210/0(16 s.	192
	19222 0	1922 0	
	199	19	
	1		
240 240		138	120 60 30 15 5
24 80		12	192 96 48 24 8
1920	1605	276	
	20	138	
	32100	1656	

Or Thirdly, If the Geodeticals be of one Nature, turn all the Lesser Denominations into Fractionary Parts of the Greater, and multiply as in the 5th. Case of Multiplication of Fractions, Book 1. Part 2. Chap. 5. or multiplying Number by Number carry out of each Multiplication so many of the next Greater Denomination as are therein, and Subscribe the Remain.

Example, Because the Geodeticals aforesaid are both one Nature, viz. Sterling Money, the 10s. 3d. and 4s. 2d. turned into Fractions, and after Multiplication of the Integers 20 by 5, proceeding to add the parts of the Multiplicand thereto: The like Product appears at K.

20l. 10s. 3d. or $20\frac{1}{2}$ and $\frac{1}{80} \times 5$ l. 4s. 2d. or $5\frac{1}{5}$ and $\frac{1}{20}$ produce 106l. 16s. 8d. $\frac{5}{8}$.

l. s. d.		l.		l. s. d.
Thus 20—10—03		or Thus 20 $\frac{1}{2}$ and 5 $\frac{1}{5}$		
5—4—2				
100	Product of 5 by 20.	100	5 by 20 is 100—	100—00—00
2—10	Five Angels.	2 $\frac{1}{2}$	5 by $\frac{1}{2}$ or half pounds—	3—10—00
1—3	Five Three pences.	0 $\frac{1}{80}$	5 by 8 $\frac{1}{80}$ abbreviated is—	0—01—03
4—00—00	Fifth part of 20 l.	4	$\frac{1}{5}$ of 20 l. which is—	4—00—00
2	Fifth part of 10 s.	0 $\frac{1}{10}$	$\frac{1}{5}$ of $\frac{1}{2}$ l. is $\frac{1}{10}$ l. or—	0—02—00
00 $\frac{3}{4}$	Fifth part of 3 d.	0 $\frac{1}{40}$	$\frac{1}{5}$ of $\frac{1}{80}$ l. or reduced, is—	0—00—03
3—04	The 120 part of 20l.	0 $\frac{1}{10}$	$\frac{1}{5}$ of 20 l. abbreviated is—	0—03—04
01	The 120 part of 10s	0 $\frac{1}{20}$	$\frac{1}{5}$ of $\frac{1}{2}$ l. is $\frac{1}{10}$ l. or—	0—00—01
00 $\frac{1}{4}$	The 120 part of 3d.	0 $\frac{1}{40}$	$\frac{1}{5}$ of $\frac{1}{80}$ l. reduced is—	0—00—04
106—15—08 $\frac{5}{8}$	The Total Product.	l. 106 $\frac{1}{2}$ $\frac{5}{8}$		106—16—8 $\frac{5}{8}$

Or, Fourthly, Multiply Number by Number, beginning at the left hand, and to find the Denomination of the several Products observe the first line shall be denominate, as the Multiplicand, the first left hand Figures of the other Lines of Production shall be denominate, as the Figures of the Multiplicand under which they stand, and all the other Figures in each line respectively are parts of one of that denomination, and are Numerators, under which for Denominator you may place or imagine placed as followeth, that is to say, under the second placed Figures to the right hand, the number of the next lesser Denomination to the greater of the Multiplicand, and under the next right hand Figures the number of the next lesser Denomination, and so accordingly under all the Numbers And to add these several Lines of Production into one Total, first begin at the left hand, and subscribe the left hand Figures under a line in the place of the greater Denominations as they stand, and then collect all the next right hand Collumne Integers together, and subscribe them as in Addition of Integers, and so do with the Integers

Integers in the other Columnes, and then all the Fractions belonging to each line of Production severally, and add them together; and what of the other Denominations are contained in them, place in their Order, and if any thing remain, reduce it to a part of the lowest Denomination of the Total. And lastly add all these subscribed Numbers into one Total.

Example.

The former Numbers thus multiplied produce for the First Line 100. 50. 15. for the Second 80. 40. 12. and for the Third 40. 20. 6. as at *L. M. N.* then for 40 in the second Line, because Shillings is the next lesser Denomination to Pounds, 20 shall be the Denominator, and the Denominator to 12 shall be 240; the Pence in 1 *l.* and so shall they be $\frac{20}{240}$ *s.* and $\frac{12}{240}$ *s.* And the like in the third line $\frac{20}{240}$ *d.* and $\frac{6}{240}$ *d.* Then collecting them as before directed for the $\frac{20}{240}$ *s.* is set 2 *s.* and $\frac{12}{240}$ is reduced to $\frac{1}{2}$ of a Penny by Abbreviation, and multiplying the Numerator by 12, and so proceeding, the other Fractions make 1 $\frac{1}{4}$ *d.* and the Numbers stand as at *O.* And for plainer Demonstration another Example is added at *P.* where 2 *l.* 10 *s.* 3 *d.* 2 *q.* is multiplied by 2 *l.* 5 *s.* 2 *d.* 2 *q.*

Denominations	<i>l.</i>	<i>s.</i>	<i>d.</i>	
Multiplicand	20	10	3	<i>O</i>
Multiplier	5	4	2	
<hr/>				
Lines of Production	<i>L.</i>	100	50	15
	<i>M.</i>		80	40
	<i>N.</i>			40
			20	240
				20
				240
<hr/>				
Collection of		100		
	50 <i>s.</i> & 80 <i>s.</i>		6	10
	15 <i>d.</i> & 40 <i>d.</i>		4	7
	$\frac{20}{240}$ <i>s.</i> & $\frac{12}{240}$ <i>s.</i>		2	$0\frac{1}{2}$
	$\frac{20}{240}$ <i>d.</i> & $\frac{6}{240}$ <i>d.</i>			$1\frac{1}{4}$
Total Product		106	16	$8\frac{1}{8}$

<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>q.</i>	
2	10	3	2	<i>P.</i>
2	5	2	2	
<hr/>				
4	20	6	4	
	10	50	15	10
	20	240	960	
	4	20	6	4
		20	240	960
		4	20	6
			20	240
				960
<hr/>				
4				
1	10			
	10			
	2			
	2	6	$3\frac{1}{2}$	
		1	$0\frac{7}{60}$	
			$12\frac{7}{40}$	
<hr/>				
5	13	8	$0\frac{3}{48}$	

Proof of Geodetical Multiplication.

Multiplication of Geodeticals is proved, if the Numbers be Integral, as Multiplication of Integers, if Fracted, as Multiplication of Fractions, that is in both by Division, as is further noted in the next Chapter.

Proof of Multiplying Fractions by the Value.

Moreover the Multiplications of Geodetical Fractions may be examined, by finding the Value of the Fractions to be multiplied, and the Product thereof, and comparing the same with the Product of their Values, multiplied as Integers; only observing to divide this Product by the Number of one of the Lesser Denominations contained in the Greater.

Example, If $\frac{2}{3}$ *l.* be multiplied by $\frac{3}{4}$ *l.* the Product will be $\frac{1}{2}$ *l.* And $\frac{2}{3}$ *l.* being 13 *s.* 4 *d.* or 160 Pence, and $\frac{3}{4}$ *l.* being 15 *s.* or 180 Pence, these multiplied produce 28800, which divided by 240, the Pence or Lesser Denomination contained in one Pound the Greater Denomination, the Quotient will be 120 Pence, or $\frac{1}{2}$ *l.* as before.

$\frac{2}{3}$ <i>l.</i> = 13 <i>s.</i> 4 <i>d.</i> or 160 <i>d.</i>	
$\frac{3}{4}$ <i>l.</i> = 15 <i>s.</i> or 180 <i>d.</i>	
$\frac{1}{2}$ <i>l.</i> or 10 <i>s.</i>	
28800 (120 (10 <i>s.</i>	
244 0 12	
2	
28800	

Proof of Multiplication in the Third and Fourth Cases.

And so also the Operations of multiplying Geodeticals in the Third and Fourth Cases of this Chapter may be compared the one with the other, the Lesser Denominations being in that Fourth Case considered as Fractions, and dividing for their Value accordingly.

ly. Because though in multiplying Integers the Multiplicand is increased in Figures and Value so many times as the Multiplier containeth Units; yet in Fractions, and that sort of Multiplying *Geodæticals*, though the Figures may be increased so many times, yet in Value the New Fraction or Product is made so much less. For a Fraction being properly less than one, and making another Number so many times less also; Must needs produce a Number so much less, as the multiplying Fraction containeth parts in it, and the Product is but the Value of a Fraction of a Fraction.

As in the last Instance, If $\frac{1}{4} l.$ be taken for the Multiplier, then $\frac{1}{4}$ of 13 s. 4 d. or $\frac{1}{4} l.$ shall be 10 s. And so is $\frac{2}{3}$ of 15 s. which is $\frac{2}{3} l.$ If $\frac{2}{3}$ be taken for the Multiplier, the Product $\frac{1}{2} l.$ or 10 s. is less than $\frac{2}{3} l.$ or 13 s. 4 d. by $\frac{1}{4}$ thereof, which is 3 s. 4 d. and less than $\frac{1}{4} l.$ or 15 s. by $\frac{1}{3}$ thereof, which is 5 s. But in Integers if they be multiplyed under the Denomination of Shillings, the Product is 20 times 10 s. or 200 s. if in Pence 28800, as before, which is 240 times as much. And this was the Reason for that Division before to equalize their Value. Nevertheless this is to be understood of Proper Fractions and Lesser *Geodæticals*, for Improper Fractions are redundant, and increase by their Units they contain after the manner of Integers.

CHAP. VI.

Division of Geodæticals.

TO Divide *Geodæticals*, observe the Case the Numbers proposed fall under, and proceed accordingly in their Division. Geodæticals divided.

1. *Case*, The Numbers given to be divided being Single *Geodæticals*, both Integral, both Fracted, or Mixt Numbers, or one an Integer or Mixt Number, and the other a Fraction. I. Single.

Then divide the Integers as Integers, and the Fractions as Fractions, and the Mixt Numbers as Mixt Numbers, Of the first sort, See Book 1. Part 1. Chap. 8. and of the other, Book 1. Part 2. Chap. 6. Rule.

1. *Example*, If 41984 l. be divided equally among 164 Men. and it be desired to know how much each should have for his Part, The Answer is 256 l. the Quotient at A. Example 1.

2. *Example*, If a Lane be 11880 Feet long, and I would know, how many Rods that is, I divide them by $16\frac{1}{2}$ the Feet in a Rod. And the Answer is 720 Rods, the Quotient at B. Example 2.

3. *Example*, If a Field be $54\frac{2}{3}$ Square Yards, and 1 side thereof be $10\frac{1}{4}$ Yards, and the other side be demanded, The Quotient answereth at C. $5\frac{1}{3}$ Yards. Example 3.

4. *Example*, If $\frac{1}{4}$ be divided by $\frac{1}{2} l.$ The Quotient is $\frac{1}{2} l.$ as at D. Example 4.

<p>A.</p> $\begin{array}{r} 9 \\ 91 \\ 41984 \overline{) 256 l.} \\ 36444 \\ \hline 166 \\ 1 \end{array}$	<p>B.</p> $16\frac{1}{2} = \frac{33}{2} \overline{) 11880} \left(720 \right. \text{ Rods}$	<p>C.</p> $10\frac{1}{4} = \frac{41}{4} \overline{) 54\frac{2}{3}} = \frac{164}{3} \left(\frac{16}{3} = 5\frac{1}{3} \right. \text{ Yards.}$	<p>D.</p> $\frac{1}{2} \overline{) \frac{1}{4}} \left(\frac{1}{2} l. \right.$
---	---	---	--

2. *Case*, One of the given Numbers being a Plural *Geodætical*, and the other a Single,

Then reduce the Plural into the Lowest Denomination, and divide the Result by the other. Or if the Single *Geodætical* be a Digit, or other small Number you may easie enough divide every one of the Denominate Numbers severally, carrying in imagination from the Greater Denomination to the next Lesser, the Number remaining on Division of the Greater, if any be, reduced and added to the Lesser, and make the Division from the Total. 2. One of the Data Single, and the other Plural. Rule.

Example, If 15 l. 13 s. 1 d. be divided equally among 13 Men, to know how much each Man had. First, 15 l. 13 s. 1 d. reduced make 3757, and divided by 13, and after by 12 and 20, gives 1 l. 4 s. 1 d. to every Man, as at E. or otherwise dividing 15 l. by 13, gives 1 l. in the Quotient, and 2 l. left, which 2 l. or 40 s. being reduced brought and added imaginary to the 13 s. makes 53 s. out of which 13 may be Example.

be had 4 times to be set in the Quotient, and 1 s. will be left, which being 12 d. and added to the 1 d. makes 13, out of which the Divisor may be had once, and the Quotient as at F.

l.	s.	d.	E.	F.
15	13	1		
20				
300				
13				
313				
12				
626				
3131				
3757				

3.
Data Plural,
and the Quo-
tient Simple.
Rule.
Example.

3. Case. Both the given Numbers being Plural *Geodeticals*, and the Quotient desired Simple, and of the lowest Denomination,

Then reduce the Numbers into their Lowest Denominations, and divide the Results as Integers.

Example, If 25640 l. 12 s. 6 d. be divided by 5 l. 4 s. 2 d. both being reduced into Pence make 6153750, and 1250, which divided yield in the Quotient 4923 d. which if occasion be, may be reduced into Shillings and Pounds, as at G.

l.	s.	d.	l.	s.	d.
25640	12	6	5	4	2
20			20		
512800			100		
12			4		
512812			104		
12			12		
1025624			208		
5128126			1042		
6153750			1250		

4.
Data Plural,
and the Quo-
tient Compound.
Rule b. Re-
duction.
Example.

4. Case, Both the given Numbers being Plural *Geodeticals*, and the Quotient required Compound,

Then after Reduction of the Numbers as above, if the Dividend be the Reduction of the Simple Product, Multiply the Divisor thus reduced by the Number of the Lesser Denomination contained in one Greater, and by this Product divide the Dividend.

Example, If 25640 l. 12 s. 6 d. were to be divided by 5 l. 4 s. 2 d. their Reductions are as before 6153750, and 1250, and 1250 the Divisor multiplied by 240, the Pence in one Pound, produce for the New Divisor 300000, by which Division being made, the Quotient is 20 l. and if the value of the remaining Fraction be gotten and added thereto, the whole *Geodetical* will be as at H. 20 l. 10 s. 3 d. as before.

l.	s.	d.	H.	l.	s.	d.
1250			6153750 (20 l.	30	75	00 (10 s.
240			3 00000	3	00	00 300 (3 d.
50000						
2500			15375		75	
			20		12	
300000			307500		150	
					75	
					900	

Rule without
Reduction.

But Secondly, If the Product given to be divided were compound without Reduction. Then first reduce the given Numbers into their least Terms, and divide them as Integers or Fractions, as the Case happens.

Example.

If 106 l. 16 s. 8 d. be divided by 5 l. 4 s. 2 d. both reduced make $\frac{22111}{8}$ and $\frac{125}{8}$ and after Division the Quotient is $\frac{1681}{8}$, or 20 l. 10 s. 3 d. is found at I. as before.

Example,

Example whereof may be seen, *Book 1. Part 1. Chap. 8. Page 36.* and by comparing the *Multiplications* in the former Chapter with the Divisions in this will be sufficiently evident without further Example.

Last sort of Division proved by Addition.

In particular the last sort of *Division* at *M.* may be proved by *Addition*, for by adding all the *Multiplees* substracted in the *Division*, the *Total* with the *Remain*, when any is, will return the *Dividend*.

	l.	s.	d.		
Products	{	104	03	04	} Multiplees
		2	12	01	
		1	03	$\frac{1}{8}$	
Total		106	16	$08\frac{1}{8}$	Dividend

Proof of Division of Fractions by the Value.

Furthermore *Division of Geodetical Fractions* may be tryed by finding the Value of the *Fractions* to be divided, and the *Quotient*, and comparing the same with the *Quotient* of their Values divided as *Integers*, the *Dividend* being first multiplied by the *Number* of one of the *Lesser Denominations* contained in the *Greater*.

Example, If $\frac{3}{4}l.$ be divided by $\frac{1}{2}l.$ the *Quotient* shall be $\frac{3}{2}l.$ and $\frac{1}{2}l.$ the *Dividend* being 10 s. or 120 d. multiplied by 240 the Pence, or *Lesser Denomination* contained in 1 l. the *Greater Denomination*, make 28800, which divided by 180 d. the Value of $\frac{1}{2}l.$ gives in the *Quotient* 160 d. which is 13 s. 4 d. or $\frac{3}{2}l.$

$\frac{3}{4}l. = 15s. \text{ or } 180d.$	
$\frac{3}{4} \bigg) \frac{1}{2} \left(\frac{2}{3} \text{ or } 13s. 4d. \right)$	$\frac{1}{2}l. = 10s. \text{ or } 120d.$
$\begin{array}{r} 240 \\ 4800 \\ 240 \\ \hline 28800 \end{array}$	$\begin{array}{r} d. \\ 10 \quad 4(4s. \\ 28800(160(13 \\ 2880 \quad 222 \\ \times \quad \times \end{array}$

In Fractions a Greater Number may divide a Lesser.

Hereby it is evident that in *Fractions* a Greater Number may divide a Lesser, though in *Integers* it cannot, and that when *Division* is said to make a Sum less in Numeration, though the *Quotient* may be of greater Denomination than the *Dividend* was: It is to be understood of *Integers*, and not of *Fractions* that are Proper *Fractions*, Seeing in this Example $\frac{1}{4}$ of a Pound the Greater Number both in Figures and Value, can divide $\frac{1}{2}$ of a Pound the Lesser Number both in Figures and Value, and bring forth a *Quotient* bigger than the *Dividend*. For 13 s. 4 d. is greater than 10 s. by $\frac{1}{3}$ thereof, and yet divided by $\frac{1}{4}$ or 15 s. which is also greater than 10 s. by $\frac{1}{2}$ thereof.

But in *Integers* and *Improper Fractions* the Case is otherwise; for in the former alwayes the Numbers will be lessened, and in the latter sometime lessened, and sometime increased, according as the Greater *Improper Fraction* is the *Divisor* or the *Dividend*, which without Example is obvious enough in every Operation.

Partis primæ Libri Secundi

F I N I S.

T H E

THE
SECOND PART
OF THE
SECOND BOOK

CHAP. I.
Of FIGURAL S.

THE next kind of Numbers to be viewed are called *Figural Numbers*, because they either do or may represent some Geometrical Figure, and are ever considered in relation to those Formes, and from thence borrowing their particular Names or Denominations, are rightly placed among Numbers generally Contract.

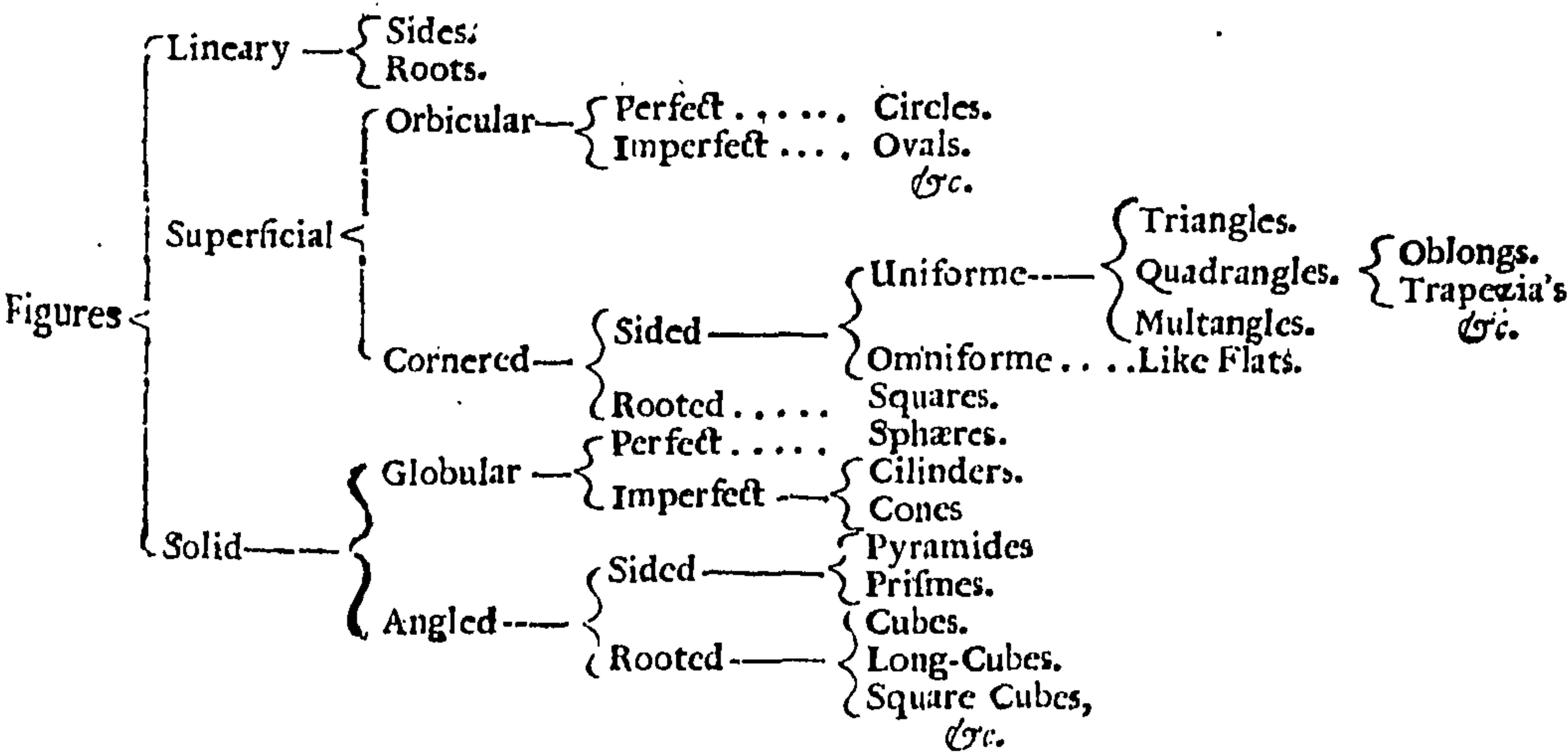
Next sort of Numbers generally contrary, are *Figurals*, why so called.

The knowledge of *Figures*, their various Formes, and how to make and measure them, belongs to *Geometry*, and is there to be sought, but the Numbers contained in the Figures, and how to Order, Increase, and Diminish them, belong to *Arithmetick*, and those of special use therein, to be found here.

What of them belongs to Geometry, and what to Arithmetick. *Figural Numbers* what.

These Numbers are those understood by the Name of *Figural Numbers*, and are as various, as the *Figures* in *Geometry*, from whence they take their Names.

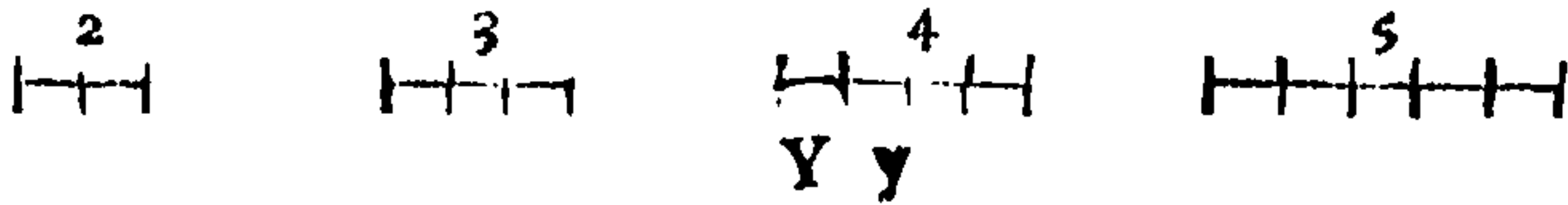
Geometrical Figures of Chief Note and most conveniently fitted to serve our turn may be thus aspected.



A Table of some Geometrical Figures.

Accordingly *Figural Numbers* are of Three sorts, viz. *Lineary*, *Superficial*, and *Solid*. *Lineary*, have comparison and relation to length only, and are therefore so called, because Unite is imagined to stand by Unit along in a Line, as in the following Figures of 2. 3. 4. 5. which of all others is to be understood.

Figurals of 3 sorts. 1. *Lineary* what.



Demonstrated. This

This Name, although it be properly referred to such Numbers as will make no other form duely; yet it may also be applyed to any Number Abstract, because all such Numbers may be taken as the sides of other Figural Numbers.

Side of a Number, what and how called.
Root of a Number what, how called, and why.

The Side (in Latine, *Latus*) is the Length of the Line containing the Side of the Figure: And if the Figure be equal-sided, or æquilateral, then is the side called a Root or *Radix* (after the Latine) by a Metaphor, because from thence other Figural Numbers arise as Branches from the Root of a Vegetable: Yet are not all equal Sides Roots, unless by *Multiplication* they can produce their Figures.

Superficial, what and how called.
Sorts of Superficial Numbers.
Oval, whence the Name.

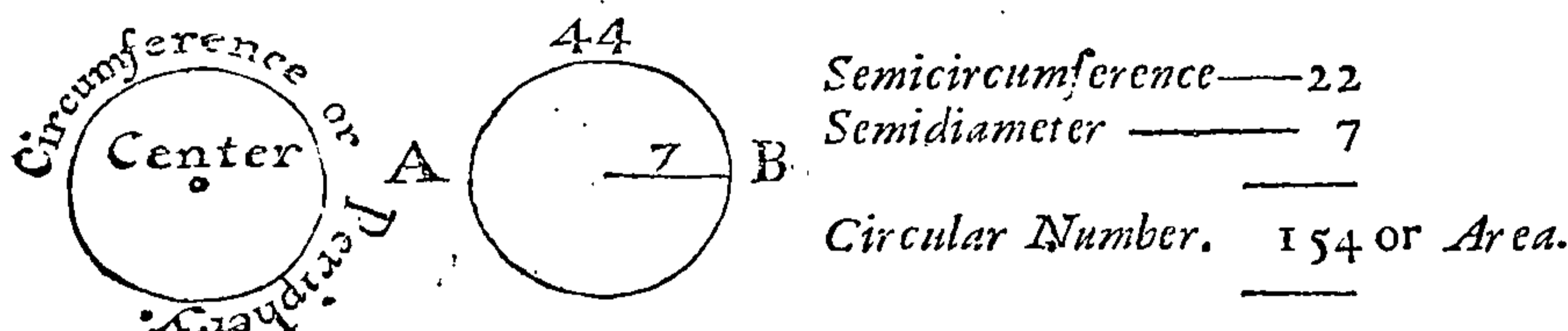
Superficial Numbers, called also *Flat*, or *Plain Numbers*, and sometime *Surfaces* are considered according to such Form as they make in *Multiplication* or *Progression*, and have both Length and Breadth.

Of *Superficies*, Some are Orbicular, and of them one sort only perfectly round, as Circles, and others imperfectly round as Ovals, so called from *Ovum*, Latine for an Egg, because they bear the resemblance thereof. Others are partly round, and partly of other Formes. The Figural Numbers of all which Figures I shall say nothing to, the Circle excepted, and of that but sparingly.

Circle what, Circumference, Periphery what, Center what. Semidiameter what. Diameter what. Circular Number what. Area of a Circle.

A Circle is a plain Figure, determined with one bowed Line called the Circumference or Periphery, in whose midst is a Point named the Center, From which all right Lines drawn to the Circumference are equal, the Circumference being alwaies æquidistant from the Center, as appeareth at A. These Lines are called *Semidiameters*. And if the Line be drawn through the Center, and have both its Extremes ending in the Circumference, this is a *Diameter*, and the Longest Line a Circle is capable to contain.

A *Circular Number* is the Superficial Content, or after the Latine the *Area* of a Circle found by multiplying the *Semidiameter* into the *Semicircumference*, as at B.



Numbers called Circular on another account.

Some Numbers sometime are called Circular, because as a Circle turns to the Point whence it began; so they being multiplyed by themselves, end in themselves; as 5 and 6, for 5 times 5 is 25, and 6 times 6 is 36, but here Circular Numbers are not taken in that Sence, but to be understood as before described.

Other Sorts of Superficies.

Other *Superficies* are Cornered or Angled, of which some are æquilateral, others inæquilateral; but none Rooted save the Square. The rest not Rooted of divers sorts, if considered *per se* simply are of one Form, if *inter se*, comparatively are of divers Formes, and therefore called *like Flats*, or *Flats* that are alike.

Uniforme.

The *Uniforme superficies* not Rooted are distinguished according to the Number of *Angles* therein: If 3 then called a *Triangle*, if 4 a *Quadrangle*, if more a *Multangle* or *Polygone*.

Triangle, what

A *Triangle* is a Figure comprehended of 3 Lines, and containeth as many *Angles*, or *Corners*.

Sorts of Triangles.

Triangles are Plain or Sphærical, according as the Lines are straight or bowed, whereof they are made; and are named both from their *Sides*, and from their *Angles*.

Æquilateral. Isoscheles.

If the 3 Sides be equal, the *Triangle* is called *Æquilateral*.

Schalenum.

If but 2 of the Sides be equal, it is named an *Isoscheles*.

Right Angled.

If all 3 Sides be unequal, it is called a *Schalenum*.

Broad Angled.

If it have one *Right Angle*, that is containing 90 Degrees of a Circle, then it is called a *Right Angled Triangle*, or *Rectangled Triangle*, and after the Greek an *Orthogon*. If it have an *Angle* greater than a *Right Angle*, it is called an *Obtuse* or *Broad Angled Triangle*, or an *Ambligonium*.

Sharp Angled.

If it have all the *Angles* less than a *Right Angle*, it is called an *Acute*, or *Sharp Angled Triangle*, or an *Oxigonium*.

And accordingly by the mixture of such *Sides* and *Angles* are the *Triangles* known asunder.

Leggs of a Triangle how called.

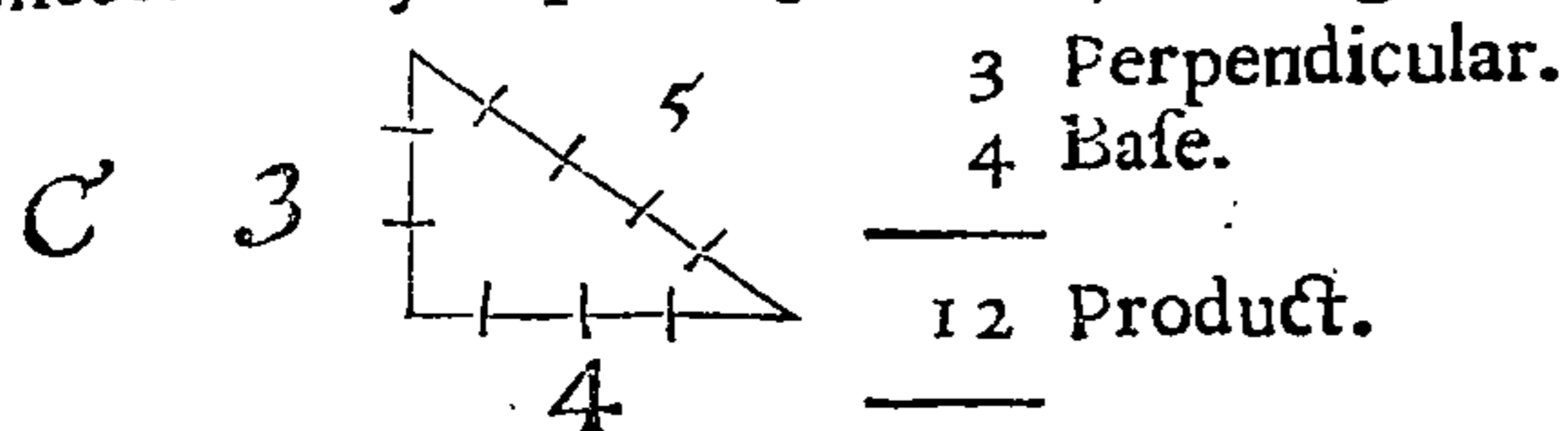
Every of the 3 Lines or Sides sometimes called the Leggs, and in Latine, *Crura*, have their distinct Names, as the *Perpendicular*, the *Base*, and the *Hypotenusa*.

Triangular Number what.

A *Triangular Number* is the *Area* of a *Triangle*, but because there is no certain Number from which, as from a Root such *Area* may be found, but differs according to the

Sides

Sides and Angles of every *Triangle*; it will not be meet to digress further than only to view the Plain Right Angled Triangle at C, and the rather, because some of the En-
fuing Discourse may depend upon the knowledge thereof.



Demonstration.

The Genus of *Quadrangles* include Squares as well as other Four-corned Figures, but because they are Rooted Figures, they take their Place by themselves hereafter. The other Species of *Quadrangled Figures* are an *Oblong*, a *Trapezium*, a *Rhombus*, and a *Rhomboid*.

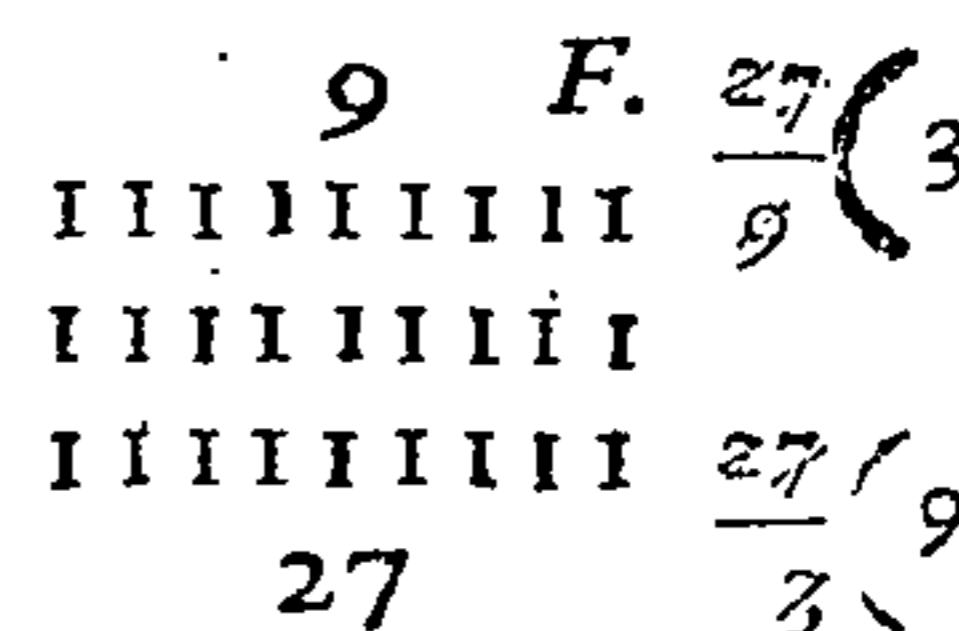
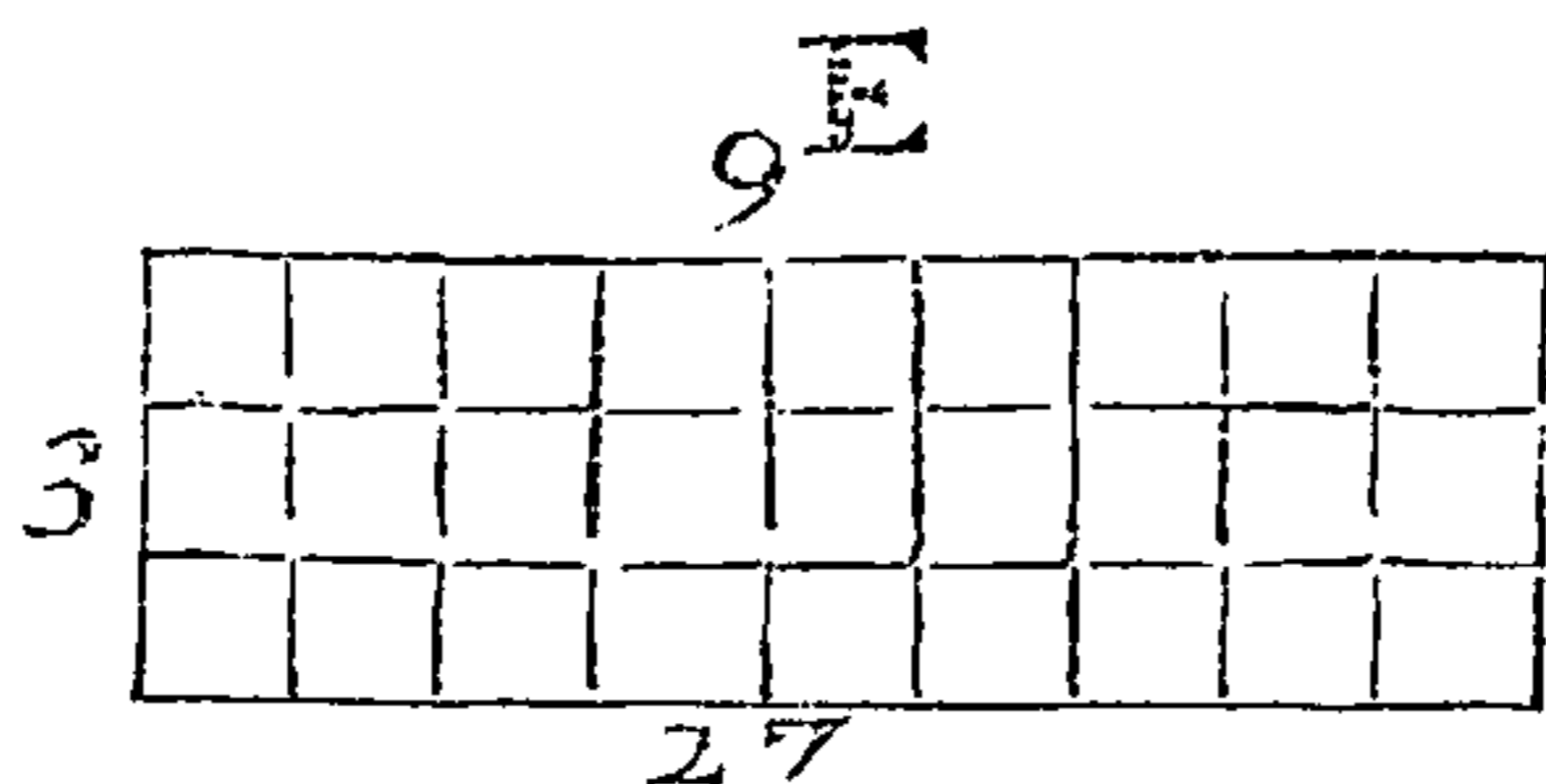
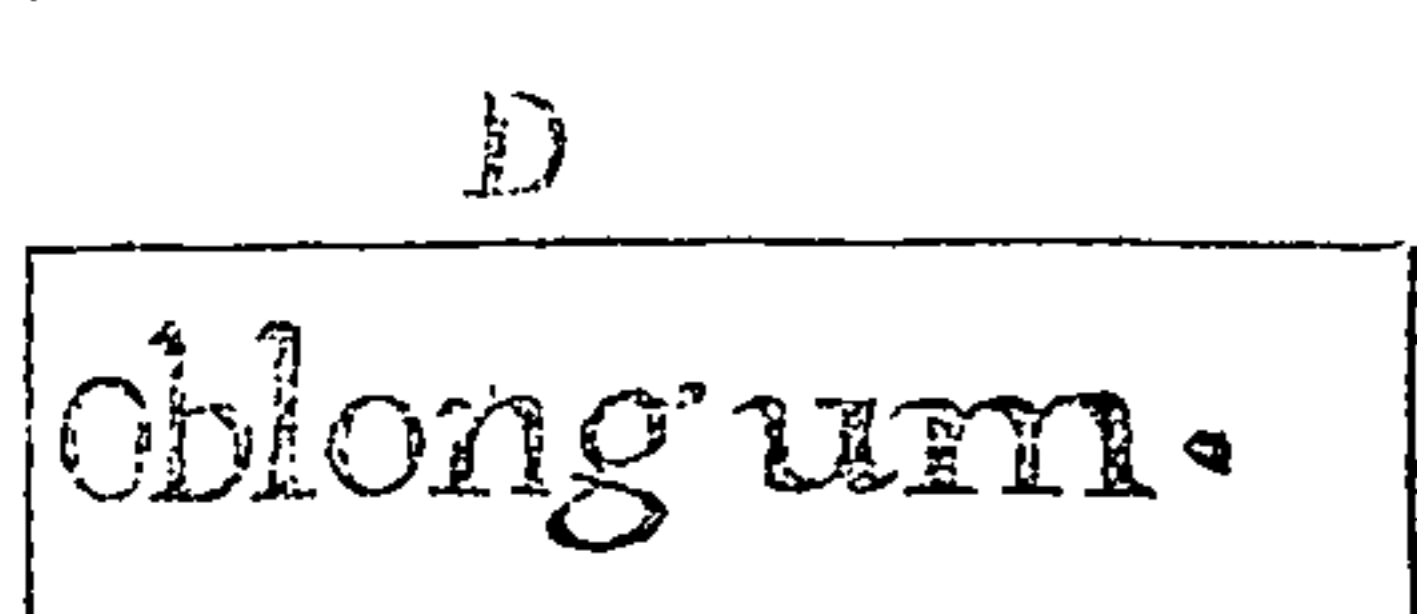
Quadrangles what, and their kinds.

An *Oblong*, called also a *Long Square*, and sometime a *Rectangle Figure* is an *Inequilateral Parallelogram* consisting of 4 *Right Angles*, but only the opposite sides equal, as the Figure at D.

Oblong what.

An *Oblong Number* is the *Area* of such Figure, which divided by the Lesser side bringeth the Greater in the Quotient, and if by Greater bringeth forth the Lesser, as 27 coming of 9 and 3, if divided by 3, giveth 9; if by 9 yieldeth 3 in the Quotient represented at E. or F.

Oblong Number what.



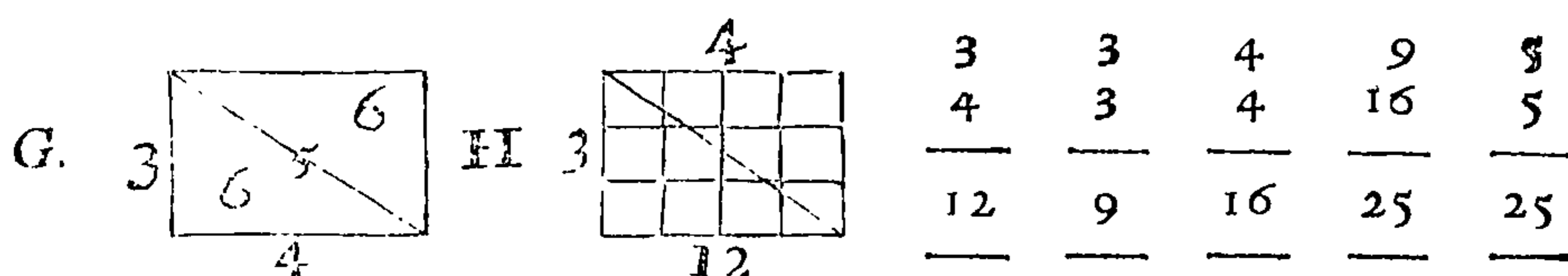
Demonstration.

If a Right Line be drawn through the Center of this Figure from one Opposite Angle to the other; this Figure will be divided into Two Right Angled Triangles, as the Figure at G. sheweth. This *Diagonal Line* is called by some a *Diameter*, and from thence the Name of *Diametral Numbers* came.

Diagonal, by some called Diameter, and thence the Name of Diametral Numbers.

A *Diametral Number* serveth to find out the length of this *Diagonal Line*, and hath two parts of that Nature, that if they be multiplied together, will make the said *Diametral Number*, and the Squares of these two parts added together will make a *Square*, whose *Root* is the Length of the *Diagonal* or *Diagonal Line* to that *Diametral Number*, as 12 is called a *Diametral Number*, because it hath Two parts, viz. 3, and 4, which produce it by *Multiplication*, and the Square of 3 is 9, and of 4 is 16, which 9 and 16 make together 25, whose *Square Root* is 5, the length of the *Diagonal Line* to the *Platform* 12, and of the *Hypotenusa* of each *Triangle* as at G. or H.

Demonstration.



A *Trapezium* hath all Four Sides unequal.

A *Rhombus* hath all four sides equal, but never a *Right Angle*.

A *Rhomboid* hath the Opposite sides equal, but never a *Right Angle*.

Trapezium, what. Rhombus what. Rhomboid, what.

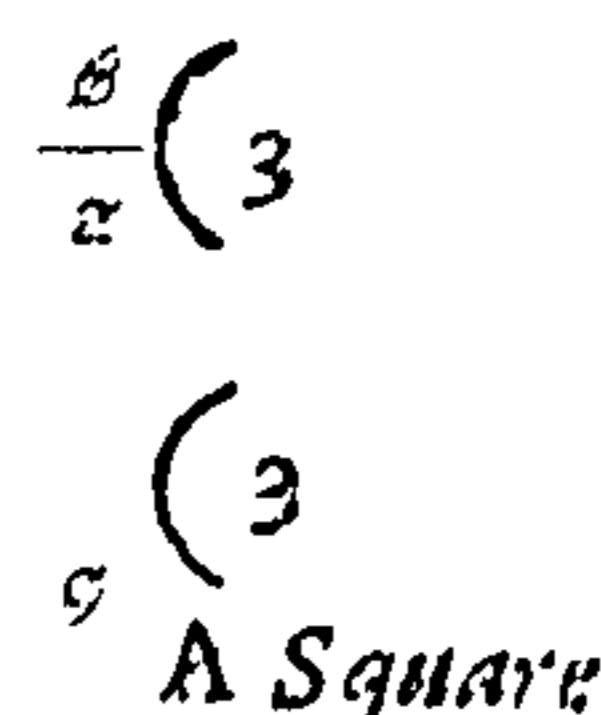
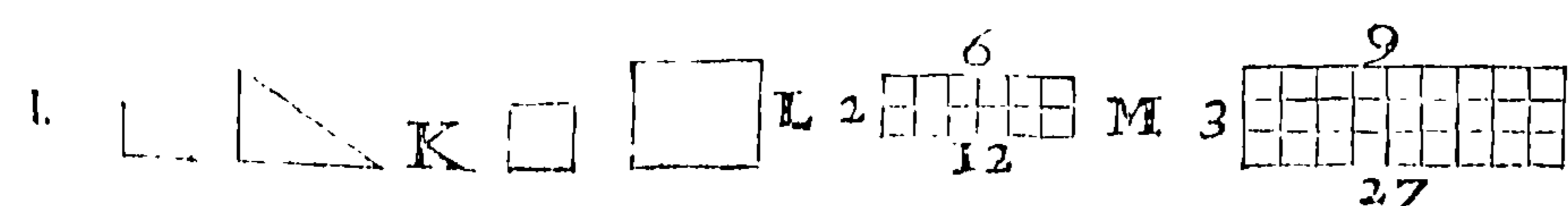
These and all *Multangles* being uncertain in the Measure of their Sides, and so consequently in their Figural Numbers are set aside here.

Those *Special Figures* called *Like Flats*, whether *Triangular*, *Quadrangular*, or otherwise, are such Plain *Homogeneous Superficies*, as bear a certain Proportion in their Sides unto each other, when compared together, as the Figures at I. and K. declare.

Omni-forme Superficies how called.

And Numbers called *Like Flats*, have the Sides of one bear like proportion to the Sides of another Platform of the same kind. As the Long Squares L. and M. If the Sides be 2 and 6 of the one, and 3 and 9 of the other, the Figures are *Like Flats*. And so the Numbers that express their Quantities, which are 12 and 27, are called *Like Flats*, because 6 is Triple to 2, as 9 is to 3.

Numbers Like Flats, what.



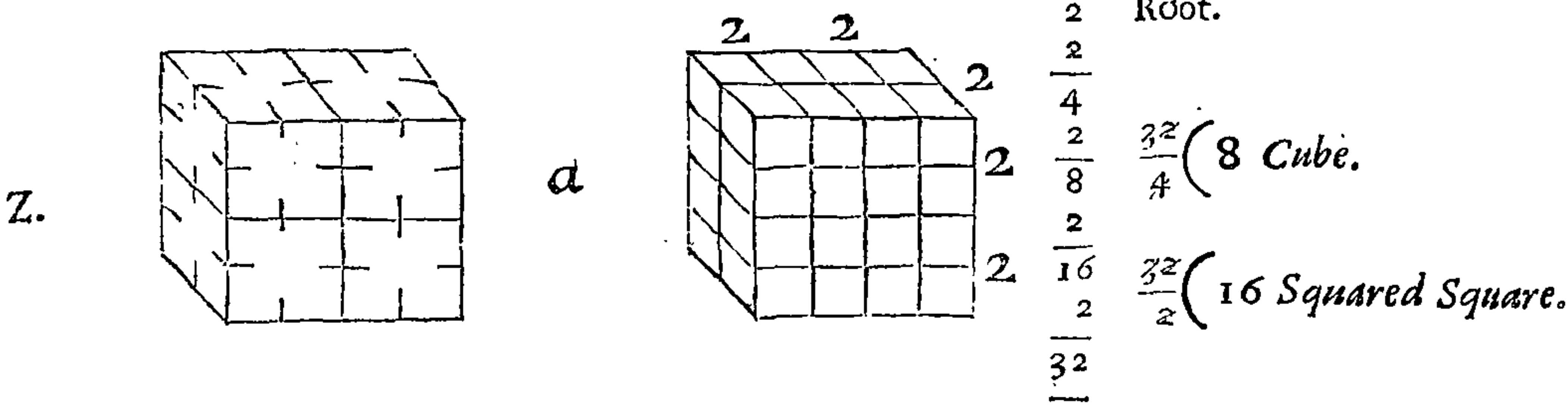
Demonstration.

A *Squared Cube* (the next in order) hath Length and Depth alike, but wants equal Breadth, yet so as every *Unite* of his Breadth begets a *Cube* both in Length and Depth, as the Figures at Z, or a from the aforesaid Root 2 make evident.

Squared Cube or Surfolide what.

A *Squared Cube Number*, in *Arithmetical Termes* is called a *Surdefolide*, or *Surfolide*, (perhaps as *Solid* upon *Solid*, *Sur* in *English* implying as much as *Super* in *Latine*) and contains the *Solid Quantity* of the Figure, being begotten by multiplying the *Square* into the *Cube*, or the *Squared Square* by the *Root*. And by *Reciprocal Division* returns them in the *Quotient*, as 32 is the *Surfolide* to the *Root* 2. If divided by 4 gives 8 the *Cube*, and by 2 brings 16 the *Squared Square* of 2 in the *Quotient*.

The Number thereof why called Surfolide.



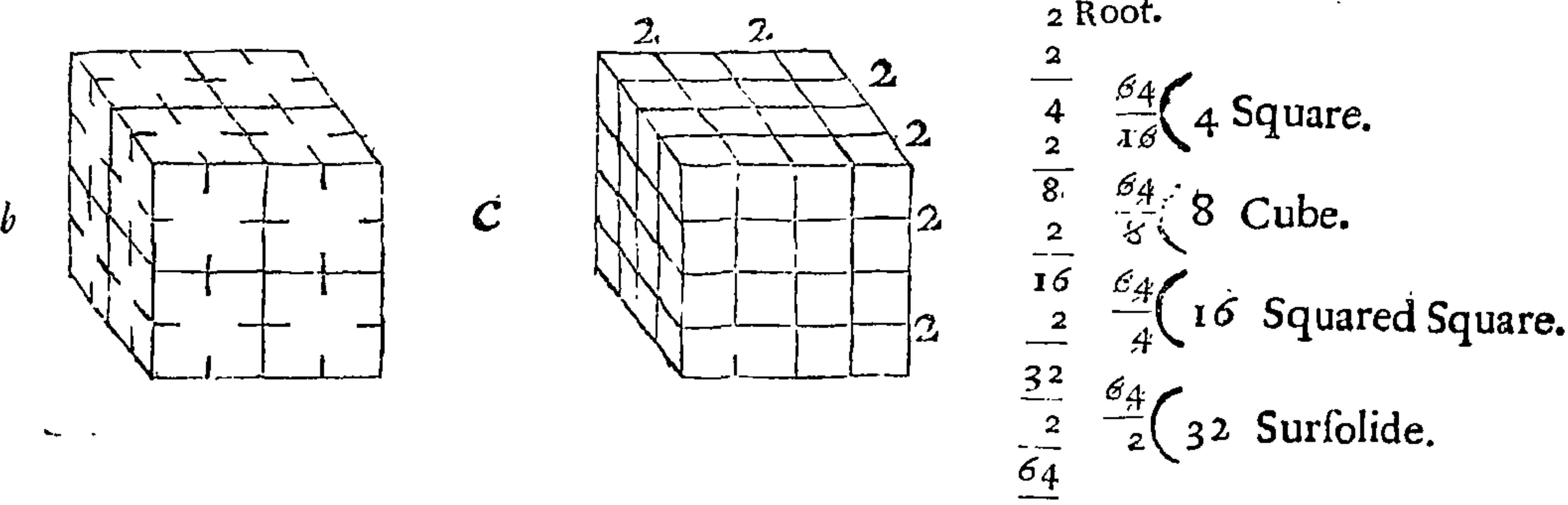
Demonstration.

A *Cubed Cube* is the Second Regular Body. *Arithmetick* takes notice of having *Square Solidity*, and is every way increased according to the *Units* originally in the *Root* there. of, as the Figures at b. or c. arising from the *Root* 2 demonstrate.

Cubed Cube or Squared Cube what.

A *Cubick Cube Number* is so called in relation to his Form only, otherwise the Number which declareth the Solidity of such Figure is called a *Zenzicube*, or a *Squared Cube Number*, made by multiplying the *Surfolide* into the *Root*, or the *Squared Square* by the *Square*, or the *Cube* by it self, and divided accordingly, brings forth the respective Numbers in the *Quotient*, as plain in 64 the *Zenzicube* of 2.

The Number and Names thereof.



Demonstration.

Further Instances may be spared, since you may proceed infinitely on *Rooted Solides*. But it is to be noted that every *Figure* receiveth its name *Geometrically*, according to his Form. And every Number containing the solid quantity thereof, being divided by his *Root*, giveth in the *Quotient*, the Number of the next lesser quantity. And these Numbers so containing the solidity in *Arithmetick*, have names not so much respecting the *Geometrical* Forms of the Figures, as the forming or producing of the Numbers themselves: For in the last Example 64. according to his Form *Geometrical* is a *Cubed Cube*, but in *Arithmetick* goes by the name of a *Squared Cube*, because the number 64, is formed by squaring 8, the *Cube* of 2, the *Root*. And so before 16, the *Squared Square* was so called, not because of his form, which was a *Long Cube*; but because it was the square of 4, which was the square of the *Root* 2.

Names Geometrically and Arithmetically.

And briefly to know how to name *Geometrically* every greater Form or higher Power (as some call them) that in order are increased by their *Roots*; observe that the fifth Body is named like the second, only doubling the *Cubes*, as *Long Cubick Cubes*. And the sixth Body like the third, as *Square Cubick Cubes*. And so the seventh Body like the fourth, tripling the *Cubes*. And so keeping the words *Long*, *Square*, and *Cube*, to every *Ternary*, the word *Cube* is added, as in the *Table of Rooted Numbers* in the next Chapter is sufficiently clear.

Reason of the Arithmetical Names.

But the Terms *Arithmetical* retained to these *Rooted Powers*, take their rise from the original *Rooted Numbers*, viz. the *Square* and the *Cube*, except the *Surfolids*, and keep this order in accompt. The first *Rooted Number* is called a *Square*, the second a *Cube*, the third a *Squared Square*, the fourth a *Surfolide*, the fifth a *Squared Cube*, the sixth a

How to name the Higher Powers Geometrically.

How to name them Arithmetically

Second Surfolide, the seventh a *Square of Squared Squares*, the eighth a *Cube of Cubes*, the ninth a *Square of Surfolides*, the tenth a *Third Surfolide*, the eleventh a *Square of Squared Cubes*, the twelfth a *Fourth Surfolide*, &c. And in *Cossical Numbers* in the next Book shall be shewn, how to proceed infinitely to name such Numbers by their *Indices*.

Names of all
Figurals by
their Quanti-
ties.

Most regular to
call the Root
the first Quan-
tity.

Observations.

1. Which have
sides.

2. Unequal
sides many,
Equal not more
than 2, unless
all.

3. Which the
Root.

4. Which of the
Solides Rooted.

5. Which a
Surde Num-
ber.

6. What the
Root is, and
how differs
from the Side.

7. Roots are
infinite, and are
differenced by
Adjectives.

8. One Number
diversly called
as he stands
related.

Yet some content themselves to call these *Figural Numbers* neither after their *Geometrical Forms*, nor yet their antient *Arithmetical Terms*; but according to their Content or Quantity. And so they call a *Square* the first quantity, a *Cube* the second quantity, a *Squared Square* the third quantity, &c. And others more regularly call the *Root* a Number of the first quantity, a *Square* the second quantity, a *Cube* the third quantity, &c.

All further needful to this Chapter may be considered in the following Observations.

1. All Angled Superficial and Sound Numbers have their sides.

2. One and the same Plain Number may have many sides unequal, but seldom more than two equal sides, except all be equal; as 36 hath 3, and 12 also 4, and 9, and 2, and 18, for the unequal sides; but hath but only 6 and 6 for the equal sides.

3. The one side which is equal to the other in *Squares*, is the *Root*, and no *Flat Number*, save only a *Square*, hath a *Root*.

4. Among *Solid Numbers* they only have *Roots*, which be made of many *Multipli- cations* of some one Number by it self, or by that which ariseth thereof.

5. That Number whose Sides cannot be expressed by a *Whole Number* is called a *Surde Number*, and is no exact *Square* nor *Cube Number*, and such are all *Prime Num- bers*, and (*Squares* only excepted) the most part of all *Compound Numbers*. For if any *Whole Number* have a *Root*, that *Root* shall be a *Whole Number*.

6. The *Root* of a Number is a Number also, and is the side of the *Figural Number*: But every *Side* is not a *Root*, only the *Equal Side*, as aforesaid, yet sometime *Root* and *Side* are used *Synonimically*.

7. *Roots* are as infinite as *Figures*; for any Number may be a *Root*, and the *Root* is always denominate according to his Number: For the *Root* of a *Square* shall be called a *Square Root*; the *Root* of a *Cube Number* is called a *Cubick Root*; so the *Root* of a *Squared Square* is a *Squared Square Root*; and the *Root* of a *Surfolide*, a *Surfolide Root*, &c.

8. One and the same *Solide Number* may be diversly named, according to the *Root* he stands related to; as 16, if it relate to the *Root* 4, is a *Square* or *Zenzike Number*, but if to the *Root* 2, is a *Zenzizenzike*. So 64, if related to the *Root* 8, is a *Zenzike Number*, and if to the *Root* 4, is a *Cubick Number*, but if to the *Root* 2, it is a *Zenzi- cube*, or a *Squared Cube Number*.

C H A P. II.

Production of Figurals.

IN *Figural Numbers* is further to be learned the *Genesis* and *Analysis*. The first of these I call *Production*, and teach in this *Chapter* the manner thereof. The other is to be found in the next. And because *Figural Numbers* principally converse with *Integers*; *Figurate Fractions* are deferred to the *Fourth Chapter*.
To lay by thole *Figural Numbers* of uncertain Product, and proceed orderly in the production of the rest most usual: Observe the Method used in the following *Sections*.

Production of Figural	Numbers not Rooted	{	Circulars	—	§.	1.	The Figurals whose Produ- ction is taught in this Chapter.	
			Oblongs	—	§.	2.		
			Diametrals	—	§.	3.		
			Like Flats	—	§.	4.		
	Rooted Numbers	{	in General	—	§.	5.		
			in Particular	Squares	—	§.		6.
				Cubes	—	§.		7.
				Higher Powers	—	§.		8.

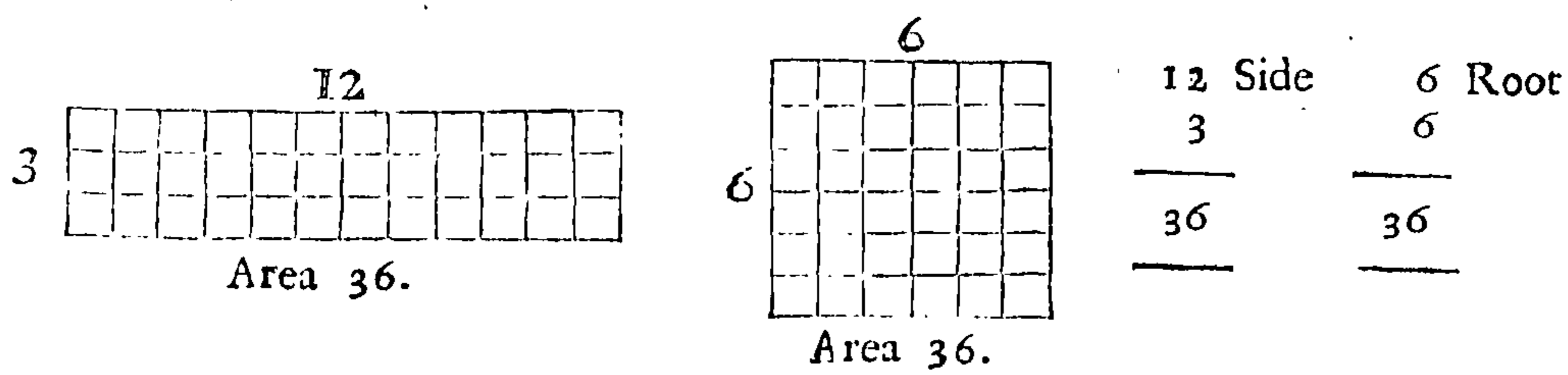
§. 1. The multitude of *Figural Numbers* both *Superficial* and *Solide* wanting *Roots*, whose *Formes* in *Geometry* deserve inspection: As they are of less concern in *Arithmetick*, so they occupy the less room here, but only 4 being touched, and that briefly. *Circular Numbers* (as in the next precedent *Chapter*) before noted in the sense here used, are but the *Areas* of the *Circles* found out as aforesaid, by multiplying the *Semidiameter* into the *Semiperiphery*, which are both usually given to produce them. If but one of them be given, the other is found by the proportion of the one to the other.

Archimedes found the Proportion of the *Diameter* of a *Circle* to the *Circumference*, to be a very small deal greater than of 7 to 22. And of late *Ludoph van Ceulen* insisting in the same steps more precisely, found it to be of 1 to 3, 141 592 653 589 793. for which 3, 1416. may be taken; but most keep the former Numbers of 7 to 22, which make the *Circumference* 3 times as big as the *Diameter*, and $\frac{1}{7}$ part more.

§. 2. *Oblongs*, or *Long Squares*, are produced by Multiplying one Side by the other adjoining. As a *Field* or other *Superficies* being 3 Rods broad, and 8 long, the *Form* of that *Superficies* is a *Long Square*, and the *Content* thereof 24 Rods; obtained by Multiplying 3 into 8.

If one Side be unknown, having the *Content* and the other Side, Divide the *Content* by the known Side, and the *Quotient* will shew the Side unknown. As 24 divided by 3, gives 8; or by 8, gives 3, in the *Quotient*.

Thole *Long Squares* whose *Area* is a *Square Number*, may be reduced from an *Oblong Form* to a perfect *Square Figure*. As a *Long Square*, whose Sides are 3 and 12, or 4 and 9, &c. and consequently the *Area* thereof 36, which because it is a *Square Number* of the Root 6, if each Side of the Platform be reduced to 6, the Figure will be a *Regular Square*, as here appeareth.



§. 3. *Diametral Numbers* were described before, and are produced as *Oblongs*, by Multiplying their proper parts together; or one Side of the *Rectangle Figure* by the other. As 60, produced by the Sides or Parts thereof, 5 and 12. Several others with their respective Sides may be seen in the *Table* following.

The

The Table of Diametral Numbers unto the Lesser Side 40.

A Table of
Diametral
Numbers.

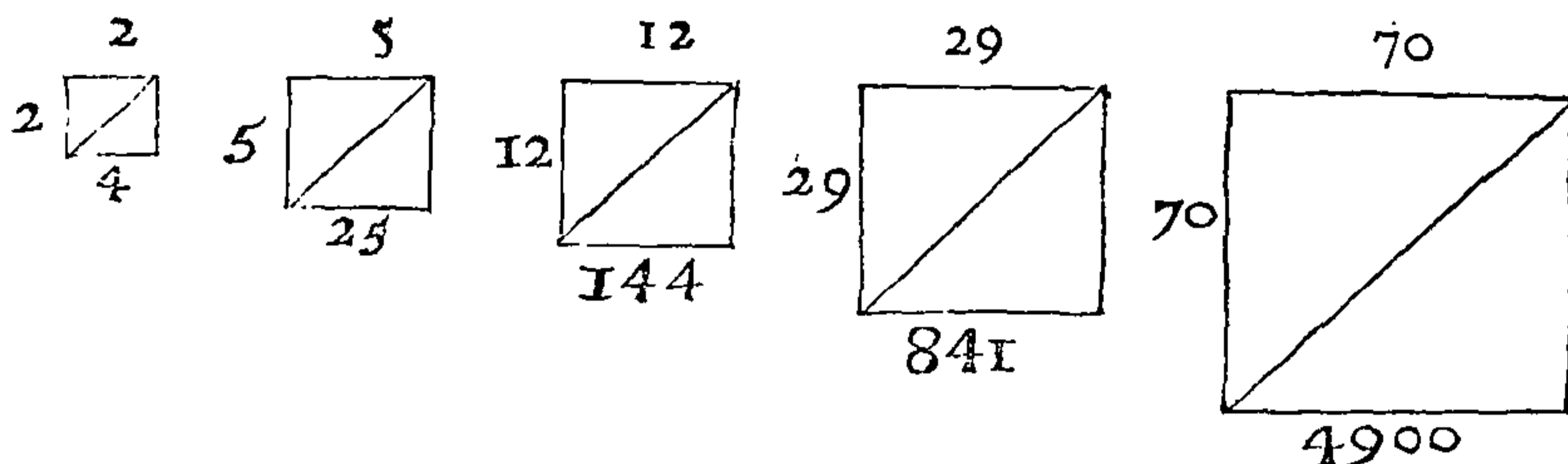
Lesser Side.	Greater Side.	Diameter or Diagon.	Diametral Number.	Lesser Side.	Greater Side.	Diameter or Diagon.	Diametral Number.
3	4	5	12	25	60	65	1500
5	12	13	60		312	313	7800
6	8	10	48	26	168	170	4368
7	24	25	168		36	45	972
8	15	17	120	27	120	123	3240
	12	15	108		364	365	9828
9	40	41	360		96	100	2688
10	24	26	240	28	195	197	5460
11	60	61	660	29	420	421	12180
	15	20	192		40	50	1200
12	35	37	420	30	72	78	2160
13	84	85	1092	31	480	481	14880
14	48	50	672		60	68	1920
	20	25	300	32	126	130	4032
15	36	39	540		255	257	8160
	112	113	1680		44	55	1452
	30	34	480	33	180	183	5940
16	63	65	1008		544	545	17952
17	144	145	2448	34	288	290	9792
	24	30	432		84	91	2940
18	80	82	1440	35	120	125	4200
19	180	181	3420		612	613	21420
	48	52	960		48	60	1728
20	99	101	1980		105	111	3780
	28	35	588	36	160	164	5760
21	72	75	1512		323	325	11628
	220	221	4620	37	684	685	25308
22	120	122	2640	38	360	362	13680
23	264	265	6072		52	65	2028
	32	40	768	39	252	255	9828
	45	51	1080		760	761	29640
24	70	74	1680		75	85	3000
	143	145	3432	40	96	104	3840
					198	202	7120
					399	401	15960

Touching *Diametral Numbers*, 3 things are further considerable.

1. Somewhat concerning the knowledge of them in general.
2. If the Lesser Side of a *Diametral Number* be given, to find out the other.
3. When a Number is propounded, to discover if it be a *Diametral Number*, and consequently to find the Sides.

For the first of these, let be minded.

1. All *Diametral Numbers* do set forth a Plain Rectangled Triangle, having all 3 Sides known; which as it is rare, and of great use in many Geometrical Conclusions, so is it to be found in no other Numbers than only in *Diametral Numbers*: For though in Geometrical Figures you may ever infallibly find a Line, that will make a Square equal to the two Squares of any other two Lines; yet the certain measure of those Sides are not known in whole Numbers. And though other Numbers may go very nigh, yet it can never be done exactly but with *Diametral Numbers*, as in the following Examples of some *Square Numbers*.



Whose Doubles I take for the Squares of the Sides unknown, and they make 8, 50, 288, 1682, and 9800. All which differ only by an Unite from being Square Numbers; for 9 is a Square, and so are 49, 289, 1681, and 9801. and their Roots 3, 7, 17, 41, and 99. But those Doubles being no Square Numbers, cannot render their Sides in whole Numbers.

2. A *Diametral Number* may have more parts, then be apt for the Sides of the Platform or Rectangle Figure it represents: For it is not every two parts of the *Diametral Number*, that by Multiplication will produce the Number, that be meet Sides. As 60 hath these parts, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30. And beginning with the two extrems, viz. 2 and 30, they will, being multiplyed together, produce 60. And so will likewise any two of those Numbers or Parts equally distant from those extrems; as 3 and 20, also 4 and 15, likewise 5 and 12, and 6 and 10, but none of them save 5 and 12 are apt Sides to find the Diameter by. For if the other Sides be multiplyed squarely, and the Squares added together, there will arise no Square Number.

3. As there are 3 Numbers, to wit, the 2 Sides and the Diameter; so alwayes if the first or least Side be odd, then shall 2 of the 3 Numbers be odd also; and the Diameter shall be the second and great odd Number.

4. All *Diametral Numbers* are even, and no odd Number can be a *Diametral Number*.

5. All odd Numbers save 1, may be the Lesser Side to *Diametral Numbers*, but even Numbers do not serve so generally. For if the Lesser Side be an even Number, he must exceed 4; and if the Greater Side be an even Number, he must be such an one as may be divided by 4.

6. If the Lesser Side be an odd Number, then ordinarily the Square of it is just equal with the summe which amounteth by Addition of the Diameter to the Greater Side. As in the former Platform of 60, where 5 being the Lesser Side, his Square is 25, and so much do the Diameter 13, and the Greater Side 12 make added together.

7. One Side may serve to divers *Diametral Numbers*, as 9 to the *Diametral Numbers* 108 and 360, begotten of the Sides 12 and 40, &c.

8. There is no *Diametral Number*, but it may be evenly divided by 12, wherefore they be all even Numbers, evenly and oddly.

9. There is no *Diametral Number*, but it endeth in 0, in 2, or in 8.

10. There is no *Diametral Number* can have any more Diameters than one, and yet may one Number be the Diameter to divers other. As 25 is the Diameter to 168 and 300, &c.

11. No Square Number can be a *Diametral Number*.

12. Though every *Diametral Number* be the Area of a Long Square, yet the Area of every Long Square is not a *Diametral Number*.

What is considerable in *Diametral Numbers*.

Of the knowledge of them.

1. *Diametral Numbers* set forth a Plain Rightangled Triangle.

2. *Diametral Numbers* may have more parts than be apt for the Sides of the Rectangle.

3. If the least Side be odd, so shall the Diameter.

4. *Diametrals* even Numbers.

5. Odd Numbers above 1, may be the least Side.

6. Lesser Side odd what the Square shall be.

7. One Side may serve divers *Diametrals*.

8. *Diametrals* divided by 12.

9. How they end.

10. They cannot have more than one Diameter, &c.

11. Squares no *Diametrals*.

12. They may be the Area of an Oblong, &c.

Of finding the Side.

Secondly, When the Lesser Side or Part of a *Diametral Number* is given to find the other thereby. This Lesser Side will be a—

Number $\left\{ \begin{array}{l} \text{Odd} \\ \text{or} \\ \text{Even.} \end{array} \right\} \left\{ \begin{array}{l} \text{Uncompound} \\ \text{or} \\ \text{Compound.} \end{array} \right.$

When the Datum is Odd and Uncompound.

Example.

If the propounded Number be an odd Uncompound Number, multiply that Number by it self, and the Product (being an odd Square Number) as near the half as may be part into two. The Number less than the half by an Unite will be even, and shall be the second or Greater Side of the *Diametral Number*. And the other part greater than the half by an Unite shall be odd, and the Length of the Diameter. As if 3 be propounded, the Square thereof is 9, the Lesser Part 4 shall be the Greater Side, and the Greater Part 5 the Diameter.

When the Datum is Odd and Compound.

Example.

If the given Number be an odd Compound Number, then hath he more Greater Sides than one; for he hath the benefit not only of the former Rule, but also he followeth the form of that Number of which he is Compound. As 9, whose Square is 81, hath not only 40, the Lesser Part for the Greater Side, and 41, for the Diameter; but being Compound of 3 followeth that form also; and therefore as 3 hath 4 for the Greater Side, so 9 being thrice 3, shall have 12 which is thrice 4, for a match Side with him, and 15, which is thrice 5, for his Diameter. Likewise 15, being compounded of 5 and 3, shall have both Forms in making of the *Diametral Numbers*. For as 3 hath 4, so 15, being 5 times 3, shall have 20, which is 5 times 4 for the second Side, and 25 for his Diameter, which is 5 times 5. Again, as 5 hath 12, so shall 15, being 3 times 5, have 36, which is 3 times 12 for his second Side, and 39, which is 3 times 13, for his Diameter.

Proportion between the Diameters and Diametrals.

Here by the way may be noted, that though both the *Diameters* and *Diametral Numbers* (as of necessity they must) vary from the former Numbers; yet is there a marvellous proportion between them. For the proportion of both the Sides in one Figure, to both the Sides in the other, being added together, will be like the proportion between the two *Diametral Numbers*. As if 3 and 4 be the Sides of a *Diametral Number* they make 12, and 9 and 12 being Sides, make 108, that is, 9 times 12. Now 9 to 3 is triple, and so is 12 to 4, and both triples added together (Addition of *Ratio's* being as Multiplication of *Fractions*) make the proportion or amounting *Ratio Noncuple*, or ninefold, and so are the two *Diametral Numbers*, 12 and 108, in proportion each to other.

When the Datum is Even.

Example.

If the Lesser Side propounded be an even Number, then square the Number as before, and of that Square take two Quarters; from one Quarter take an Unite, and put to the other; so have you two odd Numbers, the Lesser of which shall be the Greater Side of the *Diametral Number*, and the other the Diameter. As 8 squared is 64, the Quarter 16, from whence 1 taken leaves 15 for the Greater Side of the Platform, and adding 1 to 16, the total 17 shall be the Diameter.

Proportion of the Greater Sides and Diameters.

Such even Numbers as have more Greater Sides than one, yet have they the like Numbers in proportion for their Greater Sides and Diameters, as the Numbers have of which they be Compound. As 20, compound of 4 and 5, shall have the Greater Side and Diameter belonging to 5 fourfold, and so the Greater Side of 5 is 12, and of 20 is 48, which is 4 times 12, and the Diameter 52, which is 4 times 13, the Diameter to 5.

Of discovering Diametrals.

Thirdly, To discover if a Number propounded be *Diametral* or not, and consequently to find the Sides; take these 9 Directions.

1. By the ending.

1. If it end with any other Figure than 0, 2, or 8, it can be no *Diametral Number*.

2. Evenly divided by 12.

2. If it may not be evenly divided by 12, although it end as abovesaid, it is no *Diametral Number*.

3. By the Parts of the Number.

3. If the Number propounded have those two properties, then set out all the Parts thereof, so as the Lesser Part stand over the Greater Part, which being multiplied together will make the whole Number, and then examine those Parts according to the former Doctrine.

4. Take the Parts most apt.

4. Observe which of the Parts that stand for the Sides of the Platform be most apt to constitute a *Diametral Number*, and make tryal of them, for some Parts at first sight appear unapt. For if among the Parts, the Lesser Number be odd, the Square thereof must contain double to that Greater Number that is coupled with it, and 1 more. As in the *Diametral Number* 12, where the Sides are 3 and 4, there 9, the

Example.

Square

Square of 3, is double to 4, and 1 over. And if the Lesser Number be even, then must the Square of it contain the Greater Number that stands by it, 4 times and 4 more. As in the *Diametral Number* 48, is 6 coupled with 8, which 6 times 6 is 36, that is 4 more than 4 times 8; and this holds in all Numbers not Compound of other *Diametral Numbers*.

5. When the given Number hath many Parts, to save work, guess at one which seems probable, and making Proof thereby, if he be found too small, assay with the rest of the Parts greater; and if he be too big, refuse all the Parts above, and examine only the smaller Parts till a just Part be found. But if thus examining you still find the Part either too great or too little, then is the Number given no *Diametral Number*. As if 120 be the Number propounded, because it ends in 0, and may be evenly divided by 12, it is probable to be a *Diametral Number*. I therefore set out the Parts which are these.

5.
If a Part too
great or too
little be taken:

Example:

Parts of 120 { 2. 3. 4. 5. 6. 8. 10. 12. 15. 20. 24. 30. 40. 60. Parts.
coupled. { 60. 40. 30. 24. 20. 15. 12. 10. 8. 6. 5. 4. 3. 2. Numbers.

Here though I see many Parts; yet I need examine but few; because several have no likelihood of producing a *Diametral Number*. For all even Numbers under 6, cannot be the Lesser Side of any such Number, therefore the second and fourth Parts are rejected. Also all Numbers above the tenth Part are refused, because the Numbers under them are too little to answer proportionably for the Greater Side to the Parts standing over, which should be the Lesser Side, and these are greater than they. Again the third Part is set aside, as having under him too great a Number; for under 3 ought to stand no other Number than 4, to make a *Diametral Number*. Moreover, under 5 I find 24, but if I square 5, it is 25, which is but 1 more than the Number under 5, when it should be 1 more than the double. Therefore I either pitch upon the sixth, eighth or tenth Parts for the Sides of the *Diametral Number*, or else 120 cannot be *Diametral*; then examining 6, his Square is 36, but this is not 4 times 20, and 4 more; so the sixth Part is laid by also as too little. Then I square 10 its 100, but then 4 times 12 and 4 more is but 52, and therefore the tenth Part is too big. So that 8 and 15 must be the Sides, or else the Number is no *Diametral*. And squaring 8 it is 64, which is 4 times 15, and 4 more, whereby 120 is seen to be a *Diametral Number*, and hath 8 and 15 for the Sides of the Platform.

On the contrary, proving 72 by his Parts, though he end in 2, and may be divided by 12, yet doth it appear to be no *Diametral Number*.

6. By observing the proportion between the *Diametral Sides* it is easie to discern, whether the Parts be apt to constitute a *Diametral Number* or not: For the two Sides of all *Diametral Numbers* keep a constant proportion either to other in the order following, and continue in the same accordingly.

6.
Observe the
Proportion be-
tween the
Sides.

The First Order of Odd Numbers for the Least Sides.

3. 5. 7. 9. 11. 13. 15. 17. 19. 21. 23. 25. 27. &c.
4. 12. 24. 40. 60. 84. 112. 144. 180. 220. 264. 312. 364

The Orders for
placing the
Sides.

The Second Order of Even Numbers for the Least Sides.

8. 12. 16. 20. 24. 28. 32. 36. 40. 44. 48. &c.
15. 35. 63. 99. 143. 195. 255. 323. 399. 483. 575

In both these Orders the Lesser Sides stand at top, and the Greater Sides beneath, as Antecedent and Consequent.

Stifelius sets the Greater Side at top, and the Lesser Side below, and reduceth the Antecedent into Units, like Integers and Fractions.

The Order of
Stifelius:

The First Order after Stifelius.

1. $\frac{1}{3}$ 2. $\frac{2}{5}$ 3. $\frac{3}{7}$ 4. $\frac{4}{9}$ 5. $\frac{5}{11}$ 6. $\frac{6}{13}$ 7. $\frac{7}{15}$ 8. $\frac{8}{17}$ &c.

The Second Order after Stifelius.

1. $\frac{7}{8}$ 2. $\frac{11}{12}$ 3. $\frac{15}{16}$ 4. $\frac{19}{20}$ 5. $\frac{23}{24}$ 6. $\frac{27}{28}$ 7. $\frac{31}{32}$ &c.

And this for observance is best approved, because in the first Order you see both in the whole Numbers and Numerators of the Fractions the Natural Order of Numbers, as 1, 2, 3, 4, &c. And in the Denominators, the Natural Progression of odd Numbers, as 3, 5, 7, 9, &c.

But

But in the second Order the whole Numbers go in their Natural Order, and the Numerators and Denominators keep an *Arithmetical Progression* by equal distance of 4. So that in the Numerators all the Numbers be odd, in the Denominators they be all even.

7.
Parts of the
Number may be
abbreviated.

7. The great Parts of any Number to be examined may be abbreviated (like a Fraction) into its least Terms. For if the proportion hold in the least Terms it will hold in the greatest. As if 540 be proposed to find whether it be *Diametral*, I set down so many of the Parts as are necessary; thus,

Example.

2 3 4 5 6 10 12 15 18 20 &c.
270 180 135 108 90 54 45 36 30 27

Where I may abbreviate many of the Parts, as $\frac{6}{90}$, $\frac{10}{54}$, &c. but comparing their Proportions with the former, cannot find the Numbers alike proportional; but 15 and 36 are in like proportion, and so they continue if abbreviated to 5 and 12, therefore is 540 *Diametral*, and 15 and 36 the Sides thereof.

8.
What Cyphers
may be cut off.

8. From such given Numbers as end in Cyphers, cut off even Cyphers as often as you can, as 2, 4, 6, &c. and if the rest be a *Diametral Number*, so was the given Number: For if 540, or 432, &c. be *Diametral*, then 54000 and 43200 be the like.

9.
By a Square
Divisor.

9. If any Number being divided by a Square Number, make the Quotient a *Diametral Number*; then is the Number divided a *Diametral Number* also. As 48 divided by 4, (a Square Number) yieldeth 12 in the Quotient, (a *Diametral Number*) therefore is 48 a *Diametral Number* likewise.

Like Flats produced.

§. 4. *Like Flats*, because of their proportion in their Sides, are thence so termed, and therefore no one Number without relation to another can be termed a *Like Flat*. Some call them *Square-like Figures*, because they have some properties with *Square Numbers*.

To produce these Numbers, Multiply any two Square Numbers by one other Number, and the Products shall be *Like Flats*. As 4 and 9, if Multiplied by 3, give 12 and 27, which be *Like Flats*.

Also if 2 Square Numbers will admit of 1 Divisor, then divide them thereby, and the Quotients shall be *Like Flats*. As 36 and 9, divided by 3, give 12 and 3, which are *Like Flats*. And so are 4 and 9, being the Quotients of 16 and 36, divided by 4.

Properties of
Like Flats.

The 4 following Properties of *Like Flats* are collected out of *Euclid. lib. 8. prop. 18. 20. & 26. and lib. 9. prop. 1. & 2.*

1.

1. Every two Numbers *Like Flats*, have one mean Number between them in proportion to the *Lesser Flat*, as the *Greater* is to him. As 4 and 9 have 6 for a mean between them. For as 6 is $1\frac{1}{2}$ to 4, so is 9 to 6.

2.

2. One *Flat Number* beareth unto the other double that proportion their Sides do. As 4 and 9, whose Sides 3 and 2, or $\frac{3}{2}$ are in proportion *Sesquialter*, and the *Flats* themselves 9 to 4, are in double *Sesquiquarta* proportion, and so will the *Sesquialter Ratio* make doubled.

3.

3. Numbers that be *Like Flats* have such proportion together, as one of the Square Numbers used in their Composition beareth to the other. For in the former Examples, 12 to 3 is as 16 to 4, or 36 to 9, and if one of them be divided by the other, a Square is brought forth in the Quotient.

4.

4. Any two Numbers being *Like Flats* multiplied together, will produce a Square Number. As 4 and 9 make 36, so 12 and 27 make 324, the Square of 18.

Rooted Numbers produced
Generally.

§. 5. *Figural Numbers Rooted*, in General are produced by *Multiplication* thus. To Multiply any Number by it self makes a Square Number. Again, that Square Number multiplied by the Root produceth a Cubick Number. To multiply by the Root that Cubick Number, giveth a Squared Square Number. And to multiply again by the Root yieldeth a Surfolide Number. And so multiplying the last Product by the first Root, bringeth forth a Number of the next greater Quantity, and so successively. So that to produce a Square, one Multiplication will serve (one Number being accounted for the Length, and the other for the Breadth). To make a Cubick Number, two Multiplications are required, by the second whereof the Number taketh Depth, as by the first Length and Breadth. Thus to make a Squared Square, 3 Multiplications are requisite; and then there is made a Line of Cubes. A Surfolide Number produced this way must have 4 Multiplications, which make a Square, wherein every Unite is a Cube. So the fifth Multiplication maketh a Cube of Cubes, accounting every Lesser Cube for an Unite. And then the sixth Multiplication returneth the multiplied Numbers to the nature of Lineary Cubes. And the seventh to the nature of Squared Cubes. And the eighth to the nature of Cubick Cubes, and so forth infinitely as was before expressed. These 3 Names of *Long*, *Square*, and *Cubick Cube*, may be reiterated, but a fourth Form can never be devised. For further discovery inspection may be made into the *Table of Rooted Numbers* procreated after the common way thus.

*The TABLE of Rooted Numbers, consisting of Fifteen Figural Formes,
and Twelve Roots.*

Names
Geometrical.

| I | Roots. | I | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | Roots. |
|----|--------------------------------------|---|-------|----------|------------|-------------|--------------|---------------|----------------|-----------------|------------------|------------------|-------------------|--|
| 2 | Squares. | I | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 | 100 | 121 | 144 | Squares. |
| 3 | Cubes. | I | 8 | 27 | 64 | 125 | 216 | 343 | 512 | 729 | 1000 | 1331 | 1728 | Cubes. |
| 4 | Squar'd Squar. | I | 16 | 81 | 256 | 625 | 1296 | 2401 | 4096 | 6561 | 10000 | 14641 | 20736 | Long Cubes. |
| 5 | Surfolides. | I | 32 | 243 | 1024 | 3125 | 7776 | 16807 | 32768 | 59049 | 100000 | 161051 | 248832 | Squar'd Cubes. |
| 6 | Squar'd Cubes. | I | 64 | 729 | 4096 | 15625 | 46656 | 117649 | 262144 | 531441 | 1000000 | 1771561 | 2985984 | Cubick Cubes. |
| 7 | Second Surfo-
lides. | I | 128 | 2187 | 16384 | 78125 | 279936 | 823543 | 2097152 | 4782969 | 10000000 | 19487171 | 35831808 | Long Cubick
Cubes. |
| 8 | Squares of
Squar'd Squar. | I | 256 | 6561 | 65536 | 390625 | 1679616 | 5764801 | 16777216 | 43046721 | 100000000 | 214358881 | 429981696 | Square Cubick
Cubes. |
| 9 | Cubes of
Cubes. | I | 512 | 19683 | 262144 | 1953125 | 10077696 | 40353607 | 134217728 | 387420489 | 1000000000 | 2357947691 | 5159780352 | Cubes of Cu-
bick Cubes. |
| 10 | Squares of
Surfolides. | I | 1024 | 59049 | 1048576 | 9765625 | 60466176 | 282475249 | 1073741824 | 3486784401 | 10000000000 | 25937424601 | 61917364224 | Long Cubes of
Cubick Cubes. |
| 11 | Third Surfo-
lides. | I | 2048 | 177147 | 4194304 | 48828125 | 362797056 | 1977326743 | 8589934592 | 31381059609 | 100000000000 | 285311670611 | 743008370688 | Square Cubes
of
Cubick Cubes. |
| 12 | Squares of
Squared
Cubes. | I | 4096 | 531441 | 16777216 | 244140625 | 2176782336 | 13841287201 | 68719476736 | 282429536481 | 1000000000000 | 3137428376721 | 8916100448256 | Cubick Cubes
of
Cubick Cubes. |
| 13 | Fourth Surfo-
lides. | I | 8192 | 1594323 | 67108864 | 1220703125 | 13060694016 | 96889010407 | 549755813888 | 2541865828329 | 10000000000000 | 34522712143931 | 106993205379072 | Long Cubick
Cubes of
Cubick Cubes. |
| 14 | Squares of
Second Surfo-
lides | I | 16384 | 4782969 | 268435456 | 6103515625 | 78364164096 | 678223072849 | 4398046511104 | 22876792454961 | 100000000000000 | 379749833583241 | 1283918464548864 | Square Cubicks
of
Cubick Cubes. |
| 15 | Cubes of Sur-
fo lides. | I | 32768 | 14348307 | 1073741824 | 30517578125 | 470184984576 | 4747561509943 | 35184372088832 | 205891132094649 | 1000000000000000 | 4177248169415651 | 15407021574526368 | Cubed Cubick
Cubes of
Cubick Cube |

The Table explained.

Indices of Figurals, what and why so called.

Rooted Numbers produced particularly. The Square.

Theoreme of Euclide.

Example and Demonstration where the Root hath but 2 Figures.

The &c. at the foot of the Table denotes it may be increased ; but this is large enough to view the orderly production of Rooted Numbers, to the 15th Quantity and 12th Root, where is to be seen,

1. The Numbers of Quantity in the first Left Hand Column, called *Indices* in Latin, because the Power or Quantity of the Figural Number, and how far he is removed from the Root, is shewed thereby.

2. The Roots from which each Figural Number ariseth, in the head of the Table.

3. The Arithmetical Names proper to each Number, at the left side.

4. The Names according to the Forms of each Number, at the right side.

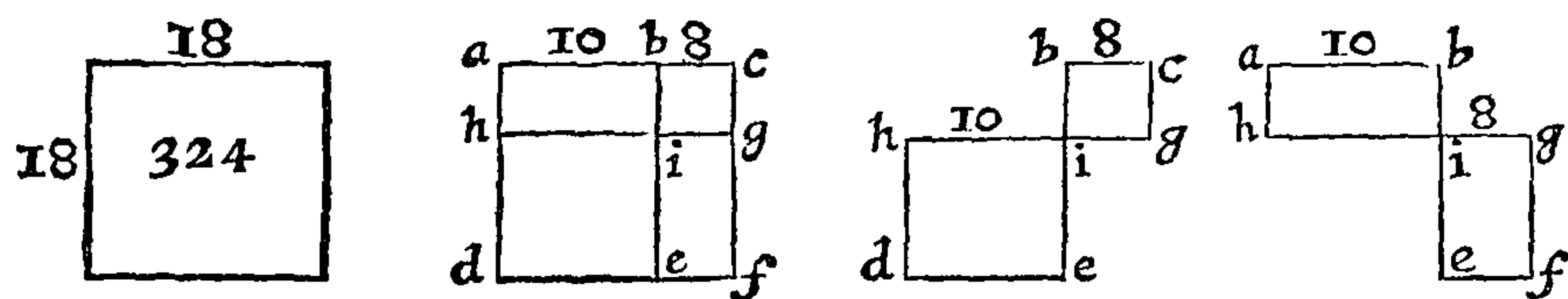
And 5. The Numbers themselves in the body of the Table, which are to be accepted for Figural Numbers of that Denomination, which you find written against them at the sides, and have that Number for their Root, that standeth over them in the head of the Table.

§. 6. Rooted Numbers, admit of some variety in production, which in particular is set forth in this, and the two following Sections.

The first Rooted Number a Square, besides multiplying the Root into it self as afore-said, may be obtained by breaking the Root into Parts, called also Segments ; and Squaring the Parts severally, and orderly adding them with the double of the Product produced by Multiplication of the Segments one into another.

This Device is grounded on that Geometrical Theoreme in Euclid. lib. 2. prop. 4. That if a Right Line be cut into two Segments, the Square of the whole Line shall be equal to the Squares of the Segments, and to the two Right-Angled Figures made of the Segments. As if the Right Line, whose Square I would produce, be 18, I cut the Line into two Segments, as suppose 10 and 8, then shall the Squares of 10 and 8, added with the double of 10, multiplyed into 8, be the Square of 18.

At the Figure following (when the Line or Side of the Square is cut into 2 Segments) there appears 4 plain Figures ; of which 2 are Squares, and 2 Rectangle Figures or Long Squares, all which added together, make the whole Square.



And of these 4 plain Figures is the whole Square obtained by the 4 Numbers of their Area ; thus,

1. The Greater Square *b. i. e. d.* is known by multiplying the Line *b. i.* (equal to *a. b.*) into it self, and so 10 squared is 100.

2. The Lesser Square *b. c. g. i.* is known by multiplying the Line *b. c.* (equal to *i. g.*) into it self, and so 8 multiplyed by 8, gives 64.

3. The Long Squares are known by multiplying the 2 Segments one into another, and then doubling the Product. As the Segment *a. b.* 10, into the Segment *b. c.* 8, which gives 80, for the Area of each Rectangle Figure, and added to the Squares are their Complement to the Square of the whole Line 18, as by the ordinary way may be proved.

| | | |
|-------------|-------------------------------------|------------|
| | 18 : Radix | Proof. |
| | 10 : 8 . Segments | 18 Radix |
| 10 8 | 100 . Square of the Greater Segment | 18 |
| 80 64 | 80 } . Rectangle Figures | |
| 100 80 | 64 . Square of the Lesser Segment | 144 |
| | 324 Zenfus | 18 |
| | | 324 Zenfus |

Where the Root hath more than 2 Figures.

In like manner, if the Root consist of many Figures, or the Line be cut into different Segments, yet is the work alike. As if 140 were the side of a Square, whose Square Number were desired, I may cut 140, into 100 and 40, or 90 and 50, or 95 and 45, or any other parts.

140 Radix

100 : 40 Segments

10000 Great Square

4000 } Rectangle Figures

4000 } Rectangle Figures

1600 Lesser Square

19600 Zenfus

140 Radix

95 : 45 Segments

9025 Great Square

4275 } Rectangle Figures

4275 } Rectangle Figures

2025 Lesser Square

19600 Zenfus

Proof.

140 Radix

140

5600

140

19600 Zenfus

Example.

Some instead of cutting the Line of the Square in but two Segments, as before, do after a sort cut the same into several Segments, even as many as the Root hath Figures; and to shorten the Multiplying work, accompt every Figure in what place so ever, or though never so great an Article, but as a Digit, and to supply the Cyphers wanting, place the Numbers gotten as aforesaid, each one place nearer to the Right Hand than the other, and some add the two Rectangle Figures together, ere they set them down under the Root. As for instance, To get the Square of 46, I accompt the Segments not 40 and 6, but 4 and 6, then place under 4 his Square 16, and multiplying the double of 4, which is 8, by 6, or 4 by 6, and double the Product (for it's all as one) the Product 48 is the summe of both the Rectangle Figures, which I place under 16, one place nearer to the Right Hand, then under the Segment 6, I set his Square 36, and the Total added together, the Square of 46 is found to be 2116, and by the former ways may be proved true.

Variety of working with 2 Figures.

Example.

Proofes.

4 6 Root
 16
 48
 36
 2116 Square

46
 40 : 6
 1600
 240
 240
 36
 2116

46 Root
 46
 276
 184
 2116 Square

And if the Root propounded consist of many Figures, then after the manner last mentioned, when you have gotten the Square of the first two Figures to the Left Hand in the same sort proceed to seek the Power or Quantity of the rest. As in seeking the Square of 46808, thus;

With many Figures.

4 6 8 0 8 Root
 16
 4 8
 36
 21 16
 73 6
 64
 21 90 24
 0
 0
 21 90 24 00
 74 88 0
 64
 Compleat 21 90 98 88 64 Square

Proof.

46808 Root
 46808
 374464
 3744640
 280848
 187232
 2190988864 Square

Example.

Touching Square Numbers, and their production, further observe.

1. A Square Number doth never end in 2, 3, 7, 8, or a single 0; but in order terminate thus, 1, 4, 9, 6, 5, 6, 9, 4, 1, 0, 1, 4, 9, 6, &c. beginning again as in the Table of Rooted Numbers is apparent.

Observations.

1. Of their Terminations.

2. The

2.
How made of
Diametrals.

3.
How made of
Like Flats.

4.
Odd Square
half lacking, 1
added makes a
Square.

5.
How made of
an Even Num-
ber.

6.
How otherwise.

7.
How b. Addi-
tion of Odd
Numbers.

8.
How by Cubes.

2. The Squares of the Parts of a *Diametral Number* added together, make a *Square Number*, as was seen before in *Diametral Numbers*.

3. Numbers called *like Flats*, multiplied together make *Square Numbers*. As 2 and 18 make 36 the Square of 6; so 3 and 48 make 144, the Square of 12.

4. If 1 be taken from any Square Number which is odd, the Square of half the remainder being added to the first Square will make a Square Number. As 9 the Square of 3, from which 1 be taken there resteth 8, the half 4 squared is 16, to which if 9 be added the total is 25 the Square of 5.

5. The Square of half any even Number, if the even Number be added to it, and 1 more, will make a Square Number. As 10 whose half is 5, the Square whereof is 25, to which 10 added, and 1 more, make the Total 36, the Square of 6.

6. If to the square of half any even Number 1 be added, and the even Number then subtracted, there will remain a Square Number. As if to 25, the Square of 5, the half of 10, there be 1 added, it will be 26, from which if 10 be subtracted, 16 a Square Number remaineth whose Root is 4.

7. Odd Numbers continually added from an Unit successively to the antecedent Squares, make the Totals, Square Numbers. As 1 and 3 is 4, so 4 and 5 is 9, and 9 and 7 is 16, &c.

8. Cubick Numbers added successively from the Unit, produce Square Numbers. As 1 and 8 is 9, so 9 and 27 is 36, &c.

Cube produced
particularly.

§. 7. The particular construction of the *Cube* (the first and least Rooted Body) is next to be seen.

A *Cube* by the fifth Section of this Chapter is produced the common way, by Multiplying the Square Number by the Root. As 2 by 2 makes 4, which 4 again multiplied by 2 makes 8, the *Cube* of 2.

This is a kind of triple Multiplication, the Root being alwayes valued in himself once. For 2 times 2 twice maketh 8, and so 3 times 3 thrice yieldeth 27, the *Cube* of 3. And 4 times 4 four times giveth 64, the *Cube* of 4, &c.

Device of
Ramus.

The production of these Numbers vary from the ordinary way: If the Root be broken into Parts, and the *Cubes* of the Parts or Segments be added to the two solid Figures, comprehended 3 times under the Square of one Segment multiplied by the other. Which Device *Ramus*, lib. 24. sect. 10. imitating that in *Euclid* of the Square, delivereth. As if there be a *Cube*, whose Root is 18 Inches, and I would know the *Cube Number* thereof, which shall declare how many solid Inches there are in that Body; I cut the Root into two Segments, as 10 and 8, and thereby doth the Body at K. appear (as much as can be aptly demonstrated in *Plano*, or by *Flats*) to be parted into 8 Bodies or Solidities, viz.

Example and
Demonstration
where the Root
bath 2 Figures.

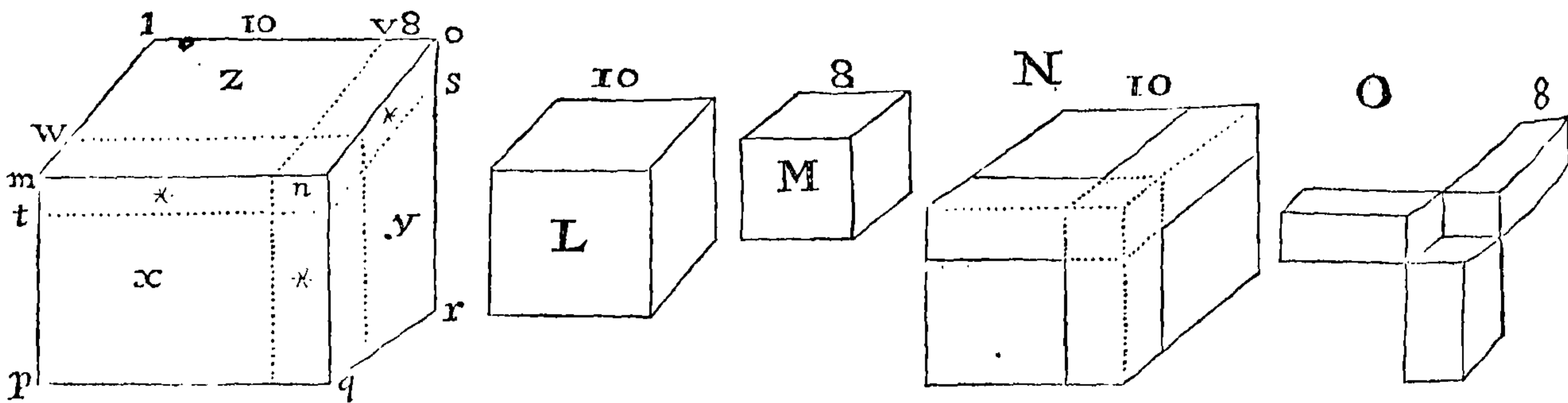
The first a *Great Cube* of the Greater Segment, which cannot well be made visible by a Paper description (*Cubes* portrayed like *Dice*, whose Form most fitly they represent, some parts will be hid from the prospect, and must be imagined) but is placed under the Greater Segment Z. and is that part of the *Cube* K. from the Line t. downward.

The second a *Lesser Cube* of the Lesser Segment, cut off in the corner from the *Cube* K. at n, joyning together the 3 Lesser Paralelipipedons to the 3 Greater.

The 3 Greater Paralelipipedons marked with x. y. z.

The 3 Lesser Paralelipipedons marked with 3 *Asterisques*.

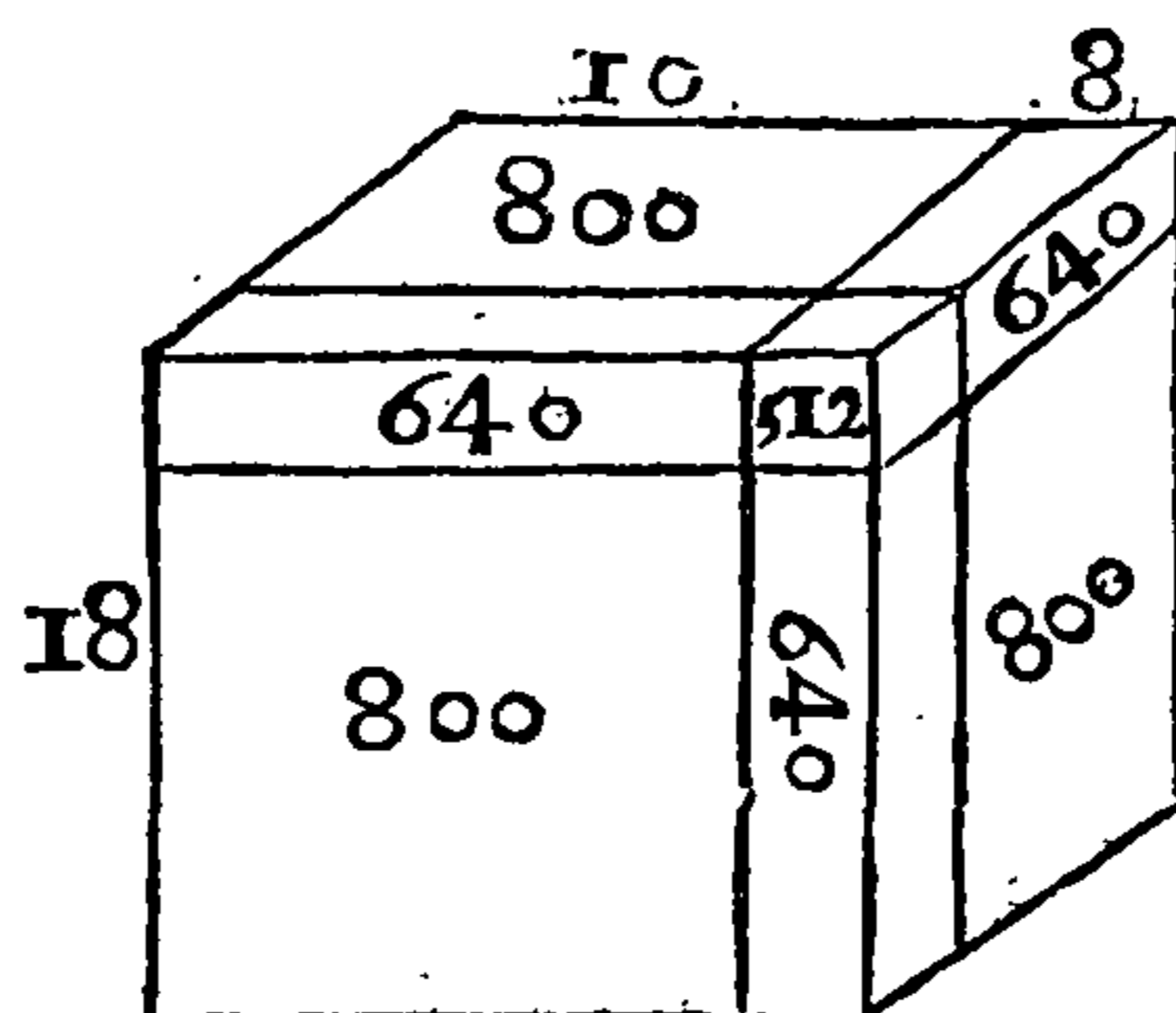
And as if the *Cube* were taken in pieces, be partly discerned at L. M. N. O.



And as all these solid Figures joyned together make the *Cube* at K. compleat, so will their several solid Numbers added together complement the *Cube Number*. For the *Cube* of the Greater Segment is 1000, obtained by multiplying the Greater Segment Cubically. And the *Cube* of the Lesser Segment also is gotten by multiplying the same Cubically,

Cubically, and so 8 Cubed is 512. Then the Greater Paralelipedons are found by multiplying the Square of the Greater Segment by the Lesser Segment; that is 100, the square of 10 by 8, maketh 800, for every one of the 3 Greater Paralelipedons. Lastly, the Lesser Paralelipedons are known by multiplying the Lesser Segment squared by the Greater Segment. As 64, the Square of the Lesser by 10 the Greater Segment, give 640 for every of the 3 Lesser Paralelipedons, all which Numbers added together render the *Cube Number* 5832. Thus,

| | |
|--------|-----------------------------|
| 18 | Radix |
| 10 : 8 | Segments |
| 1000 | Cube of the Greater Segment |
| 800 | 3 Greater Paralelipedons |
| 800 | |
| 800 | |
| 640 | 3 Lesser Paralelipedons |
| 640 | |
| 640 | |
| 512 | Cube of the Lesser Segment |
| 5832 | Cubus |



Proof by the Common way.

| | |
|------|--------|
| 18 | Radix |
| 18 | |
| 144 | |
| 18 | |
| 324 | Zensus |
| 18 | |
| 2592 | |
| 324 | |
| 5832 | Cubus |

Hereby is the Theoreme of *Ramus* also clear, That the *Cube* of the whole Line is equal to the *Cube* of the Segments, and the 2 solid Figures comprehended 3 times under the Square of his Segment, and the remaining Segment.

And it may further be noted, that as in the vulgar way of production you first get Plain Numbers, and then Multiply those Plains by the Roots, to bring forth the solid Numbers: So may you here proceed likewise; for if you get the Squares of the Segments, and the Rectangle Figures, as before in the precedent *Section* was taught, and then Multiply those 4 Plain Numbers by the Segments severally, you will produce 8 Solids, the Total whereof will be the *Cube Number*. As in the former Number the 4 Plain Numbers in the Square of 18, were 100, 80, 80, 64, which Multiplied by 10, give the several Products of 1000, 800, 800, 640, and when Multiplied by 8, give 800, 640, 640, 512, all which added together produce the *Cube*, as before, 5832.

| |
|------------------------|
| 100 . 80 . 80 . 64 |
| Greater 10 Segment. |
| 1000 . 800 . 800 . 640 |

| |
|-----------------------|
| 100 . 80 . 80 . 64 |
| Lesser 8 Segment. |
| 800 . 640 . 640 . 512 |

If there be more Figures in the Root than two, yet this course is still kept. seeking the *Cube* of 468; thus,

| | |
|-----------|-----------------------------|
| 468 | Radix |
| 400 : 68 | Segments |
| 64000000 | Cube of the Greater Segment |
| 10880000 | 3 Greater Paralelipedons |
| 10880000 | |
| 10880000 | |
| 1849600 | 3 Lesser Paralelipedons |
| 1849600 | |
| 1849600 | |
| 314432 | Cube of the Lesser Segment |
| 102503232 | Cubus |

| | |
|--------|----------------|
| Proof. | 468 Root |
| | 468 |
| | 3744 |
| | 2808 |
| | 1872 |
| | 219024 Square |
| | 468 |
| | 1752192 |
| | 1314144 |
| | 876096 |
| | 102503232 Cube |

As in Where the Root hath more than 2 Figures.

Example.

Variety of
working with
2 Figures.

Example.

Some shorten the work, accompting the Segments as Digits only, and to supply the place of Cyphers (as before in the Square) place the Numbers orderly to the Right Hand. As where the Root hath but 2 Figures, the first Right Hand Figure of the Cube of the Greater Segment under the first Left Hand Figure of the Root. And next that one place nearer to the Right Hand, the summe of the 3 Greater Paralelipipedons gotten by tripling the Square of the Left Hand Digit, and Multiplying that triple by the Right Hand Digit of the Root. And under this one place nearer to the Right Hand is set the summe of the Lesser Paralelipipedons obtained by tripling the Square of the Right Hand Digit, and Multiplying that triple by the Left Hand Digit of the Root. And lastly, the Cube of the Right Hand Digit of the Root (or Lesser Segment) placed under him compleat the work. As if the Cube of 56 be demanded, I place under 5 his Cube 125, and next 450, which is the Product of 6 into 75, the triple of 25, the Square of 5; and next 540, the Product of 5 into 108, the triple of 36, the Square of 6; and lastly, 216 the Cube of 6, in their orderly places before directed, and adding them, find the Cube sought to be 175616.

| 5 6 Root | 56 | Proofes. | 56 Root |
|--|--|----------|---|
| <div> <div>125</div> <div>450</div> <div>540</div> <div>216</div> </div> | <div>50 : 6</div> <div>125000</div> <div>150000</div> <div>150000</div> <div>150000</div> <div>180000</div> <div>180000</div> <div>180000</div> <div>216</div> | | <div>56</div> <div>336</div> <div>280</div> <div>3136 Square</div> <div>56</div> <div>18816</div> <div>15680</div> <div>175616 Cube</div> |
| 175 616 Cube | 175616 | | |

With many Fi-
gures.

If the Root consist of many Figures, then after in the former manner you have wrought for the Cube of the first 2 Figures to the Left Hand, go forward to seek the quantity of the residue. As in seeking the Cube of 46808, this is the work.

Example.

| | 4 6 8 0 8 | | Proof. |
|----------|---|------|---|
| Cube of | <div> <div>64</div> <div>288</div> <div>432</div> <div>216</div> </div> | 46 | <div>46808 Root</div> <div>46808</div> <div>374464</div> <div>3744640</div> <div>280848</div> <div>187232</div> <div>2190988864 Square</div> <div>46808</div> <div>17527910912</div> <div>175279109120</div> <div>13145933184</div> <div>8763955456</div> <div>102555806746112 Cube</div> |
| Cube of | <div> <div>97336</div> <div>50784</div> <div>8832</div> <div>512</div> </div> | 468 | |
| Cube of | <div> <div>102503232</div> <div>525657600</div> <div>898560</div> <div>512</div> </div> | 4680 | |
| Compleat | 102 555 806 746 112 Cube | | |

How Cubes
are gotten by
Addition of
Odd Numbers.

How Cube
Numbers end.

Further in general may be noted, That Cubick Numbers are begotten by Addition of odd Numbers from an Unit successively. As 3 and 5 make 8, so 7, 9, and 11, the next odd Numbers are 27, the next Cube to 8; also 13, 15, 17, 19, make 64, and the like of others; and so many Units in the Root, so many odd Numbers in the Cube.

One thing more is remarkable in the termination of the Cube, viz. that he may end in any Figure, but he maketh exchange in some. For if he have 0, 1, 4, 5, 6, or 9, in

in the first place of his Root, he will have the same in the first place of the Cube. But if 2 be first in the Root he will exchange for 8 in the first place of his Cube ; and if 8 be the first of his Root, 2 shall be the first of the Cube. And in like sort do 7 and 3 make exchange in the Cubick Number.

§. 8. The varieties of production of *Higher Powers*, or *Rooted Solides*, greater than the *Cube* shall close up this *Chapter*. And because it would be tedious to recite, and consequently to remember many particulars, take this General Rule to know how many ways every such *Higher Power* may be produced.

Mark their *Indices*, or how many degrees the Number you would produce is removed from the Root, as whether it be the second, third, fourth, fifth, &c. Quantity (accompting the Root alwayes for the first, as before in the *Table of Rooted Numbers*), and couple each two Numbers together, the one next increasing from the Root, with the other next decreasing from the Quantity or Number sought to be produced, and so proceeding, at last there will be either an even Couple, or an odd Number. If they be all even Couples, then so many wayes may the Number desired be produced by Multiplying each two Numbers together that answer to their coupled *Indices*. And if among the coupled *Indices* there be an odd Number which hath none to match him, then may the Number you seek, by Multiplying the *Figural Number* answering to this odd *Index* into it self, be produced, as well as by the Coupled Numbers. As to know how many wayes the *Third Surfolide*, or *11th Rooted Figural Number* (accompting the Root for the Prime or Original) may be produced, I couple their *Indices* as at *P*, and supposing the Root 2, place the correspondent *Figural Numbers* as at *Q*. and find he may be produced 5 wayes, by Multiplying, 1. The Root into the Square Surfolide. 2. The Square into the Cubed Cube. 3. The Cube into the Zenzizenzenzike. 4. The Squared Square into the Second Surfolide. And 5. The Surfolide into the Squared Cube.

By their Indices.

1. Example:

| Indices Coupled. | | | | | | Numbers Placed. | | | | | |
|-------------------------------|----|---|---|---|---|------------------------|------|-----|-----|-----|----|
| P. | 10 | 9 | 8 | 7 | 6 | Q. | 1024 | 512 | 256 | 128 | 64 |
| | 1 | 2 | 3 | 4 | 5 | | 2 | 4 | 8 | 16 | 32 |
| <hr/> | | | | | | <hr/> | | | | | |
| 11 11 11 11 11 | | | | | | 2048 2048 2048 768 128 | | | | | |
| <hr/> | | | | | | <hr/> | | | | | |
| Index of the Third Surfolide. | | | | | | 128 192 | | | | | |
| | | | | | | <hr/> | | | | | |
| | | | | | | 2048 2048 | | | | | |
| | | | | | | <hr/> | | | | | |

The Third Surfolide of 2 produced.

Likewise 8 several varieties of producing 65536, the Zenzizenzenzenzike of 2, or the 16th *Figural Rooted Number*, by this way are found. For besides the 7 Couples of *Indices*, the 8th *Index* is odd, and his Number Multiplied squarely will effect as much as the other. See the Operations at *R*. and *S*.

| R. | | | | | | | | S. | | | | | | | |
|-------------------------|----|----|----|----|----|----|---|---|-------|-------|------|------|------|------|-----|
| Added | 15 | 14 | 13 | 12 | 11 | 10 | 9 | Doubled | 32768 | 16384 | 8192 | 4096 | 2048 | 1024 | 512 |
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | | 2 | 4 | 8 | 16 | 32 | 64 | 128 |
| <hr/> | | | | | | | | <hr/> | | | | | | | |
| 16 16 16 16 16 16 16 16 | | | | | | | | 65536 65536 65536 24576 4096 4096 4096 1536 | | | | | | | |
| <hr/> | | | | | | | | <hr/> | | | | | | | |
| | | | | | | | | 4096 6144 6144 1024 1280 | | | | | | | |
| | | | | | | | | <hr/> | | | | | | | |
| | | | | | | | | 65536 65536 65536 512 512 | | | | | | | |
| | | | | | | | | <hr/> | | | | | | | |
| | | | | | | | | 65536 65536 65536 | | | | | | | |
| | | | | | | | | <hr/> | | | | | | | |
| | | | | | | | | 65536 65536 | | | | | | | |
| | | | | | | | | <hr/> | | | | | | | |

And further, because this Rule depends upon the proportions between the *Indices* and their Quantities ; for as the one increafe by Addition, so the other by Multiplication (as in procefs of this Treatise hereafter may be seen) ; therefore whatever *Indices* added together, will make the Total, the *Index* of the *Figural Number* sought, those *Figural Numbers* answering to the added *Indices*, Multiplied one into another shall produce the *Figural Number* desired, as in the last Example, because 5, 5, and 6, make 16, and also 4, 4, and 8, and 11, 4, and 1, and several other Numbers do the like, the *Figurals* of those Quantities Multiplied together, as at *T. V. W.* will produce the *Figural Number* of 16, and the Root being 2, will be 65536, as before.

| T. | | V. | | W. | |
|-------|----|-------|----|-------|----|
| 32 | 5 | 16 | 4 | 2048 | 11 |
| 32 | 5 | 16 | 4 | 16 | 4 |
| 64 | | 96 | | 12288 | |
| 96 | | 16 | | 2048 | |
| 1024 | | 256 | | 32768 | |
| 64 | 6 | 256 | 8 | 2 | 1 |
| 4096 | | 1536 | | 65536 | 16 |
| 6144 | | 1280 | | | |
| | | 512 | | | |
| 65536 | 16 | 65536 | 16 | | |

Of the ending
of
Squared
Squares.

Surfolides.

Others higher.

Proof of Pro-
duction of
Rooted Num-
bers.

Some have observed the Terminations of these *Higher Powers*, and find, The Squared Square never endeth in 2, 3, 4, 7, 8, or 9, but either in 0, 1, 5, or 6, and thus according to the Natural Order of Numbers in their Roots, 6, 1, 6, 5, 6, 1, 6, 1, 0, 1, and then begin again.

The Surfolides imitate their Roots. For if the Root be a Digit, then hath the Surfolide the same Digit in his first place : But if his Root be an Article, then as the Surfolide is the fifth Quantity, so hath he 5 times so many Cyphers together in the Right Hand places as the Root had, and the next signifying Figure after these Cyphers is the first Figure significative of his Root. And if the Root be a mixt Number, yet still is the first Figure of the Surfolide, the first of the Root.

The next Figural Number, as Surfolides follow the manner of their Roots, so do they the manner of the Squares. And the next to them the manner of the Cubes. And the next the manner of the Squared Squares. And then they begin again, and are like the Surfolides, &c. All which may fully be seen in the foregoing *Table of Rooted Numbers*.

The Proof of Production of Rooted Numbers (besides the varieties proved one by another) must be deferred till Extraction of their Roots be learned, which shall be the Subject of the next *Chapter*.

C H A P. III.

Extraction of Roots.

Extraction of
Roots.

Of what made.

Divisor to seek.

Sides of the
Numbers not
Rooted, found
by Division.

Roots by Ex-
traction.

THE *Genesis of Figural Numbers* now finished, I come to their *Analysis*, vulgarly called *Extraction of Roots*.

As Production of *Figural Numbers* was made up of Multiplication and Addition, so is Extraction of their Roots of Substraction and Division. But whereas in Division of Integers, the Divisor is known, here it is to seek, every remove requiring a new Divisor.

The Sides of those *Figural Numbers* not Rooted, mentioned in the former *Chapter*, being sufficiently known by their Production to be the Factors of such Products by one single Multiplication, render their Invention easie by one single Division of those produced Area's by the Side known, and need no further remembrance here. But the finding of the Root of a *Figural Number* made of several Multiplications, is much more difficult. For the clear understanding whereof, and all needful thereto, see,

| | | | |
|---------------------|---|---------------------------------------|-------|
| Extraction of Roots | { | in General | §. 1. |
| | | Squares | §. 2. |
| | { | Cubes | §. 3. |
| | | Compounds of both | §. 4. |
| | | All Higher Powers, by the Table | §. 5. |
| | | Surdes, and to denominate the Remains | §. 6. |
| | | in Particular of | |

§. 1. To the Extraction of *Roots* in General, 2 things are necessary.
1. To have the *Figural Numbers* of every Digit perfectly in mind : For it were superfluous to seek Rules for them, since they may be easier remembered then Rules for their production, and readily found in the *Table of Rooted Numbers* : As for the Square and Cube, thus ;

What necessary
to Extraction of
Roots.
1.
To know the
Figural Num-
ber for every
Digit.

| | | | | | | | | | | |
|---------|---|---|----|----|-----|-----|-----|-----|-----|---------------------|
| Roots | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |
| Squares | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 | The like of others. |
| Cubes | 1 | 8 | 27 | 64 | 125 | 216 | 343 | 512 | 729 | |

2. To prick the *Figural Numbers* given whose Root is to be extracted, according to their Quantities, beginning at the Right Hand. As the Square, because a Number of the second Quantity, prick every second Figure, and leave one unprickt. The Cube prick every third Figure, and leave two unprickt, and so accordingly upward. And let it be noted that so many Pricks as the Number will admit of, so many Figures must be in the Quotient for the Root to consist of.

2.
To prick the
Number accord-
ing to the
Quantity.

§. 2. In Particular, the Square Root of a Number is extracted commonly thus :

To Extract the
Square Root.
Rules.
1.

1. When the Number is placed with a Crooked or Rectangle Line to separate the Quotient or Root from the Square, and pricked as abovesaid, then seek the greatest Square Number contained in the Figure or Figures that belong to the last Prick to the Left Hand, which Square Number set thereunder, and subtract therefrom, and the Root of that Square set in the Quotient, cancelling the Figure or Figures standing over this Square, and if there be Remainders set them in order at top, and so is the work for that Prick ended.

2. Double this Root, and set him down for a Divisor, thus ; if he be a Digit, place him under the next unprickt Figure inclining to the Right Hand ; and if an Article or Mixt Number, then set the first Figure or Cypher of this doubled Root in the unprickt place as before, and the other Figure one place nearer to the Left Hand, and divide with this Number for a new Quotient Figure, which must be no greater than that the Square of the same Figure may be also subtracted from the remaining Numbers left to the next Right Hand Prick after you have, as in common Division, subtracted the Product of your Divisor multiplied into this Quotient Figure, still cancelling the Figures from which any thing is taken, and setting down the Remainder if any be.

2.

3. Then set down the Square of this Quotient Figure, the first Figure thereof, if more than one, under the next Prick inclining to the Right Hand, and the other Figure one place nearer to the Left Hand, and subtract it, cancelling the Figures after Subtraction, and setting the Remainders, if any, at top as before.

3.

4. If your Number admit of more Pricks than 2, then for every Prick exceeding 2 must you repeat the work in the second and third Directions, to double the Quotient and divide thereby, and subtract the Square of each Figure in their order.

4.

For further Explanation ; suppose I would prove whether 46808 be the Square Root of 2190988864, then having placed and pricked the Number, I find 21 to belong to the last or Left Hand Prick, I therefore inquire the Greatest Square in 21, and it is 16, whole Root is 4, which placed in the Quotient, and subtracting 16 from 21, there rests 5 at top, as at *A*.

Example.

Then I double the Root 4, and it is 8, which placed under the unprickt Figure, and dividing thereby, I can take but 6 in the Quotient, or else I shall not leave enough to the next to make Subtraction of his Square ; for should I take 7, I should leave but 3 to the 0, which would be but 30, and the Square of 7 is 49 ; therefore I first place 6 in the Quotient, and take 6 times 8, which is 48, out of 59, there rests 11, as at *B*.

Then do I square 6, and it is 36, which I set under 110, the Numbers belonging to the Prick, and subtract it, as at *C*.

Then do I double the whole Quotient 46, and the summe 92 is the next Divisor to be placed under 749, and dividing thereby find the Divisor may be taken 8 times from 749, and yet leave enough to take the Square of 8, from the remaining Figures, the next pricked Figure being joyned thereto, I therefore set 8 in the Quotient, and subtracting the Divisor 8 times, there resteth 13 ; and the Square of 8, which is 64, subtracted leaves 74, uncanceled as at *D*.

Then I again repeat this last work, and double the whole Root 468, and place the summe 936, in order beginning under the next unprickt Figure, and finding the Divisor bigger than the uppermost Number, I set a Cypher in the Quotient, and also under the next Prick to the Right Hand ; for the Square of 0 is 0, and cancel the Divisor, as at *E*.

Laſtly, Doubling the whole Quotient 4680, the next Diviſor will be 9360, thereby 8, gotten for the Quotient, and the Diviſor ſubſtracted 8 times, and afterward the Square of 8, as before, there will be nothing left remaining, as at F. ; whereby 2190988864, is ſeen to be a Square Number, and hath 46808 for his Root. And thus is the production of the Square in the former Chapter proved true. And reciprocally the truth of Extraction thereby.

A.

$$\begin{array}{r} \overset{5}{\cdot} \\ 2190988864 \mid 4 \\ \underline{16} \end{array}$$

B.

$$\begin{array}{r} \overset{51}{\cdot} \\ 2190988864 \mid 46 \\ \underline{168} \end{array}$$

C.

$$\begin{array}{r} \overset{17}{\cdot} \quad \overset{514}{\cdot} \\ 2190988864 \mid 46 \\ \underline{1686} \\ 3 \end{array}$$

D.

$$\begin{array}{r} \overset{1717}{\cdot} \quad \overset{51434}{\cdot} \\ 2190988864 \mid 468 \\ \underline{168624} \\ 396 \end{array}$$

E.

$$\begin{array}{r} \overset{1717}{\cdot} \quad \overset{51434}{\cdot} \\ 2190988864 \mid 4680 \\ \underline{16862460} \\ 3963 \\ 9 \end{array}$$

F.

$$\begin{array}{r} \overset{1717}{\cdot} \quad \overset{51434}{\cdot} \\ 2190988864 \mid 46808 \text{ Root} \\ \underline{1686246004} \quad 46808 \\ 3963366 \\ 99 \quad 374464 \\ \quad 3744640 \\ \quad 280848 \\ \quad 187232 \end{array}$$

Square 2190988864 Proof

How to work
and prove it
by Addition.

Example.

Gnomon
what.

Extraction of the Square Root may be proved by Addition : If the ſubſtracted Numbers be orderly placed one under another, and then added, the Total will return the Square whoſe Root was extracted. As in the former Example, after the ſecond Figure is placed in the Quotient, ſet him under the Diviſor, and multiply the Diviſor thereby, and the amounting Product joyned with the Square of the Quotient Figure, makes the ſecond Number to be ſubſtracted, called a *Gnomon* ; and ſo continue this work according to the Number of Pricks on the Great Square, and then add all the *Gnomons* together with the Left Hand Square firſt ſubſtracted.

| | | | | | | |
|--------|----|----|----|----|--------|-------------|
| | | | | | 87474 | |
| Zenus | 21 | 90 | 98 | 88 | 64 | 46808 Radix |
| | 16 | 8 | | | | |
| | | 3 | | | | |
| | | 4 | | | | |
| | | | 36 | | | |
| Gnomon | | | 16 | | | |
| | | | | 92 | | Diviſor. |
| | | | | 6 | | |
| | | | | 73 | | |
| | | | | 64 | | |
| Gnomon | | | | 74 | 24 | |
| | | | | | 946 | Diviſor. |
| | | | | | 0 | |
| | | | | | 000 | |
| Gnomon | | | | | 0000 | |
| | | | | | 9260 | Diviſor. |
| | | | | | 8 | |
| | | | | | 748 | |
| | | | | | 64 | |
| Gnomon | | | | | 748864 | |

Proof.

| | |
|------------|--|
| 16 | |
| 516 | |
| 7424 | |
| 0000 | |
| 743864 | |
| <hr/> | |
| 2190988864 | |
| <hr/> | |

Some omit to place the Divisors underneath, and others multiply the Root by 20, and take the Product for the Divisor; but set the Cypher under the next Right Hand Prick, and by the new Quotient Figure multiply this Divisor, to which the Square added shall make the Total the Gnomon to be subtracted, and then multiply again by 20, this whole Quotient, to get a new Divisor, if there be more Pricks, reiterating this work till all be finished; as in the former instance. Again thus;

57474
2190988864 | 46808
16
516
7424
6000
748864
Collection 2190988864 Proof

44464684680

420202020

1680920936093600

688

48073600000748800

3664064

51674240000748864

Example.

§. 3. The Cube Root is extracted with some variety, but the most received way is thus.

1. When the Number is placed and pricked as before directed, from the Figures belonging to the last Prick to the Left Hand, subtract the Greatest Cube Number you can have out thereof, and cancelling the same Figures, if there be any Remains set them at top, and the Root of this Cube set in the Quotient, and so have you done with this Prick: For this work, as in the Square, is wrought but once

2. Triple the Root, and multiply this triple by the Root, the Number that ariseth shall be the Divisor, and set one place nearer to the Right Hand, by which inquire for a new Quotient Figure from the Figures standing there over, and when found out place in the Quotient; and as in Division of Integers, subtracting the Product of your Divisor multiplied by the new Quotient Figure, cancel the Figures from which any thing is subtracted, and set the Remainders, if any be, at top.

3. Square the last Quotient Figure, and multiply this Square by the triple of the former Quotient Figure, and place the Product one place nearer to the Right Hand, and subtract it from the upper Figures, still cancelling and setting the Remains at top, as before.

4. Under the next Right Hand Prick, set the Cube of the last Quotient Figure, and subtract it likewise: For if this Cube, and the Number last above mentioned, cannot be subtracted, the last Quotient Figure is too big, and a less must be taken.

5. If there be more Pricks on the given Cube, reiterate the work in the second, third and fourth Directions.
- Rules.

For Example: If I would know what is the Cube Root of 102503232, when I have placed and pricked the Number, I find 102 to belong to the Left Hand Prick, the Greatst Cube I can have from thence is 64, of which the Cube Root is 4, therefore I set 4 in the Quotient, and taking 64 from 102 leave 38 over the same, as at G.

Then 4 tripled is 12, which multiplied by 4 is 48 for a Divisor, which standing under 385, I see I may take him 6 times from thence, wherefore putting 6 in the Quotient, and subtracting 48 the Divisor, 6 times, I cancel the Number 385, and set the Remainder over the same in order as at H.

Then I square 6, and it is 36, which multiplied by 12, the triple of 4, makes 432, placed under 570 and subtracted, leaves remaining 538, and then placing the Cube of 6, which is 216, under 5383, and subtracting it, leave 5167, as at I.

Then because there is yet one Prick remaining, I square the Quotient 46, and it is 2116, which I triple, and it is 6348; and this I take for a new Divisor under 51672, and by it get 8 in the Quotient, and subtracting this new Divisor 8 times, leave behind 888, as at K.

Then do I square 8, and it is 64, which multiplied by the triple of 46, produceth 882, this I subtract from the next place to the Right Hand, as at L.

Lastly, the Cube of 8 subtracted, cuts off all the Figures, whereby it appears, 468 is the Cube Root of 102503232, and the whole work stands as at M. And the truth of this Extraction is proved by the Production of the Cube according to the former Copies, as this by Extraction reciprocally.

$$\begin{array}{r} 38 \\ \hline G. \quad 102503232 \mid 4 \\ 64 \end{array}$$

$$\begin{array}{r} 9 \\ 387 \\ \hline H. \quad 102503232 \mid 46 \\ 648 \\ 4 \end{array}$$

$$\begin{array}{r} 51 \\ 936 \\ 38787 \\ \hline I. \quad 102503232 \mid 46 \\ 64826 \\ 431 \\ 42 \end{array}$$

$$\begin{array}{r} 518 \\ 9368 \\ 387878 \\ \hline K. \quad 102503232 \mid 468 \\ 648268 \\ 4314 \\ 423 \\ 6 \end{array}$$

$$\begin{array}{r} 518 \\ 93685 \\ 3878781 \\ \hline L. \quad 102503232 \mid 468 \\ 6482682 \\ 43143 \\ 4238 \\ 68 \end{array}$$

$$\begin{array}{r} 518 \\ 93685 \\ 3878781 \\ \hline M \quad 102503232 \mid 468 \\ 64826822 \quad 468 \text{ Root} \\ 431431 \\ 42385 \quad 3744 \\ 68 \quad 2808 \\ 1872 \\ \hline 219024 \text{ Square} \\ 468 \\ \hline 1752192 \\ 1314144 \\ 876096 \end{array}$$

Proof 102503232 Cube

How to work
and prove it by
Addition.

Cubical Extraction may also be proved by Addition : If when you have gotten the second Quotient Figure, you place him under the Divisor, and multiply the Divisor thereby, and under this Product, one place nearer to the Right Hand, the Product of the Multiplier squared and multiplied by the tripled Root, and under this one degree nearer to the Right Hand, the Cube of the Multiplier, and subtract all these Numbers added into one Total Gnomon from the given Cube, cancelling the Figures you make Subtraction from, and if you please those underneath, except the Gnomon. And so continue this work till all the Pricks be done with, then adding all the uncanceled Gnomons with the first subtracted Cube orderly placed, and you will have the Cubical Number returned ; as appeareth by the former Example wrought and proved this way.

Example.

| | | | |
|--------|-------------|-----------|--|
| | 5
28 167 | | |
| Cubus | 102503232 | 468 Radix | |
| | 64 | | |
| | 48 | | |
| | 6 | | |
| | 288 | | |
| | 432 | | |
| | 216 | | |
| Gnomon | 33336 | | |
| | 6345 | | |
| | 3 | | |
| | 51784 | | |
| | 81 | | |
| | 512 | | |
| Gnomon | 516713 | | |

Proof.

$$\begin{array}{r} 64 \\ 33336 \\ 5167232 \\ \hline 102503232 \end{array}$$

Some

Some after pricking the Number, and subtracting the Greatest Cube out of the last Prick, and getting the Divisor, and thereby a second Figure in the Quotient as before; by this Quotient Figure multiply the Divisor, and subtract the Product, then triple the first Quotient Figure, and to the Right Hand of the triple set the last Quotient Figure, and the Product of this Number multiplied by the Square of the last Quotient Figure, place under the given Number, so that the Right Hand Figure thereof may stand under the next Prick to the same Hand, and subtract. And so reiterate this manner of work till all be finished.

As in the former Number, after 64, as before, is subtracted, the Divisor 48 I multiply by 6, and the Product 288 subtract from 385, then I triple 4 the first Quotient Figure, and by the 12 amounting, I set 6 the second Quotient Figure; this 126 I multiply by 36, the Square of 6, and the Product 4536 I withdraw from 9703, and have at top 5167: Then getting the new Divisor, as before, 6348, and multiplying by the new Quotient 8, there is produced 50784, after Subtraction of which to the triple of 46, which is 138; I adjoyn 8, and increase this 1388 by 64, the Square of 8, and the Product 88832, abated from the given Cube, leaveth nothing as before:

| | | | | | |
|----------------------------------|-----------------------|------|--------|------|-----------|
| | | | Proof. | | |
| <div>5
9188
287078</div> | | | | | |
| Cubus | 1 2503232 468 Radix | | 126 | 1388 | 64 |
| | 64 : : | | 36 | 64 | 288 |
| | 48 : : | | | | 4536 |
| | 6 : : | | | | 50784 |
| Gnomon | { 233 : : | 756 | 5552 | | 88832 |
| | { 4536 : : | 378 | 8328 | | |
| | { 6348 : : | | | | |
| | 6348 : : | 4536 | 88832 | | 102503232 |
| | 6 : : | | | | |
| Gnomon | { 5784 : : | | | | |
| | { 88832 : : | | | | |

Others after the Greatest Cube out of the Left Hand Prick is subtracted, and by the Divisor a second Figure placed in the Quotient, as before, triple the whole Quotient, and multiply that triple by the first Quotient Figure, and again the Product by the latter Quotient Figure, and to that subjoyn the Cube of the said latter Figure one place nearer to the Right Hand, and then deduct the Total out of the given Cube, repeating the like work for every Prick.

As in the former Example thus: After 6 is found for the second Figure of the Quotient, I triple 46, and it's 138, which multiplied by 4, gives 552, that again by 6 produceth 3312, then 216, the Cube of 6, subjoyned makes 33336, for the Gnomon to be subtracted from 38503, so rests 5167 to that Prick; this reiterated for the next leaves 0, as before.

| | | | | | |
|------------------------|-----------------------|-------|---------|------|-----------|
| | | | Proof. | | |
| <div>5
38167</div> | | | 46 | 468 | |
| | | | 3 | 3 | |
| Cubus | 1 2503232 468 Radix | | 138 | 1404 | 64 |
| | 64 : : | | 4 | 46 | 33336 |
| Divisor | 48 : : | | 552 | 8424 | 5167232 |
| | 48 : : | | 6 | 5616 | |
| Gnomon | 33336 : : | 3312 | 64584 | | 102503232 |
| | 33336 : : | 216 | 8 | | |
| Divisor | 6348 : : | 33336 | 516672 | | |
| | 6348 : : | | 512 | | |
| Gnomon | 5167232 : : | | 5167232 | | |

Tap in his *Seamans Kalendar* implyes 300 and 30 in the work; thus: After the Number is pricked, and the Cube of the Left Hand Prick subtracted as before, the Square of the Root found multiplied into 300, shall be the Divisor, to be placed so as the Right Hand Cypher thereof shall stand under the next Prick, and a new Quotient Figure gotten thereby, multiply the Divisor, and then multiply the first Quotient Figure by 30, and that Product by the Square of the second Quotient Figure, and add these, with the Cube of this last Quotient Figure, into one total Gnomon; and for every Prick do the like.

Example.

As in the former Number, first 64, the Cube of 4, subtracted from 102, I set down 300, and 30, and against 300, to the Left Hand the Square of 4, which is 16, and to the Left Hand of 30, the Root it self 4, then multiplying 300 by 16, the Product 4800 is the first Divisor, by which 6 gotten in the Quotient, I place it on the Right Hand of 300, and the Square of 6 on the same Hand of 30, and then multiplying all the Numbers in the upper row, I have 28800, and the Product of those in the lower row is 4320, which added with 216, the Cube of 6, make the Gnomon 33336; and so proceed to deal with the Numbers remaining to the other Prick, as here appeareth.

| | | | | | | |
|---------|-----------|-----------|--------------|--|--|--|
| | | | | | | |
| | | | | | | |
| | | | | | | |
| Cubus | 102303232 | 468 Radix | 16 . 300 : 6 | | | |
| | 64 : : | | 4 . 30 . 36 | | | |
| | 4800 : : | 300 | 30 | | | |
| | | 16 | 4 | | | |
| Gnomon | 33336 : : | | 28800 | | | |
| | | 1800 | 120 | | | |
| Divisor | 634800 | 300 | 36 | | | |
| | | | 216 | | | |
| Gnomon | 5167232 | 4800 | 720 | | | |
| | | 6 | 360 | | | |
| Proof | 102503232 | 28800 | 4320 | | | |

Adding the uncanceled Gnomons with the Cube first subtracted, as they stand will return the Great Cube, and prove the work true in this, as the other varieties.

To extract the Root of Compounds.

§. 4. Of the Prime or Original Figurate Rooted Numbers, a *Square* and a *Cube*: Several Higher Quantities are Compound, as their Names *Arithmetical* denote. And perhaps this may be the reason why they retain Names different from their *Geometrical* Forms, that it might be an easie memento to the speedy Extraction of their Roots: For according to the Composition so shall you draw the Root from thence, whether Zenzick or Cubick, and so often as the Name is found in the Composition, in their order beginning at the Left Hand.

Compounds of 3 sorts.

These two Names *Square* and *Cube*, or *Zenzick* and *Cubick*, make three sorts of Nominal Compounds, viz.

Either Squares with Squares, as Zenzizenzikes, Zenzizenzizenzikes, &c.

Or 2. Cubes with Cubes, as Cubicubicks, Cubicubicubicks, &c.

Or 3. Squares with Cubes, as Zenzicubes, Zenzizenzicubes, Zenzicubicubes, &c. Examples of each sort follow.

Extraction of the first sort.

A Zenzizenzike is the least and first Number of the first Composition, in which, because the Zenzick is twice repeated, I extract the Square Root of the given Number, and from that Root (which also will be a Square Number) I extract the Square Root again; and so is this last Root, the Zenzizenzike, or Squared Square Root of the first given Number.

1 Example.

As if I would extract the Zenzizenzike Root of 796594176, I first extract the Square Root, which is 28224, and from this Square Number extract the Square Root again, and so have 168, which I accept for the Squared Square Root desired.

| | | | |
|--------------|-----------|----------|-----------|
| | | 11225 | 3 |
| | | 37235671 | 1666 |
| Zenzizenzike | 796594176 | 28224 | 168 Radix |
| | 444644446 | 12624 | |
| | 6556641 | 336 | |
| | 5 | | |

2 Example.

And so for the Zenzizenzizenzikes, or of greater Quantity, it is but to reiterate the Zenzick Extraction. As to know the Zenzizenzizenzike Root of 16983563041, I first extract the Square Root, which is 130321, and then the Square Root of that, which is 361, and also the Square Root thereof, and so take 19 for the Root desired.

$$\begin{array}{r}
 260 \\
 54716 \quad 447 \quad 28 \\
 \text{Zenzizenzenzike } 16983563041 \mid 130321 \mid 361 \mid 19 \text{ Radix} \\
 12960096441 \quad 96621 \quad 121 \\
 2266006 \quad 37 \quad 8 \\
 226
 \end{array}$$

Numbers of the second Composition have the Cubick Root extracted in like manner. *Extraction of the second sort.*
 As to know the Cubicubike Root of 10604499373, the first Extraction gives the Cubick Root 2197, which is also a Cube Number, and his Root extracted is 13, the Cubicubike Root of the given Number. *Example.*

$$\begin{array}{r}
 10132 \\
 13520423 \\
 244376024 \quad 122 \\
 \text{Cubicube } 10604499373 \mid 2197 \mid 13 \text{ Radix} \\
 8261339333 \quad 1377 \\
 113202894 \quad 22 \\
 517813 \\
 1432 \\
 3
 \end{array}$$

The like is to be done for Numbers of greater Quantity under this Composition; as Cubes of Cubick Cubes to extract the Cube Root 3 times, &c.

When the Number is of the third Composition, as in Zenzicubes, Zenzizenzicubes, Zenzicubicubes, &c. so often extract the Zenzick Root as that Name is in the Composition, and so likewise the Cubick Root, and in such order as they stand compounded. *Extraction of the third sort.*

For in a Zenzicube, first extract the Square Root, this Root shall be a Cube, whose Cube Root extracted shall be the Zenzicube Root of the given Number. As 729 is a Zenzicube, whose Square Root is 27, which is a Cube Number, and hath 3 for his Root. *Example.*

In a Zenzizenzicube, extract the Square Root twice, and the Cube once at last. As in 4096, whose Square Root first extracted is 64, the Square Root of which is 8, and 8 is the Cube of 2; so 2 is the Radix Zenzizenzicubick of 4096. *Example.*

But in a Zenzicubicubick extract the Square Root once at first, and then the Cube Root twice. As in 262144, the Square Root is 512, the Cube Root whereof is 8, which is also a Cube Number, and hath 2 for his Root. *Example.*

All these Extractions may be proved by common Production, and if the Numbers subtracted in the Work be collected into Gnomons, by Addition, as aforesaid. *Proof.*

§. 5. Because the Surfolides are excluded out of the foregoing Compositions; whose Roots nevertheless it is requisite to know how to extract, when occasion shall require; and because it would be tiresome to set down particular Rules for every Quantity of the Higher Powers, and troublesome to the Memory to retain them, with the varieties of work, seeing as a Figural Number may be produced divers wayes, his Root may be many wayes extracted. And besides the particular Rules for each increasing according to their Quantities, and so in effect are as endless and various as the Numbers themselves, it will be more commodious to perform the Extraction of the Roots of all the Higher Powers by some one General Direction, which by the help of the Table following is perfectly to be done. *To extract the Roots of the Higher Powers by the Table following.*

And indeed all the Particular Rules that are given for Extraction of Roots, even those of the Square and Cube, have their ground in the Table, and come from thence; as might very easily be demonstrated. And by comparing the Tabulary Numbers and Operation therewith, hereafter in this Section set forth, with the Particular Rules here set down for the Zenzizenzike, Surfolide, and Zenzicube, will be sufficiently clear without further illustration. *Particular Rules grounded on the Table.*

Particular Rules for the Zenzizenzike.

Particular Rules for the Squared Square.

1.

1. After the Number is pricked according to his Quantity from the Left Hand Prick, subtract the greatest Zenzizenzike Number, and set the Zenzizenzike Root thereof in the Quotient.

2. Multiply

2. Multiply this Root Cubically, and quadruple the Product for a Divisor to be placed one place nearer to the Right Hand, and a new Quotient Figure gotten thereby, Multiply the Divisor, and reserve this Product to be added with the 3 Numbers in the next Directions to make up the Gnomon.
3. Square the first Quotient Figure, sexcuple the Square, and multiply the Product by the Square of the last Quotient Figure.
4. Cube the last Quotient Figure, and Multiply the Cube by the quadruple of the first Quotient Figure.
5. Take the Zenzizenzike Number of the last Quotient Figure, and with the 3 Numbers last above-mentioned, add them (duely placed one nearer than another to the Right Hand) into one total Gnomon, and subtract the same from the given Number: And if the Number have more Pricks than 2, the work in the second, third, fourth, and fifth Directions is to be repeated.

Example.

Thus the former Number 796594176 pricked, and 1 the greatest Squared Square Number in 7 subtracted, I multiply the Cube of 1, which is 1 by 4, and 4 is Divisor; which though standing under 69, yet will afford but 6 for the Quotient. This 6 multiplied into 4, gives 24. Then 1 squared is 1, and multiplied by 6 is 6, and again by 36, the Square of the last Quotient Figure, makes 216. And then 216, the Cube of 6, by 4 the quadruple of 1, the Product is 864. Lastly, 1296, the Zenzizenzick of 6, added with 24, 216, 864, makes the Gnomon for the second Prick.

Then the Cube of 16 quadrupled is 16384, the next Divisor, whereby 8 is gotten for a new Quotient Figure, which multiplying the Divisor gives 131072. Then the Square of 16, sexcupled and multiplied by the Square of 8, makes 98304. And the Cube of 8, by the quadruple of 16, produceth 32768. And the Zenzizenzike of 8 is 4096, added to the other Numbers make the next Gnomon, which subtracted as the former leave 0 remaining.

| | | | | | |
|--------------|--|--|-----------|---|---|
| | $\begin{array}{r} x \\ 64123 \end{array}$ | | | | |
| Zenzizenzike | $\begin{array}{r} 796594176 \end{array}$ | | 168 Radix | | |
| Gnomons | $\begin{array}{r} 1 \quad : \quad : \\ 55536 \quad : \\ 141234176 \end{array}$ | | | | |
| Proof | $\begin{array}{r} 796594176 \end{array}$ | | | | |
| | | | | $\begin{array}{r} 1 \\ 24 \\ 216 \\ 864 \\ 1296 \end{array}$ | $\left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} 55536$ |
| | | | | $\begin{array}{r} 131072 \\ 98304 \\ 32768 \\ 4096 \end{array}$ | $\left. \begin{array}{l} \\ \\ \\ \end{array} \right\} 141234176$ |
| | | | | $\begin{array}{r} 796594179 \end{array}$ | |

Particular
Rules for the
Surfolide.

Particular Rules for the Surfolide.

1. The Number being pricked according to his Quantity, take from the Figures belonging to the Left Hand Prick, the greatest Surfolide Number therein, and place the Surfolide Root thereof in the Quotient.
2. Multiply the Zenzizenzike Number of the Root by 5, (or quintuple it) and the Product shall be the Divisor to be set one place nearer to the Right Hand, by which get a new Quotient Figure, and Multiply the Divisor thereby, and reserve the Product for the first Number of the Gnomon.
3. Cube the first Quotient Figure, decuple the Cube (or Multiply it by 10) and Multiply the Product by the Square of the last Quotient Figure.
4. Square the first Quotient Figure, decuple the Square, and Multiply the Product by the Cube of the second Quotient Figure.
5. Quintuple the first Quotient Figure, and Multiply the Product by the Zenzizenzike of the next Quotient Figure in the Root.
6. Add these 4 last mentioned Numbers with the Surfolide of the last Quotient Figure, orderly placed every one nearer by one place to the Right Hand than the other, into one Gnomon, and subtract the summe from the given Number. And for every other Prick on the given Number, go over again with the work in these 5 last Directions.

Example.

As to extract the Surfolide Root of 28153056843, which marked as directed, it doth appear that 1 is the greatest Surfolide, and Root also of 2. Then the Zenzizenzike of 1 is 1, which quintupled is 5, this 5 is Divisor to 18, by which I get but 2 for the Quotient, this multiplied into 2 makes 10. Then the Cube of 1 is but 1, which

which decupled is 10, and multiplied by 4 is 40. And the Square of 1 decupled is 10, and multiplied by 8, the Cube of 2, gives 80. And the quintuple of 1 is 5, which multiplied by 16, the Zenzizenzike of 2, yields 80. Lastly, 32, the Surfollide of 2, added with 10, 40, 80, and 80, in their order make up the Gnomon 148832, for the second Prick. And by reiterating the work for the next, get the Gnomon 3269856843.

132698

Surfollide 28153056843 | 168 Radix

Gnomons { 148832 :
3269856843

Proof 28153056843

1

10
40
80
80
32

148832

311040
155520
38880
4860
243

3269856843

28153056843

Particular Rules for the Zenzicube.

1. The Number pricked according to his Quantity, deduct the greatest Zenzicube Number from the Left Hand Prick, and put the Zenzicube Root thereof in the Quotient.

2. Sexcuple the Surfollide Number of the Root, this Product shall be Divisor, by which get another Figure for the Quotient, and multiply the Divisor thereby, reserving this Number for the Gnomon.

3. Multiply the Zenzizenzike Number of the first Figure of the Root by 15, and the Product again by the Square of the next Figure of the Root.

4. Multiply the Cube of the first Quotient Figure by 20, and the Product again by the Cube of the next Quotient Figure.

5. Multiply the Square of the first by 15, and that again by the Zenzizenzike of the second Quotient Figure.

6. Multiply the first Figure of the Root by 6, and the Product by the Surfollide of the second Figure of the Root.

7. Let all these five last mentioned Numbers be added with the Zenzicube of the second Quotient Figure, orderly placed one nearer to the Right Hand than the other, into one Gnomon, and subtract the same from the given Zenzicube ; and repeat the work of these six last Directions for every other Prick.

Particular
Rules for the
Squared
Cube.

1.
2.
3.
4.
5.
6.
7.

As in extracting the Zenzicube Root of 244140625 ; first out of 244, I take the greatest Zenzicube I can, which is 64, whose Root is 2, the Surfollide of which 32 sexcupled is 192, for Divisor, by which 5 is gotten for the Quotient ; this 192 multiplied by 5 is 960. And 16, Zenzizenzike of 2 by 15 is 240, and again by 25, the Square of 5 is 6000. And 8, the Cube of 2, by 20 is 160, which by 125, the Cube of 5, is 20000. And 4, Zenzike of 2, by 15 is 60, which by 625, the Zenzizenzike of 5, is 37500. And 2, the Root by 6 is 12, that by 3125, the Surfollide of 5, is 37500. Lastly, 15625, the Zenzicube of 5, makes up the Gnomon.

Example:

180

Zenzicube 244140625 | 25 Radix

Gnomon 64 :
180140625

Proof 244140625

64

960
6000
20000
37500
37500
15625

180140625

244140625

A Table for
Extraction of
Roots.

The Table for Extraction of Roots.

| | | | | | | | | | | | |
|--------------------|----|-----|-----|-----|-----|-----|-----|-----|----|----|--|
| Zenzizenzicube. | | | | | | | | | | | |
| 12 | 66 | 220 | 495 | 792 | 924 | 792 | 495 | 220 | 66 | 12 | |
| Third Surfollide. | | | | | | | | | | | |
| 11 | 55 | 165 | 330 | 462 | 462 | 330 | 165 | 55 | 11 | | |
| Square Surfollide. | | | | | | | | | | | |
| 10 | 45 | 120 | 210 | 252 | 210 | 120 | 45 | 10 | | | |
| Cubicube. | | | | | | | | | | | |
| 9 | 36 | 84 | 126 | 126 | 84 | 36 | 9 | | | | |
| Zenzizenzizike. | | | | | | | | | | | |
| 8 | 28 | 56 | 70 | 56 | 28 | 8 | | | | | |
| Second Surfollide. | | | | | | | | | | | |
| 7 | 21 | 35 | 35 | 21 | 7 | | | | | | |
| Zenzicube. | | | | | | | | | | | |
| 6 | 15 | 20 | 15 | 6 | | | | | | | |
| Surfollide. | | | | | | | | | | | |
| 5 | 10 | 10 | 5 | | | | | | | | |
| Zenzizenzike. | | | | | | | | | | | |
| 4 | 6 | 4 | | | | | | | | | |
| Cube. | | | | | | | | | | | |
| 3 | 3 | | | | | | | | | | |
| Squa. | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| Root. | | | | | | | | | | | |
| 1 | | | | | | | | | | | |

The Table ex-
plained.

The Table Explained.

The little Iffuants at Top denote the Table may be increafed as occafion requires, though this reaching to Zenzizenzicubes be large enough for Example.

On the exterior parts on either fide afcending from the Root, are Numbers declaring the Quantities of the Powers whole Names are placed diftinctly on the Head of each Quantity.

The Numbers in the Body of the Table are thus gotten, after the Numbers of the Quantities are placed in the exterior parts, add the two Numbers belonging to the Cube together, which being 3 and 3, they make 6, to fill up the middle Square belonging to the Zenzizenzike, between the 4 and 4, fignifying his Quantity which 6 and 4 added make 10, for the two middle Squares of the Surfollide ; and this 10 added to 5 is 15, for the Squares of the Zenzicube next the middle, and for that 10 to 10 gives 20 : And thus adding one Number with another alternately, the other Numbers are found ; and the Table may be enlarged *ad infinitum*.

Use of the
Table.

The Use of the Table.

To extract the Root of any Number by help of the Table ; after the Number is pricked according to his Quantity as before taught, and the Greater Number of that Quantity whole Root you would extract, fubtracted out of the Figures belonging to the Left Hand Prick, and the Root thereof fet in the Quotient, as before ; then fet apart this Root with his Square, Cube, Zenzizenzike, &c. until you come to the Number before fubtracted. And in order under them, beginning at the Root, place the Numbers found in the Table belonging to the Quantity whole Root you are extracting. Then Multiply the Numbers ftanding one over another, one into another, the Multiplication next the Right Hand fhall be the Divifor, which after you have gotten another Figure of the Root by to be fet in the Quotient, fet him down under the Divifor, and his Square under the next Multiplication to the Left Hand, and his Cube under

1. *Example:* To extract the Square Root of 28224.

| | | | | | |
|-----------|-----------------|--|------------------|--|--------------------|
| | 126 | | Rad. Zen. | | Rad. Zen. |
| | • • • | | I I | | 16 &c. |
| Zenzike | 28224 168 | | 2 | | 2 Tabulary Number |
| | I : : | | | | |
| Gnomons { | 156 : | | 2 First Divisor. | | 32 Second-Divisor. |
| | 2024. | | | | |
| | <u> </u> | | | | |
| Proof | 28224 | | Zen. Rad. | | Zen. Rad. |
| | <u> </u> | | 36 6 | | 64 8 |
| | | | <u> </u> | | <u> </u> |
| | | | 36 12 | | 64 256 |
| | | | <u> </u> | | <u> </u> |
| | | | 12 | | 256 |
| | | | 36 | | 64 |
| | | | <u> </u> | | <u> </u> |
| | | | 156 Gnomon. | | 2624 Gnomon |

*Example of the
Cube.*

| | | | | | | | | | | |
|---------|---------|-----|------|------|---------------|--|------|--------|-------------------|--|
| | 3648 | | Rad. | Zen. | Cub. | | Rad. | Zen. | Cub. | |
| | . | . | I | I | I | | 16 | 256 | 4096 | |
| Cubus | 4741632 | 168 | 3 | 3 | | | 3 | 3 | Tabulary Numbers. | |
| | I | | | | | | | | | |
| Gnomons | 3096 | | 3 | 3 | First Divisor | | 48 | 768 | Second Divisor. | |
| | 645632 | | | | | | | | | |
| Proof | 4741632 | | Cub. | Zen. | Rad. | | Cub. | Zen. | Rad. | |
| | | | 216 | 36 | 6 | | 512 | 64 | 8 | |
| | | | | | | | | | | |
| | | | 216 | 108 | 18 | | 512 | 3072 | 6144 | |
| | | | | | | | | | | |
| | | | | 108 | | | | 3072 | | |
| | | | | 216 | | | | 512 | | |
| | | | | | | | | | | |
| | | | | 3096 | Gnomon. | | | 645632 | Gnomon | |

*Example of the
Squared
Square:*

| | | | |
|--------------|---|-----------|-----------|
| | | x | |
| | | 84123 | |
| Zenzizenzike | | 796594176 | 168 Radix |
| Gnomons | { | 55536 | |
| | | 141234176 | |
| Proof | | 796594176 | |

[illegible]

4. *Example.*

Example of the Surfolide.

4. Example. To extract the Surfolide Root of 28153056843.

| | | | | | | | | | |
|-----------|--|--|--|--|---|--|--|--|--|
| | | | | | 132688 | | | | |
| Surfolide | | | | | 28153056843 123 Radix | | | | |
| Gnomons | | | | | $\begin{array}{r} 148832 \\ 3269856843 \\ \hline 28153056843 \end{array}$ | | | | |
| Proof | | | | | 28153056843 | | | | |

| | | | | | | | | | |
|------|------|------|-------|----------------|------|------|-------|--------|-----------------|
| Rad. | Zen. | Cub. | Zenz. | Sur. | Rad. | Zen. | Cub. | Zenz. | Sur. |
| 1 | 1 | 1 | 1 | 1 | 12 | 144 | 1728 | 20736 | Gr. |
| 5 | 10 | 10 | 5 | | 5 | 10 | 10 | 5 | Tabulary Number |
| 5 | 10 | 10 | 5 | First Divisor. | 60 | 1440 | 17280 | 103680 | Second Divisor |

| | | | | | | | | | |
|------|-------|------|------|------|------|-------|-------|--------|--------|
| Sur. | Zenz. | Cub. | Zen. | Rad. | Sur. | Zenz. | Cub. | Zen. | Rad. |
| 32 | 16 | 8 | 4 | 2 | 243 | 81 | 27 | 9 | 3 |
| 32 | 60 | 80 | 40 | 10 | 243 | 4860 | 38880 | 155520 | 311040 |

| | |
|----|--------|
| 40 | 155520 |
| 80 | 38880 |
| 80 | 4860 |
| 32 | 243 |

| | | | |
|--------|--------|--------|------------|
| Gnomon | 148832 | Gnomon | 3269856843 |
|--------|--------|--------|------------|

§. 6. A *Surde Number*, sometime called *Irrational*, is as much as to say a Number from which it is not possible to take the Root, but there will remain something, which declares that the Number given was not a perfect Figural Number, and the Side thereof cannot therefore be expressed by an Integer. As the Square Root of 12 is 3, and there will remain 3; to the Cube Root of 28 is 3, and 1 will remain.

The Extraction of the Roots of *Surde Numbers* differs nothing from the wayes already set forth, and may be proved by the common way of Production, or Addition of the Gnomons, adding in the Remain to the Product or Total. But all the difficulty is to denominate the Remain, to know what part of the whole is signified thereby, seeing the Divisors are alwayes uncertain. Some Authors for the Square double the Root; others add 1 to the double for the Denominator; and others double the Remainder for the Numerator, and to the quadruple of the Root add 1 for the Denominator. And for the Cube, triple the Square of the Root, and add thereto the triple of the Root, and 1 more for the Denominator; yet all these, and several other Rules, fail to find out a true Denominator exactly, but that which comes nearest the truth is thus.

Adjoyn Cyphers to the Right Hand of the given Number according to his Quantity. As if a Square, as many times two Cyphers, as you please: If a Cube, as many times three Cyphers, as you please, &c. and continue the Extraction of the Root to the end of the Cyphers, and the more Cyphers are adjoyned the nearer the true Root you come. Then divide the Quotient by an Unit, and half the Numbers of Cyphers you added if it were a Square, and the third part if a Cube, the fourth part if a Squared Square, &c.

1. *Example*, in a Square Surde. Suppose a Square Plot of Ground were 18 Perches, and I would know the Side thereof: If then I add 2 Cyphers, and extract the Root, the Quotient is 42, which divided by 10, the Root is $4\frac{1}{5}$. And if I add 4 Cyphers, and divide the Quotient 424 by 100, the Root is $4\frac{6}{25}$. But if I add 6 Cyphers, the Quotient will be brought nearer the truth, and be $4\frac{121}{500}$ besides the small Fractions still left upon the Extractions, which denominate after the other wayes used in Authors, and added will somewhat increase the Number.

| | | Proof of both. | |
|---|--|--|--|
| $\begin{array}{r} 2 \overline{) 36} \\ \underline{40} \\ 164 \end{array}$ | $\begin{array}{r} \text{Radix} \\ 4 \frac{1}{5} \text{ Perches} \end{array}$ | $\begin{array}{r} 2 \overline{) 24} \\ \underline{40} \\ 168 \\ \underline{160} \\ 84 \end{array}$ | $\begin{array}{r} 42 \\ \underline{424} \\ 84 \\ 168 \\ \underline{160} \\ 84 \end{array}$ |
| | | | |
| | | | |
| | | | |

2. *Example*, in a Surde Cube. It is noted of the *Greeks*, that through their great Luxuriousness and Riot, they had brought Contagious Diseases upon them, and consulting their Oracles for redress thereof, received answer, *That when they would double their Altar* (which was of a Cubick Form) *they should be delivered from those Plagues*; meaning, the best Method to deliver Realms from such contagion breeding Vices, was to abate of their Voluptuousness, and apply themselves to Literature. But now suppose the Altar was 4 Feet Square every way, and the Altar were doubled, what must the Side be? Here if I double the Cube of 4, which is 64, it is 128, from which because I cannot extract the Cube Root without leaving a Remain, I assay to come near the Truth, and adjoyn 6 Cyphers, and extracting the Root yet 503 in the Quotient, and 736473 remaineth, then dividing 503 by 1, and 2 Cyphers, which are the third part of 6, I have 5 Feet, and $\frac{3}{100}$ of a Foot for the Root or Side, besides the odd Remain, which denominated after the common way, will be $\frac{736473}{760537}$ of $\frac{1}{100}$ or $\frac{736473}{76053700}$.

| | | | | |
|--------|-----|--------------|---------------------------------|---------------|
| | | 3 736473 | Radix | 503 |
| Root | 4 | 128 000000 | 5 03 (5 $\frac{3}{100}$ Feet | 503 |
| | 4 | 125 : : 1 00 | | 1509 |
| Square | 16 | 75 : : | | 25150 |
| | 4 | 0000 : : | | 253009 |
| Cube | 64 | 7500 : : | | 503 |
| | 4 | 2263527 | | 759027 |
| Double | 128 | | Remain | 12650450 |
| | | | | 736473 added. |
| | | | | 128000000 |

C H A P. IV.

Figurate Fractions.

To Figurate
Fractions,
And Mixt
Numbers.

When a Fraction is given to be multiplied Figurally, Multiply the Numerator by himself into the Quantity desired, and the Denominator likewise.

If an Integer and a Fraction be given, reduce them into an Improper Fraction, and then Multiply Figurally the Numerator by himself, and also the Denominator into the Quantity desired.

Examples.

As { To Square $\frac{3}{4}$ is $\frac{9}{16}$, and $2\frac{3}{4}$ reduced is $\frac{11}{4}$ Squared is $\frac{121}{16}$.
To Cube $\frac{3}{4}$ is $\frac{27}{64}$, and $2\frac{3}{4}$ reduced is $\frac{11}{4}$ Cubed is $\frac{1331}{64}$.

To extract the
Roots.

To extract the Root of a Fraction : First, extract the Root of the Numerator, and then the Root of the Denominator of the same Quantity.

Examples.

As to extract the { Square Root of $\frac{9}{16}$ renders $\frac{3}{4}$.
Cube Root of $\frac{27}{64}$ renders $\frac{3}{4}$.

No Roots in
Heterogeneous
Fractions.

But if the Numerator and Denominator be Heterogeneous, that is, not both of one Nature, though the one be a Square, and the other a Cube, as $\frac{16}{27}$, or the like ; yet can the Fraction have no Root extracted, but must remain as a Surde broken Number ; of which more hereafter in the next Book.

Partis Secundæ & Libri Secundi

F I N I S.

A R I T H.

ARITHMETICK.

The Third BOOK,

CONCERNING
Numbers specially contract;

In Six PARTS.

WHEREIN

DECIMALS

ASTRONOMICALS

LOGARITHMES

COSSICKS

SURDES

SPECIES

} are {
Discovered.
Anatomized.
Overlooked.
Characterized.
Surveyed.
Inspected.

AND THEIR

SIMPLE ELEMENTS.

CHAP. I.

Of DECIMALS.

IN the former Books enough hath been said of *Abstract* and *Generally Contract*, or *Vulgar denominate Numbers*, to make repetition here would be tautological. The next sort of Numbers I shall therefore fall upon are *Numbers specially Contract*, which are Numbers challenging to themselves some special Operations, and restrained by some Denomination either implied or expressed : Implied if the Denominators be certain ; expressed if otherwise
Denominators are certain in *Decimals*, *Astronomicals* and *Logarithmes*, and therefore omitted : Denominations uncertain in *Cossicks*, *Surdes* and *Species*, and therefore expressed. Of these in order, and first of *Decimals*.

Numbers specially contract follow those generally so. What Numbers specially Contract are. Denominators where certain where uncertain.

Decimals

Decimals their converse and practice.

Whence the Name, and what it signifies. Denominator of Decimals what, and how known.

Several ways of distinguishing the Decimal.

Cöma, a good way. Seperatrix what.

Decimals converse very much with Geodeticals, and practice to work both Integers and Fractions together in Addition, Subtraction, Multiplication and Division; thereby facilitating many laborious and intricate works in Common Fractions.

They take their Name from the Latin word *Decimus*, signifying a Tenth, or Tenth part; because as the Integers above the Unit increase by Tens towards the Left Hand, so the Decimal Fractions below the Unit decrease by Tens towards the Right Hand; the Denominator of every Decimal Fraction being always an Unit with Cyphers: As 10, 100, 1000, &c. that is to say, an Unit and so many Cyphers as there are Figures in the Numerator. And therefore because the Denominator doth alway consist of one place more than the Numerator, and is thereby certainly known, the Denominator is omitted, and the Numerator only used. For if the Numerator consist of 3 Figures or Places, the Denominator shall be 1000, which is 3 Cyphers and an Unit; and if the Numerator have 4 Figures, the Denominator shall be 10000, &c.

For distinguishing of the Decimal Fraction from the Integers, it may truly be said, *Quot Homines, tot Sententiæ*; every one almost fancying severally. For some call the Tenth Parts, *Primes*; the Hundredth Parts, *Seconds*; the Thousandth Parts, *Thirds*, &c. and mark them with *Indices* equivalent over their heads. As to express 34 Integers and $\frac{1426}{10000}$ Parts of an Unit, they do it thus, 34. 1. 4. 2. 6. Or thus,

(1) (2) (3) (4)
34. 1. 4. 2. 6. Others thus, 34,1426^{'''}; or thus, 34,1426⁽⁴⁾. And some thus, 34. 1. 4. 2. 6. setting the Decimal Parts at little more than ordinary distance one from the other, and they read them thus, 34 Integers, 1 Prime, 4 Seconds, 2 Thirds, 6 Fourths: Or 34 Integers 1426 Fourths. Others distinguish the Integers from the Decimal Parts only by placing a Cöma before the Decimal Parts thus, 34,1426; a good way, and very useful. Others draw a Line under the Decimals thus, 34 $\frac{1426}{10000}$, writing them in smaller Figures than the Integers. And others, though they use the Cöma in the work for the best way of distinguishing them, yet after the work is done they use a Rectangular Line after the place of the Unite, called *Seperatrix*, a separating Line, because it separates the Decimal Parts from the Integers, thus 34,1426. And sometimes the Cöma is inverted thus, 34¹⁴²⁶, contrary to the true Cöma, and set at top. I sometime use the one, and sometime the other, as cometh to hand.

For further discovery of the orderly progression of Decimals behold the following Tables, one with Figures and Letters, and the other with Letters, Figures and Cyphers, sometime called *Numeration Tables*.

A Table for Numeration of Decimals.

Table with Letters and Figures.

| Integers. | | | | | | | | | Unit. | Decimals. | | | | | | | | |
|-----------|----|----|----|----|----|----|----|----|-------|-----------|----|----|----|----|----|----|----|----|
| 9. | 8. | 7. | 6. | 5. | 4. | 3. | 2. | 1. | 0. | 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. |
| M | M | M | M | M | M | M | C | X | V | X | C | M | M | M | M | M | M | M |
| M | M | M | M | C | X | | | | | | | | X | C | M | M | M | M |
| M | C | X | | | | | | | | | | | | | | X | C | M |

The Units place.

Another.

Table with Letters, Figures and Cyphers.

| Integers. | | | | | | | | | Unit. | Decimals. | | | | | | | | |
|-----------|----------|---------|--------|-------|------|-----|----|----|-------|-----------|-----|------|-------|--------|---------|----------|-----------|------------|
| I. | H. | G. | F. | E. | D. | C. | B. | A. | V. | a. | b. | c. | d. | e. | f. | g. | h. | i. |
| 9. | 8. | 7. | 6. | 5. | 4. | 3. | 2. | 1. | 0. | 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. |
| 100000000 | 10000000 | 1000000 | 100000 | 10000 | 1000 | 100 | 10 | | | 10 | 100 | 1000 | 10000 | 100000 | 1000000 | 10000000 | 100000000 | 1000000000 |

The Units place.

In these *Tables* (for both are as one) there are four Progressions, two *Arithmetical* in the Figures above, which are called *Indices*, proceeding both wayes from Unity by the difference 1, though contrarily; two *Geometrical*, in the lower continued Proportions from Unity both wayes, by the *Ratio* 10. So that as the Series of the Numbers from the Units place are continued in a decuple proportion from the Right Hand towards the Left, and increase in their value, as before was said; so their value decreaseth in a subdecuple proportion from Unity towards the Right Hand.

The Tables explained. Indices what.

Example, by the latter *Table* suppose, 4. 3. 2. 1. 0. 1. 2. 3. 4. be a Number given, and stand with his Letters thus, $\begin{array}{cccccccc} D. & C. & B. & A. & V. & a. & b. & c. & d. \\ 4. & 3. & 2. & 1. & 0. & 1. & 2. & 3. & 4. \end{array}$; then shall *D*, be equal to 10 *C*, and *C*. equal to 10 *B*, and *B*. equal to 10 *A*, and *A*. equal to 10 *V*, and *V*. equal to 10 *a*, and *a*. equal to 10 *b*, &c.

Again, if *A*. be equal to 10 *V*, then shall *a*. be equal to $\frac{1}{10}$ of one *V*, and if *B*. be equal to 100 *V*, then shall *b*. be equal to $\frac{1}{100}$ of one *V*; and so if 1 Prime be $\frac{1}{10}$ of 1 Integer, every Second shall be $\frac{1}{100}$ Part, and every Third $\frac{1}{1000}$ Part, &c. of that Integer. And hence it is that *Decimals* are set down in a retrograde order to *Integers*: As if I were to set down Eight Hundred in *Integers*, it is thus 800, but in *Decimals* thus, $\frac{800}{1000}$.

The *Unit*, or one Integer, is alwayes understood to be divided into Parts, bearing denomination or name of the place of the last Figure to the Right Hand in the Decimal Fraction.

As 0, $\frac{1}{10}$ signifieth One Tenth Part, and 0, $\frac{2}{10}$ Two Tenth Parts, as if they were writ at large thus, $\frac{1}{10}, \frac{2}{10}$, the denomination of the place being Tenths, as the first place from the Unit towards the Right Hand, and noted as above with 10 in the *Table*. In like manner shall 0, $\frac{12}{100}$ signifie Twelve Hundred Parts, and 0, $\frac{34}{100}$ Thirty Four Hundred Parts, as if written at large thus, $\frac{12}{100}, \frac{34}{100}$, the denomination of the last place being noted with 100, under the last Figures, 2 and 4. The like is to be understood of others.

Note further, That Cyphers before Integers, or after Decimals, that is to the Left Hand of the one, and to the Right Hand of the other, signifie nothing at all; but after the Integers and before the Decimals, they are significant, for making up the places whereby the value of the other Figures are estimated. As 0001, signifieth but 1 in Integers; and, $\frac{1}{1000}$, but $\frac{1}{1000}$ in Decimals: Therefore in writing of Decimal Parts, mark well the Units place, by some one or other of the distinguishing Notes before mentioned, and let the void places (if any be) be filled up with Cyphers, and the place of the Unit set down, though there be no Integers. As to set down 3 Fourths, and 4 Fifths, thus 0,00034.

Cyphers before Integers or after Decimals signifie nothing.

Units place to be well noted.

To remove the *Seperatrix* one place nearer to the Right Hand increaseth the Number ten times in value, and toward the Left diminisheth it as much: For 0,125 shall be ten times less than 01,25.

Removing the Seperatrix what effect.

The *Indices* are very considerable, because of singular use almost in all Contract Numbers, as well as Decimals, and serve here to find out the true value of the Products in Multiplication, and Quotients in Division, as hereafter may be seen.

Indices very considerable.

The *Index* of the Unit is 0, as in the former *Tables*. And the *Index* of any other place may easily be known thus:

Indices how known.

In Integers, abate one from the number of places from the Units place, and the Remain is the *Index*. As the *Index* of 8 in this Number 8921, is 3, of 9 is 2, of 2 is 1, and 0, the *Index* of 1, because he standeth in the Units place.

In Decimals, the just number of the places from Unity is the *Index*. As the *Index* of 8 in this Number 0,8921, is (1), and in this Number 0,1298 is (4), of 9 in the former is (2), in the latter (3), of 2 in the former (3), in the latter (2), of 1 in the former (4), in the latter (1).

These *Indices* are expressed by the Learned *Oughtred* in his *Clavis limati*, affirmatively and negatively; that is, of Integers affirmatively by the sign +, signifying more; and

How expressed by Mr. Oughtred.

H h h

and of Decimals negatively, by the sign —, signifying *less*. And where the sign — is not set, the sign + is understood, though not expressed. And so 34,1426 stands thus, 34,1426.

Signs of a
Number either
Negative or
Affirmative.

When a Number is wholly negative or wanting, the sign *less* shewing its defect, is set at the Left Hand, thus, —34,1426; but if affirmative, the Number is placed without the sign *more*, as 34,0000, as well as with it thus, +34,0000. And though the Number be not wholly affirmative, but negative in the Decimals, yet it may be set without, thus, 34,1426, as well as +34,1426. And Decimals themselves albeit wholly negative, yet not taken privatively may be marked or not, as 0,1426; or thus, +0,1426.

These Signs
helpful.

Decimals Pure
or Mixt, Simple
or Compound.

How auxiliary the right understanding of these signs + and — is to the Simple Elements of Decimals and all Contract Numbers will be seen hereafter. It remains therefore, as necessary to this *Chapter*, that Decimals be counted Pure, when set without Integers. Mixt, when set with them. Simple, when affirmative or negative in their Signs. Compound, when their Signs are both affirmative and negative.

C H A P. II.

Reduction of Decimals.

Decimals reduced, how many ways, and the use thereof.

To turn Common Fractions into Decimals.

REDUCTION of *Decimals* serveth to reduce *Fractions* or *Geodeticals* into *Decimals*, or *Decimals* into them, that without impediment they may be wrought together, and Operation being ended their value may be found.

§. 1. To reduce *Common Fractions* into *Decimals*: Adjoyn to the Right Hand of the Numerator of the Fraction as many Cyphers as you please, and divide by the Denominator, the Quotient shall be the Decimal, and denominate according the Number of Cyphers adjoyned; that is to say, the Right Hand Figure or Cypher of the Quotient, as the Right Hand Cypher adjoyned to the Number before Division, and so the other Figures or Cyphers in order, proceeding towards the Left Hand accordingly. So that if I adjoyn but one Cypher, then the *dexter* Figure of the Quotient shall be Primes, if 2 Cyphers, Seconds, if 3 Cyphers, Thirds, &c.

Example in a
single Fraction
turned into a
Perfect Decimal.

As to set $\frac{3}{4}$ in Decimals, I adjoyn to 3 the Numerator, 2 Cyphers, and divide by 4 the Denominator, and the Quotient is 75, which is 7 Primes, 5 Seconds, there being but 2 Cyphers adjoyned, therefore the Quotient can be but Seconds.

$$\begin{array}{r} 3 \\ 4 \end{array} \quad \begin{array}{r} 2 \quad (1)(2) \\ 3,00 \\ 4 \end{array} \left(75 \right)$$

When any Number to be reduced into a Decimal cannot be brought into a perfect Decimal, but there will be some Remain upon the Division, then the more Cyphers are adjoyned the nearer the true Decimal you come, but seldom above 7 or 8 Cyphers need to be used; for that the Quotients of such Divisions come so near the truth that what is wanting is very inconsiderable.

Example in a
single Fraction
turned into one
Imperfect.

As to reduce $\frac{2}{3}$ to Decimals, if 5 Cyphers be adjoyned the Quotient is 66666, divided by 3; and if the Division be continued (more Cyphers being adjoyned) the same Quotientary Numbers will still be increased, because the Fraction cannot be reduced into a perfect Decimal.

$$\begin{array}{r} 2 \\ 3 \end{array} \quad \begin{array}{r} 22222 \quad 2 \quad (1)(2)(3)(4)(5) \\ 2,00000 \\ 3 \end{array} \left(66666 \right)$$

Reason of Reduction.

So any other proper single Fraction whatsoever may be turned into a Decimal, the Reason being the same: For as the Denominator of any Fraction is to the Numerator, so shall any Denominator be to a Numerator which shall have the same value to the Denominator given, as the first Numerator had to his Denominator, as is evident in the

the foregoing Examples: For as 4 to 3, so is 100 to 75; and as 3 to 2, so is 100000 to 66666 $\frac{2}{3}$.

If the Fractions given to be reduced into Decimals be conjunct or divided Fractions, then first by Reduction of Fractions reduce them into one Denomination, and abbreviate them into their least Terms, and if Conjunct add them together, and then turn them into Decimals, as above. Example of both. *Fractions conjunct and divided reduced into Decimals.*

To reduce $\frac{3}{4}$ and $\frac{1}{5}$ into a Decimal: First by Reduction of Fractions they make being reduced $\frac{15}{20}$ and $\frac{4}{20}$, and added $\frac{19}{20}$, then adjoining to 19, 2 Cyphers, and dividing by 20, the Quotient 9 Primes, 5 Seconds, is the Decimal. *Examples of both.*

To reduce $\frac{3}{4}$ of $\frac{1}{5}$ into a Decimal: First by Reduction of Fractions they are brought to $\frac{3}{20}$, and by adjoining 00, to 3, and the summe divided by 20, gives 1 Prime, 5 Seconds, for the Decimal.

$$\begin{array}{r} 19 \\ \hline 15 \quad 4 \\ \times \quad \times \\ \hline 3 \quad 1 \\ \hline 4 \quad 5 \\ \hline 20 \end{array} \quad \begin{array}{r} \times \\ 19,00 \\ \times \\ 20 \end{array} \begin{array}{r} (1)(2) \\ 95 \end{array} \quad \begin{array}{r} 3 \\ \hline 3 \quad 1 \\ \hline 4 \quad 5 \\ \hline 20 \end{array} \quad \begin{array}{r} \times \\ 3,00 \\ \times \\ 20 \end{array} \begin{array}{r} (1)(2) \\ 15 \end{array}$$

§. 2. To reduce Decimals into Common Fractions. If the Denominator be given to find a Numerator: then multiply the given Decimal by the given Denominator, and the Numbers exceeding the place of Primes shall be the Numerator to the given Denominator. And if the Numerator be given to find a Denominator, then by the given Numerator multiply the Denominator of the Decimal, and divide the Product by the given Decimal. Example of both. *To reduce Decimals into Common Fractions.*

Suppose 0,75 be the given Decimal, and 4 the Denominator given, to which a Numerator is desired; then multiplying 0,75 by 4, there exceeds the *Seperatrix* 3, which shall be the Numerator, and so 0,75 reduced to a Common Fraction shall be $\frac{3}{4}$. For as 100 to 75, so is 4 to 3. *Examples:*

And if 3 the Numerator were given, and 0,75 the Decimal to find a Denominator, then I multiply 100 the Denominator of the Decimal by 3, and the Product 300 divide by 75 the Decimal, the Quotient is 4 the Denominator to 3. For as 75 to 100, so is 3 to 4.

$$\begin{array}{r} 0,75 \\ \times 4 \\ \hline 3,00 \end{array} \quad \begin{array}{r} 3 \\ \times 4 \\ \hline 12 \end{array} \quad \begin{array}{r} 1,00 \\ \times 3 \\ \hline 3,00 \end{array} \quad \begin{array}{r} 3,00 \\ \times 4 \\ \hline 12,00 \end{array} \quad \begin{array}{r} 3 \\ \times 4 \\ \hline 12 \end{array}$$

§. 3. To reduce Geodeticals into Decimals, observe whether the Geodetical be of one Denomination or more, single or plural. *To reduce Geodeticals into Decimals.*

If the Geodætical be single, have but one Denomination, and that next the greatest Denomination of the Integer, then adjoyn to the Right Hand thereof so many Cyphers as you please, and divide the summe by the whole Number of Units of that Denomination contained in the Integer, and the Quotient is the Decimal, which shall be denominated according to the number of Cyphers adjoyned, as was shewed before. *1. Case. Single and next the Great Integer.*

As to reduce 9 Shillings, which is the next denomination to Pounds, into Decimals; I adjoyn 2 Cyphers to 9, and this 900 I divide by 20, the whole number of Shillings contained in one Pound, or greatest Integer, and the Quotient is 45, of which 5 shall be Seconds, because 2 Cyphers were adjoyned to 9, and the 4 shall be Primes; so is

0,45 the Decimals for 9 s.

Also to reduce 9 Months into Decimals, 00 adjoyned to 9, and divided by 12, the Months in a Year, the Quotient 0,75 is the Decimal desired.

$$\begin{array}{r} 900 \\ \times 20 \\ \hline 18000 \end{array} \begin{array}{r} (1)(2) \\ 45 \end{array} \quad \begin{array}{r} 900 \\ \times 12 \\ \hline 10800 \end{array} \begin{array}{r} (1)(2) \\ 75 \end{array}$$

2. Case.
Single, and not
next the great
Integer.

If the single Geodætical given be not next to the greatest Denomination of the Integer, then the Decimal may be had either working as above, accompting one of the next Denomination for the Integer, or by dividing the given Geodætical increased with his Cyphers by the parts of the greatest Integer.

Example.

As to reduce 9 d. into Decimals, because Pence is not the next Denomination to Pounds, the great Integer, I either divide 9 with Cyphers by 12, the parts of one Shilling, which is the Denomination next to Pence, and so have 1 75, the Decimal of a Shilling; or by 240, the Pence in a Pound the greatest Integer, and thereby get 375, which is the Decimal of a Pound for 9 d.

$$\begin{array}{r} 6 \quad (1)(2) \\ 9,00 \quad (75) \\ 12 \end{array}$$

$$\begin{array}{r} x \\ 182 \quad (2)(3)(4) \\ 9,0000 \quad (375) \\ 24440 \\ 22 \end{array}$$

3. Case.
Plural.

If the given Geodætical to be reduced consist of more Denominations than one, reduce the given Numbers by Geodætical Reduction into their lowest Denomination, and then work as above.

Examples.

As to reduce 3 s. 3 d. into Decimals: 3 s. 3 d. reduced into Pence, make 39 Pence, to which 4 Cyphers adjoyned, and the summe divided by 240, the Pence in one Pound, the Quotient is 1625, which is 1 6 2 5, because the Right Hand Cypher was of the same Denomination, there being 4 Cyphers adjoyned.

So 1 s. 4 d. 2 q. reduced into Decimals make 6875, being first reduced into Farthings make 66, and then divided with 0000 adjoyned by 960, the Farthings in one Pound.

$$\begin{array}{r} s. \quad d. \\ 3-3 \\ 12 \\ \hline 36 \\ 3 \\ \hline 39 \end{array}$$

$$\begin{array}{r} x \\ 1562 \\ 39,0000 \quad (+) \\ 244440 \\ 222 \end{array}$$

$$\begin{array}{r} s. \quad d. \quad q. \\ 1-4-2 \\ 12 \\ \hline 16 \\ 4 \\ \hline 66 \end{array}$$

$$\begin{array}{r} 74 \\ 8428 \\ 66,00000 \quad (6875) \\ 966660 \\ 999 \end{array}$$

4. Case.
Many Plural.

If Plural Geodæticals, of one or more Denominations, be given to be reduced into Decimals; first add them as Geodæticals are to be added, then reduce them into their lowest Denomination, and turn them into Decimals, as before.

Example.

As to reduce into Decimals, 3 s. 2 d., 2 s. 6 d. and 1 s. 1 d. being added the total is 6 s. 9 d. and in Pence 81, then by adjoyning 4 Cyphers, as above, and dividing, I get in the Quotient 3375 for the Decimal of all the 3 given Numbers.

$$\begin{array}{r} s. \quad d. \quad s. \quad d. \\ 3-2 \quad 6-9 \\ 2-6 \quad 12 \\ 1-1 \quad \hline \hline 6-9 \quad 69 \\ \hline \hline 81 \end{array}$$

$$\begin{array}{r} x \quad x \\ 982 \quad (1)(2)(3)(4) \\ 81,0000 \\ 244440 \\ 222 \end{array}$$

The end of
Tables.

Tables easily
made.

The like may be understood of other Geodæticals of Weight, Measure, Time, &c. as well as those instanced of Coyn; but some to ease this continued work, prepare Tables for each Denomination, out of which the Decimals taken for each given Number, may soon, by common Addition, be brought into one total. And these Tables are easily made and understood, considering (as was before shewed in the First Chapter of this Book) that every Integer is equal to 10 Primes, therefore half that Integer shall be equal to 5 Primes, and the quarter of that Integer equal to 2 Primes, 5 Seconds; and so the Decimals for the even aliquot parts of the Integer found, the rest of the intermediate

intermediate places may be easily had by Addition or Substraction, &c. as the case requireth.

Example in the *Table of English Coyn.* If 1 Pound or Integer (containing 20 Shillings) in Decimals set thus, 1,00, be 10 Primes, then $\frac{1}{2}$ l. or 10 s. must be 5 Primes, and $\frac{1}{4}$ l. or 5 s. shall be 2 Primes, 5 Seconds; and the fifth part of 5 s. which is 1 s. shall give the fifth part of 2', 5'', which is 5''; then if I substract 5'', the Decimal of 1 s. from 2', 5'', the Remain, 0,2', is the Decimal of 4 s. the half of which is 2 s. and the Decimal thereof 0,1', being half of the Decimal 0,2', and so also if I double 5'', the Decimal of 1 s. I have 0,1', as before. And in like manner all the other intermediate places for the 19 s. may be filled with Decimals.

Then taking 0,05'', the Decimal of 1 s. and breaking it in two parts, which is 0,025''', you have the Decimal of 6 d.; and again 0,0125''', half the latter, the Decimal of 3 d., and so of others, as the following *Tables* make conspicuous.

DECIMAL TABLES,

FOR
Sterling-Money.

Decimal Tables for

Sterling Money.

| Shillings. | | Pence and Farthings. | |
|---------------------|-----------|----------------------|-----------|
| Geodæticals. | Decimals. | Geodæticals. | Decimals. |
| <i>l.</i> <i>s.</i> | | <i>s.</i> <i>d.</i> | |
| 1, or 20 | 1,00 | 1, or 12 | 0,050000 |
| 19 | 0,95 | 11 | 0,045833 |
| 18 | 0,90 | 10 | 0,041667 |
| 17 | 0,85 | 9 | 0,037500 |
| 16 | 0,80 | 8 | 0,033333 |
| 15 | 0,75 | 7 | 0,029167 |
| 14 | 0,70 | 6 | 0,025000 |
| 13 | 0,65 | 5 | 0,020833 |
| 12 | 0,60 | 4 | 0,016667 |
| 11 | 0,55 | 3 | 0,012500 |
| 10 | 0,50 | 2 | 0,008333 |
| 9 | 0,45 | 1 | 0,004167 |
| 8 | 0,40 | <i>d.</i> <i>q</i> | |
| 7 | 0,35 | 1, or 4 | 0,004167 |
| 6 | 0,30 | 3 | 0,003125 |
| 5 | 0,25 | 2 | 0,002083 |
| 4 | 0,20 | 1 | 0,001041 |
| 3 | 0,15 | | |
| 2 | 0,10 | | |
| 1 | 0,05 | | |

Troy-Weight.

Penny-Weights.

| Ounces. | | Penny-Weights. | |
|---------------------------|-----------|-----------------------|-----------|
| Geodæticals. | Decimals. | Geodæticals. | Decimals. |
| <i>lb.</i> <i>Ounces.</i> | | <i>Ounce, Pennyw.</i> | |
| 1, or 12 | 1,000000 | 1, or 20 | 0,083333 |
| 11 | 0,916667 | 19 | 0,079167 |
| 10 | 0,833333 | 18 | 0,075000 |
| 9 | 0,750000 | 17 | 0,070833 |
| 8 | 0,666667 | 16 | 0,066667 |
| 7 | 0,583333 | 15 | 0,062500 |
| 6 | 0,500000 | 14 | 0,058333 |
| 5 | 0,416667 | 13 | 0,054166 |
| 4 | 0,333333 | 12 | 0,050000 |
| 3 | 0,250000 | 11 | 0,045833 |
| 2 | 0,166667 | 10 | 0,041667 |
| 1 | 0,083333 | 9 | 0,037500 |
| <i>Pen.w.</i> <i>Gra.</i> | | 8 | 0,033333 |
| 1, or 24 | 0,004166 | 7 | 0,029166 |
| 12 | 0,002083 | 6 | 0,025000 |
| 6 | 0,001041 | 5 | 0,020833 |
| 3 | 0,000520 | 4 | 0,016667 |
| 1 | 0,000173 | 3 | 0,012500 |
| | | 2 | 0,008333 |
| | | 1 | 0,004166 |

Troy-Weight.

The

The *Table of Troy Weight* for Ounces, may serve for Pence, if One Shilling be accompted the Integer : And also for Months, One Year being accompted the Integer.

The Decimal Table for Avoirdupois Weight, accompting the Hundred Weight for the Integer.

A Decimal Table for Avoirdupois Weight.

| Pounds. | | |
|-----------------------|-----------|--|
| Geodæticals. | Decimals. | |
| C. lb. | | |
| 1, or 112 | 1,000000 | |
| $\frac{2}{4}$, or 84 | 0,750000 | |
| $\frac{3}{2}$, or 56 | 0,500000 | |
| $\frac{3}{4}$, or 28 | 0,250000 | |
| 27 | 0,241071 | |
| 26 | 0,232143 | |
| 25 | 0,223214 | |
| 24 | 0,214286 | |
| 23 | 0,205357 | |
| 22 | 0,196429 | |
| 21 | 0,187500 | |
| 20 | 0,178571 | |
| 19 | 0,169643 | |
| 18 | 0,160714 | |
| 17 | 0,151786 | |
| 16 | 0,142857 | |
| 15 | 0,133929 | |
| 14 | 0,125000 | |
| 13 | 0,116071 | |
| 12 | 0,107143 | |
| 11 | 0,098214 | |
| 10 | 0,089286 | |
| 9 | 0,080357 | |
| 8 | 0,071429 | |
| 7 | 0,062500 | |
| 6 | 0,053571 | |
| 5 | 0,044643 | |
| 4 | 0,035714 | |
| 3 | 0,026786 | |
| 2 | 0,017857 | |
| 1 | 0,008929 | |

| Ounces. | | |
|--------------|-----------|--|
| Geodæticals. | Decimals. | |
| lb. ʒ. | | |
| 1, or 16 | 0,008929 | |
| 15 | 0,008371 | |
| 14 | 0,007812 | |
| 13 | 0,007254 | |
| 12 | 0,006696 | |
| 11 | 0,006138 | |
| 10 | 0,005580 | |
| 9 | 0,005022 | |
| 8 | 0,004464 | |
| 7 | 0,003906 | |
| 6 | 0,003348 | |
| 5 | 0,002790 | |
| 4 | 0,002232 | |
| 3 | 0,001674 | |
| 2 | 0,001116 | |
| 1 | 0,000558 | |

Or thus, accompting 1 lb. for the Integer.

| | |
|---------------|----------|
| lb. ʒ. | |
| 1, or 16 | 1,0000 |
| 15 | 0,9375 |
| 14 | 0,8750 |
| 13 | 0,8125 |
| 12 | 0,7500 |
| 11 | 0,6875 |
| 10 | 0,6250 |
| 9 | 0,5625 |
| 8 | 0,5000 |
| 7 | 0,4375 |
| 6 | 0,3750 |
| 5 | 0,3125 |
| 4 | 0,2500 |
| 3 | 0,1875 |
| 2 | 0,1250 |
| 1 | 0,0625 |
| $\frac{3}{4}$ | 0,046875 |
| $\frac{1}{2}$ | 0,031250 |
| $\frac{1}{4}$ | 0,015625 |

The Table for Ounces serves for Nails in a Yard.

The last Table for Ounces may serve for the Nails in one Yard, which are 16, as well as the Ounces in one Pound, and so the Decimals the same, accompting one Yard the Integer instead of one Pound.

The Table for the Dayes of one Month, accompting the Integer One Year, or 365 Dayes.

| Dayes. | Decimals. | Dayes. | Decimals. |
|--------|-----------|--------|-----------|
| 1 | 0,002740 | 16 | 0,043836 |
| 2 | 0,005479 | 17 | 0,046575 |
| 3 | 0,008219 | 18 | 0,049315 |
| 4 | 0,010959 | 19 | 0,052055 |
| 5 | 0,013698 | 20 | 0,054795 |
| 6 | 0,016438 | 21 | 0,057534 |
| 7 | 0,019178 | 22 | 0,060274 |
| 8 | 0,021918 | 23 | 0,063014 |
| 9 | 0,024658 | 24 | 0,065753 |
| 10 | 0,027397 | 25 | 0,068493 |
| 11 | 0,030137 | 26 | 0,071233 |
| 12 | 0,032877 | 27 | 0,073973 |
| 13 | 0,035616 | 28 | 0,076712 |
| 14 | 0,038356 | 29 | 0,079452 |
| 15 | 0,041096 | 30 | 0,082192 |

A Decimal Table for the Dayes in a Month.

In like manner taking the Decimal for one Day, and breaking it into parts, I may have the Decimals for Hours; as for 12 Hours 0,00137, for 6 Hours 0,000685, &c. And in the same method Tables may be made for other Geodæticals, and by these Tables the Decimals may be speedily had. As if 16 s. 9 d. were to be turned into Decimals; in the Table of *Coyne* against 16 s. is found 8 Primes, and against 9 d. is 3 Seconds, 7 Thirds, and 5 Fourths, which added together make the total Decimal 0,8375. So if 17 C. 13 lb. 5 3. be represented in Decimals, the Table of *Avoirdupois* Weight sheweth the Decimal of 13 lb. to be 0,116071, and the Decimal of 5 3. 0,00279, both which together represent the Decimal Number thus, 17,118861.

To get the Decimals for Hours.

Use of the Tables.

§. 4. To reduce Decimals into Geodæticals. If the Decimal be found in the Table, then the correspondent Number, without further work, effects the desire: But if not, multiply the Decimal given by the denominate parts contained in the Integer, and what exceeds the Prime Line shall be the Number desired, and shall be denominate according to the denomination of the Multiplier. One Example will make all plain. As if 0,9875 be given, and it is desired to know how much the same is in *English* Money: Then if I multiply by 20, the Shillings in one Pound, I find 19 exceed the Prime Line or *Seperatrix*, which shall be 19 Shillings, because 20 the Multiplier was Shillings, and the remaining Decimal 0,75, multiplied by 12 the parts of one Shilling, the Product is 9, which shall be Pence, because 12 Pence are the denominate parts of a Shilling that were multiplied by. But if I multiply the Decimal first given by 240, the Pence in one Pound, I find 237 exceed the Prime Line, which 237 shall be Pence because the Multiplier 240 was Pence.

To reduce Decimals into Geodæticals.

Example.

$$\begin{array}{r}
 0,9875 \\
 \times 20 \\
 \hline
 s. \ 19 \ 7500 \\
 \times 12 \\
 \hline
 1 \ 5000 \\
 7 \ 500 \\
 \hline
 d. \ 9 \ 0000
 \end{array}$$

$$\begin{array}{r}
 0,9875 \\
 \times 240 \\
 \hline
 39 \ 5000 \\
 197 \ 50 \\
 \hline
 d. \ 237 \ 0000
 \end{array}$$

$$\begin{array}{r}
 2 \\
 11 \ 9 \ d. \\
 237 \ 19 \ s. \\
 \hline
 12
 \end{array}$$

Many times in imperfect Decimals, when there is left in the place of Primes near another Integer, and upon a Division almost the Divisor is left remaining, another Unit is commonly taken for the same. And so is done in many of the Tabellary Numbers before. And when it happeneth may be done in all the Cases of the Four Sections of this Chapter.

What to be done with Imperfect Decimals.

The Reductions of the first and second Sections are alternate Proofs of each other; and so are they of the third and fourth Sections.

Proof of Reduction of Decimals.

G H A P.

C H A P. III.

Indices Added and Subtracted.

Addition and
Subtraction
of Indices.

SO much in the *First Chapter* of this *Third Book* hath been said of *Indices*, as may be sufficient to give a right understanding of them, that they are Numbers which as their Name implyeth, do shew the number or distance of places any Number or Species to which they are annexed is from the Unit, and how many places such Number or Species hath ; so as there needs no further explanation here.

What shewed
by their Addi-
tion.

Addition of *Indices*, discovers the number of places any *Factor* or *Product* should consist of, and thereby consequently sheweth the true nature of the *Product*.

What by their
Subtraction.

Subtraction of *Indices*, declareth the true nature of any *Quotient* after *Division* of *Mixt Numbers*, or Numbers of several distances from the Unit.

Indices to be added will be either alike, that is, both *Integers*, or both *Decimals*; or else unlike, that is, the one an *Integer* and the other a *Decimal*.

How added if
alike.

If they be both alike, then add the *Indices* as *Integers* are added, and the summe shall be the *Index* of the same kind the added *Indices* were.

Examples.

Examples.

Integers.

Decimals.

| | | | | |
|---------|----------|--------|------------|------------|
| Addends | { 4
3 | 5
3 | (4)
(3) | (5)
(3) |
| Totals | 7 | 8 | (7) | (8) |

How added if
unlike.

If the *Indices* be unlike, then subtract the Lesser Number out of the Greater, the difference shall be the *Index* of the same kind with that Number which was the greatest.

Examples.

Examples.

Integers and Decimals.

| | | | | |
|---------|------------|----------|----------|---------------|
| Addends | { 7
(2) | 2
(7) | (2)
7 | (7)
2 |
| Index | 5 | (5) | 5 | (5) remaining |

In both these Cases the Operation and Reason thereof is so plain they need no explanation.

Indices to be subtracted are to be placed one over the other, as the Numbers to which they belong ought to be placed ; for it matters not though the uppermost be the least.

These *Indices* also will be either alike or unlike.

How subtract-
ed if alike.

If both the *Indices* be alike, take the difference for the *Index* desired, and his nature is thus discerned : If the Subtrahend or Number to be subtracted be the least, the difference remaining shall be an *Integer* in *Integers*, and a *Decimal* in *Decimals* : But if the greatest the contrary, to wit, in *Integers* a *Decimal*, and in *Decimals* an *Integer*.

Examples.

Examples.

Integers.

Decimals.

| | | | | |
|-------------|--------|--------|------------|------------|
| Subtrahends | 5
4 | 4
5 | (5)
(4) | (4)
(5) |
| Remains | 1 | (1) | (1) | 1 |

How subtract-
ed if unlike.

If the *Indices* be unlike, add them together, and take the summe, and as the upper Number from which Subtraction is to be made, so shall this *Index* be, whether *Integer* or *Decimal*.

Examples.

Examples.

Integers and Decimals.

| | | | | | | | | |
|-------------|----------|----------|----------|----------|----------|----------|----------|--------------|
| Subtrahends | 5
(2) | (5)
2 | 2
(5) | (2)
5 | 4
(4) | (4)
4 | (3)
0 | 0
(3) |
| Index | 7 | (7) | 7 | (7) | 8 | (8) | (3) | 3 amounting. |

In the Examples of Subtraction, where the *Indices* are alike, the work is plain ; for taking the difference between 4 and 5, the Remain 1 is the *Index*, which is either an Integer or a Decimal, according as the Number to be subtracted was greater or lesser than the other.

And in the latter Examples the Addition of both the *Indices* make the remaining *Index* denominate alwayes as the higher *Index* or Number from which Subtraction is made. And the Reason of both is evident, for that the more is subtracted the less of necessity must remain ; and the lesser Number either in Nature or Figure subtracted, the greater will remain.

In these last Examples, c, or the place of the Unit, is seen to be accompted as the *Index* of an Integer.

C H A P. IV.

Addition of Decimals.

THE variety of adding Decimals is occasioned as the Numbers given to be added are Simple or Compound. Simple whether Pure or Mixt, being orderly placed every one under his like Denomination, are added as Integers.

| Examples. | Pure. | Mixt. | Examples. |
|-----------|----------|----------|-----------|
| | 0,990625 | 352,7250 | |
| | —0,5875 | —51,025 | Addends. |
| Simple | 0,725000 | 240,0375 | —20,050 |
| | 1,715625 | 592,7625 | —71,075 |
| | —0,7125 | | Totals. |

Compound Decimals are added by subtracting, as in Integers, the Lesser Number out of the Greater, and to the Remain which shall be the Total, subscribe the sign of the Greater Number.

| Examples. | Pure. | Mixt. | Examples. |
|-----------|--------|---------|-----------|
| | +0,950 | +34,125 | |
| | —0,825 | —34,125 | Addends. |
| Compounds | +0,825 | —3,950 | +3,950 |
| | +0,825 | +30,175 | —30,175 |
| | —0,825 | | Totals. |

If Compounds given to be added be many, let the Numbers of each sign be added with their fellows, as Simple Decimals, and then the Totals, as Compounds. As if 34,125 —0,95, were to be added with 1,45 —3,5, the + being added with + and — with —, the Totals are 35,575 —4,45, then taking the — from the +, the remaining Total is 31,125.

| | | |
|--------|-------|---------|
| 34,125 | —0,95 | +35,575 |
| 1,45 | —3,5 | —4,45 |
| 35,575 | —4,45 | +31,125 |

Besides the Proof of these works by Subtraction, noted in the next Chapter, the truth may be brought to the Test by reducing your Decimals into Geodæticals, and adding them in their order, the Totals of both Operations will agree when rightly done. As in the last Example.

| Decimals. | | Geodæticals. | | | | | | | | | | | |
|------------------------|-------|--------------|-----------|-----------|---------|-----------|-----------|-----------|----|-----------|-----------|-----------|--|
| | | <i>l.</i> | <i>s.</i> | <i>d.</i> | | <i>l.</i> | <i>s.</i> | <i>d.</i> | | <i>l.</i> | <i>s.</i> | <i>d.</i> | |
| 34,125 | —0,95 | 34 | —02 | —06 | lacking | 00 | —19 | —00 | or | 33 | —03 | —06 | |
| 1,450 | —3,50 | 1 | —09 | —00 | lacking | 03 | —10 | —00 | | —2 | —01 | —00 | |
| <hr/> | | <hr/> | | <hr/> | | <hr/> | | <hr/> | | <hr/> | | <hr/> | |
| 35,575 | —4,45 | 35 | —11 | —06 | wanting | 04 | —09 | —00 | | 31 | —02 | —06 | |
| —4,450 | | —4 | —09 | —00 | | | | | | | | | |
| <hr/> | | <hr/> | | <hr/> | | <hr/> | | <hr/> | | <hr/> | | <hr/> | |
| +31,125 Correspondents | | +31 | | —02 | | —06 | | Proof. | | | | | |
| <hr/> | | <hr/> | | <hr/> | | <hr/> | | <hr/> | | <hr/> | | | |

C H A P. V.

Subtraction of Decimals.

Decimals Sub-
tracted.
Simple.

THE variety in subtracting Decimals, as well as their Addition, is according to the Simplicity or Composition of the Numbers given to be subtracted. Simple Decimals Pure or Mixt, being orderly placed every Denomination under his like, are subtracted by withdrawing the nether Number from the upper, as Integers borrowing 10, if need be, and subscribing the Remain, except the Subtrahend be the greatest Number, then take the Lesser Number from the Greater, and change the Sign to the difference.

Examples.

| Examples. | Pure. | Mixt. | Pure. | Mixt. |
|-----------|----------|----------|--------|---------------------|
| Simple | 0,990625 | 352,7250 | 0,725 | —1,725 |
| | 0,725000 | 240,0375 | 0,950 | —1,950 Subtrahends. |
| | <hr/> | <hr/> | <hr/> | <hr/> |
| | 0,265625 | 112,6875 | —0,225 | +0,225 Remains. |
| | <hr/> | <hr/> | <hr/> | <hr/> |

Compound.

Compound Decimals having contrary Signs, for their Subtraction require Addition; so the Totals of the Numbers shall be the Remains; to which is to be affixed the upper Numbers sign, that is, the sign of the Number from which Subtraction is to be made.

Examples.

| Examples. | | Pure. | | | |
|-----------|---|--|--|--|---|
| Compound | { | $\begin{array}{r} -0,950 \\ -0,125 \\ \hline \end{array}$ | $\begin{array}{r} -0,950 \\ +0,125 \\ \hline \end{array}$ | $\begin{array}{r} +0,125 \\ -0,950 \\ \hline \end{array}$ | $\begin{array}{r} -0,125 \\ +0,950 \\ \hline \end{array}$ Subtrahends. |
| | | $\begin{array}{r} +1,075 \\ \hline \end{array}$ | $\begin{array}{r} -1,075 \\ \hline \end{array}$ | $\begin{array}{r} +1,075 \\ \hline \end{array}$ | $\begin{array}{r} -1,075 \\ \hline \end{array}$ Remains. |
| | | | | | |
| | { | Mixt. | | | |
| | | $\begin{array}{r} +34,125 \\ -3,950 \\ \hline \end{array}$ | $\begin{array}{r} -34,125 \\ +3,950 \\ \hline \end{array}$ | $\begin{array}{r} +3,950 \\ -34,125 \\ \hline \end{array}$ | $\begin{array}{r} -3,950 \\ +34,125 \\ \hline \end{array}$ Subtrahends. |
| | | $\begin{array}{r} +38,075 \\ \hline \end{array}$ | $\begin{array}{r} -38,075 \\ \hline \end{array}$ | $\begin{array}{r} +38,075 \\ \hline \end{array}$ | $\begin{array}{r} -38,075 \\ \hline \end{array}$ Remains. |

Compounds
when many.
Example.

If Compounds given to be subtracted be many, let the Numbers of each sort be subtracted as Simple Decimals, and then the Remains one from the other as Compounds. As if 3,5 —1,45 were to be subtracted from 22,475 —0,8, the + taken from the + leaves +18,975, and the — from the — leaves +0,65, and this Remain added to the other makes the Remain at last +19,625. Otherwise if I take each — from the respective + to which they are joyned, the 2 Numbers will be +21,675 and +2,05, and then Subtraction made as in Integers, the Remain will be as before.

Proof of
Decimal
Subtraction.

The Proof of Addition is by Subtraction, and of Subtraction by Addition alternately, as before in Integers and Geodæticals hath at large been seen, with this difference only here, that Simple Decimals reserve their Proof by the Simple Operations, and the Compounds by the Compound Operations respectively: So as Addition of the Simple shall be proved by Simple Subtraction, and Compound Addition by Compound Subtraction, and accordingly Subtraction Simple or Compound by the Simple or Compound Addition.

Besides

Besides this Proof, the truth of Decimal Subtraction as well as Addition may be proved by turning the Decimals into Geodæticals, and subtracting them in their order; will the Remains of both Operations agree, if done aright. As in the last Example.

| Decimals. | | Geodæticals. | | | | | |
|-----------|-------------------------|--------------|-----------|-----------|-----------|-----------|-----------|
| | | <i>l.</i> | <i>s.</i> | <i>d.</i> | <i>l.</i> | <i>s.</i> | <i>d.</i> |
| 22,475 | — 0,80 | 22 | — 09 | — 06 | wanting 0 | — 16 | — 0 |
| 3,500 | — 1,45 | 3 | — 10 | — 00 | wanting 1 | — 09 | — 0 |
| <hr/> | | <hr/> | | <hr/> | | <hr/> | |
| Thus | 18,975 + 0,65 | 18 | — 19 | — 06 | more 0 | — 13 | — 0 |
| | 0,65 | + | 0 | — 13 | — 00 | | |
| <hr/> | | <hr/> | | <hr/> | | <hr/> | |
| | 19,625 Correspondents | 19 | — 12 | — 06 | | | |
| <hr/> | | <hr/> | | <hr/> | | <hr/> | |
| Or | + 22,475 | + | 22 | — 09 | — 06 | | |
| | — 0,800 | — | 0 | — 16 | — 00 | | |
| <hr/> | | <hr/> | | <hr/> | | <hr/> | |
| | + 21,675 | + | 21 | — 13 | — 06 | | |
| <hr/> | | <hr/> | | <hr/> | | <hr/> | |
| Thus | + 3,50 | + | 3 | — 10 | — 00 | | |
| | — 1,45 | — | 1 | — 09 | — 00 | | |
| <hr/> | | <hr/> | | <hr/> | | <hr/> | |
| | + 2,05 | + | 2 | — 01 | — 00 | | |
| <hr/> | | <hr/> | | <hr/> | | <hr/> | |
| | + 19,625 Correspondents | + | 19 | — 12 | — 06 | | |
| <hr/> | | <hr/> | | <hr/> | | <hr/> | |

CHAP. VI.

Multiplication of Decimals.

THE varieties of multiplying Decimals are sorted under the like double head of Simple and Compound. Decimals multiplied.

Multiplication of Simple Decimals is so ordered, that the Product may be procured compleat or contracted, as occasion shall require. Simple and

To procure the Product compleat, multiply Number by Number, as in Integers, and add the Decimal *Indices* of the Multiplicand and Multiplier (or the *Indices* of the Right Hand Figures of them) together, and the summe shall be the *Index* of the Product, and the sign thereof alwayes +. The Product compleat.

| Examples. | Pure. | Mixt. | Examples. |
|------------|--------------------|-------------|------------|
| 0,9875 (4) | — 0,125 (3) | — 3,625 (3) | 13,625 (3) |
| 0,8875 (4) | — 0,025 (3) | — 2,5 (1) | 12,75 (2) |
| <hr/> | | <hr/> | |
| 49375 (8) | 625 (6) | 18125 (4) | 68125 (5) |
| 69125 | 250 | 7250 | 95375 |
| 79000 | | | 27250 |
| 79000 | + 0,003125 | + 9,0625 | 13625 |
| <hr/> | | <hr/> | |
| 0,87640625 | Compleat Products. | | 173,71875 |
| <hr/> | | <hr/> | |

Not only the Total *Index* of the Product by adding the *Indices* as abovesaid is found, but the true place in the Product of any two Numbers multiplied may be had by adding the particular *Indices* of the particular *Factores*. As in the last Example; if 5 in the Multiplicand be multiplied by 1 in the Multiplier, to know of what place the *Factus* or Product will be, or how far distant from Unity, and whether a Decimal or Integer; the *Index* of 5 is (3), and the *Index* of 1 there is 1, added make the Total *Index* (2), and so shall the Product 5 fall in the place of Decimal Seconds. Also the Product of 2 in the Multiplicand, whose *Index* is (2), multiplied into 2 in the Multiplier, whose *Index* is 0, because he standeth in the Units place, makes the Product Decimal Seconds, shewed by Addition of their *Indices* as afore said.

Sometime

Simple and the
Product con-
tracted.

Sometime it happens that all the Figures of the Product need not be expressed, but only some of those towards the Left Hand, because those towards the Right Hand are of small value, and in many Operations inconsiderable, and so the Product is contracted into a less number of places.

Contraction 3
ways.

1.
By Nepaires
Bones.

To contract the Product proceed in one of these three ways.

First, by *Nepaires Bones*, having set the Multiplicand on the *Bones*, and by the Index of the *Bones*, having the several Products answering the Figures of the Multiplier, take off each *Bone* so many Figures as will be sufficient, having respect to the Tens that rise on the next Right Hand *Bone*, and placing them orderly one under the other, add them into one Total Product.

Example,

As if 321,125 were to be multiplied by 43,25, where though there be 5 Decimals in both the *Factores*, yet needing but 3 in the Product, I Tabulate 321,125 on the *Bones*, and take out the Multiples, and in that appertaining to the multiplying, omit two places, and one place in the Multiple of 2, and add the Residue in order, as followeth.

| | | | | | | |
|---|----|----|---|---|----|---|
| 1 | 3 | 2 | 1 | 1 | 2 | 5 |
| 2 | 6 | 4 | 2 | 2 | 4 | 0 |
| 3 | 9 | 6 | 3 | 3 | 6 | 5 |
| 4 | 12 | 8 | 4 | 4 | 8 | 0 |
| 5 | 15 | 10 | 5 | 5 | 10 | 5 |
| 6 | 18 | 12 | 6 | 6 | 12 | 0 |
| 7 | 21 | 14 | 7 | 7 | 14 | 5 |
| 8 | 24 | 16 | 8 | 8 | 16 | 0 |
| 9 | 27 | 18 | 9 | 9 | 18 | 5 |

84225.
963375
1284500
16056..

321,125
43,25

16056|..
64225|..
963375|..
1284500|..

13888,656

2.
By cutting off
the Right
Hand Figures.

But Secondly, for that the whole Art of *Arithmetick* may be performed by the Pen, and many times the *Bones* are not at hand, multiply all as Integers, and cut off from the particular Multiples or Total Product, so many of the Right Hand Figures as are useless, respecting the Tens that rise in the next Right Hand File. And so the work of the former Example stands.

Example.

Thus, 321,125
43,25

16056|25
64225|0
963375|
1284500|

13888,656

Or thus, 321,125
43,25

16056|25
64225|0
963375|
1284500|

13888,656|25

3.
By Mr.
Oughtred's
retrograde
way.

Thirdly, Mr. *William Oughtred* in his *Clavis* proceeds in a retrograde order thus; If the Product be desired Pure, that is, to be wholly Integral without any Decimal Fraction, then place the Unity of the Multiplier, or the Lesser Number, under the Unity of the Multiplicand, or Greater Number, and so the Integers in order under the Decimals, and the Decimals under the Integers counterchanged. And if the Product be desired Mixt, then place the Unity of the least Number under that Decimal which the Product is desired to be. And if the Product be desired less than Pure, by so much remove the Units place of the Lesser Number towards the Left Hand. And the Numbers thus placed, begin to multiply by each Figure of the least Number or Multiplier at that Figure of the Multiplicand, or greatest Number, which stands thereover, and the Products of the Left Hand Multipliers let be set first, and so the rest in order by a Perpendicular Line.

Example.

As for further instance in the former Example thus, I place 3 in the Lesser Number being in the place of Unity, under the place of Thirds, which is 5 in the Greater Number,

Number, and the other Numbers in order alternately, and then beginning to multiply by 5 in the Lesser with 1 in the Greater Number which stands over him, adding in the Article that will arise; if he be multiplied by 2, the next Right Hand Figure in the Greater Number, I subscribe 6 under the Units place in the Lesser Number, and so proceed towards the Left Hand, and the like is done with the other Multiplying Figures, save because the last 4 hath no Figure over him, the Product arising by him is set one place distant from the Unity towards the Left Hand : And the work stands thus;

$$\begin{array}{r}
 321,125 \\
 \underline{52,34} \\
 16056 \\
 64225 \\
 963375 \\
 1284500 \\
 \hline
 13888,656
 \end{array}$$

And if the Product had been desired purely Integral, or less than Pure, the Products would have been found thus by,

The second way.

| | |
|----------|-----------------|
| 321,125 | 321,125 |
| 43,25 | 43,25 |
| 16 05625 | 1 605625 |
| 64 2250 | 6 42250 |
| 963 375 | 96 3375 |
| 12845 00 | 1284 500 |
| 13889 | 1389 |
| Pure. | Less than Pure. |

Mr. Oughtreds way.

| | |
|---------|-----------------|
| 321,125 | 321,125 |
| 52,34 | 52,34 |
| 16 | 1 |
| 64 | 6 |
| 963 | 96 |
| 12845 | 1284 |
| 13889 | 1389 |
| Pure. | Less than Pure. |

In all these is added more than really found in the Right Hand File, an Unit in the pure Products, and two Units in the others, respect had to the Tens, or almost Tens arising from the Figures cut off, as is usual in such Cases.

An Unit or more added, and why.

Compound Decimals convert into Simple, if by Addition or Subtraction they may, and then multiply them as before : Otherwise multiply every Number of the Multiplier by every Number of the Multiplier, beginning at the Left Hand, as in Geodætics, Book 2, Part 1, Chap. 5. Case 4. And for the Signs, when Numbers of like Signs, as + with +, or — with —, are multiplied together, the Product shall be +; but unlike Signs, as + with —, or — with +, shall make the Product —.

Compound multiplied.

As if 5,9 — 2,8 were to be multiplied by 3,4 — 1,2, I either convert them into Simple by subtracting the — out of the +, and then multiply the Remains 3,1 by 2,2; as at A. : Or otherwise multiply like Geodætics, as at B. ; and by adding + with +, and — with — in the Multiples, and then subtracting the — from the +, the Product of both Operations are alike 6,82.

Example:

| | | | | |
|-------------|-------------|----------------------|--|---------|
| A. | | B. | | |
| + 5,9 — 2,8 | + 3,4 — 1,2 | 5,9 — 2,8 | | |
| — 2,8 | — 1,2 | 3,4 — 1,2 | | |
| + 3,1 | + 2,2 | 236 112 | | + 20,06 |
| | | 177 84 | | + 3,36 |
| 3,1 (1) | | 20,06 — 9,52 | | + 23,42 |
| 2,2 (1) | | | | — 16,60 |
| 62 2 | | 118 56 | | + 6,82 |
| 62 | | 59 28 | | |
| 6,82 | | — 7,08 + 3,36 | | |
| | | 20,06 — 16,60 + 3,36 | | |

Multiplication of Decimals is proved by their Division, as is mentioned in the next Chapter. But besides that Proof, if the Decimals be turned into Geodæticals, and multiplyed, the Products of both will correspond when the work is without Error: As in the last Example at *A*.

Or if the Geodæticals be reduced into Shillings, and their results 62, and 44 multiplied, and then returned again into Pounds by 20 times 20, as in Multiplication of Geodæticals in like Case before was taught, there will be obtained 6 *l.* 16 *s.* $\frac{2}{5}$ equivalent to 6,82, as before.

| | | | | |
|-----|-------------|------------|--|------------------------------|
| l. | s. | s. | | (3 |
| 3—2 | or 62 | 20 | | 27 28(6 l. |
| 2—4 | or 44 | 20 | | 4 00 |
| | <u>248</u> | <u>400</u> | | 2(1 |
| | 248 | <u>400</u> | | 65 60(16 s. $\frac{2}{5}$. |
| | <u>2728</u> | | | 4 00 |

Division of Decimals.

THE varieties of dividing Decimals are ordered according to the Simplicity or Composition of the given Numbers.

Division of Simple Decimals runs into a Biverty of dividing a Greater Number by a Lesser, or a Lesser by a Greater.

In the first Case, where a Greater Number is to be divided by a Lesser, divide Number by Number as in Integers, and to find the true *Index* of the Quotient, subtract the *Indices* Decimal of the Divisor from the Dividend, or the *Indices* of the Right Hand Figures of them the one from the other, and the Remain shall be the *Index* of the Quotient, and the sign thereof +.

To divide 0,87640625 by 0,8875, the Quotient will be 9875, which shall be Decimal Fourths, because the *Index* of the Dividend is (8), from which (4) the *Index* of the Divisor subtracted, the Remain is (4).

So $-\text{0,003125}$ divided by $-\text{0,125}$, shall make the Quotient Decimal Thirds; for (3) the *Index* of the Divisor subtracted from (6) the *Index* of the Dividend leaves (3) remaining.

Examples

Examples in Mixt Decimals.

As 173,71875 divided by 13,625, the Quotient 1275, shall be 12,75; for by sub-
tracting (3) the Decimal Index of the Divisor from (5) the Decimal Index of the
Dividend, the Remain is (2).

Also if 758 Yards of Cloth cost 284 l. 5 s., or in Decimals 284,250, and the one
be divided by the other, the Quotient will be 375, which by subtracting 0, the Index
of 758, from (3) the Index of the Dividend, will be found Decimal Thirds, or 7 s.
6 d. for the price of one Yard.

Likewise if 7748,288 be divided by 0,2864, the Quotient shall be 27042, and
by subtracting the Index (4) of the Divisor from (3) the Index of the Dividend, the
Remain 1 shews the Index of the Right Hand Quotient Figure to be removed one place
distant from the Unit towards the Left Hand, and therefore Cyphers are placed to fill
up the void places.

Examples in
Mixt.

68

1021

374612

173,71875

13,625

13,625

13,625

13,625

13,625

(2)

(5)

(3)

(2)

37

5689

284,250

758

758

758

758

(3)

(3)

(0)

(3)

25

2016072

7748,288

2864

2864

2864

2864

2864

2864

1

I

(3)

(4)

1

The second Case, where a Lesser Number is to be divided by a Greater, trebly
branches it self. As first by supplying the defect of the Dividend with Cyphers. Se-
condly, by contracting the Divisor. Thirdly, by both. So as in Multiplication a Pro-
duct may be contracted, in Division a Quotient may be increased negatively, as far as
shall be needful, and is sometime phrased to divide by an Irrational or Infinite Number.
First Branch, adjoyn to the Right Hand of the Dividend as many Cyphers as shall
be requisite, and divide as before (accompting the Index of the Dividend, as it now
stands with the Cyphers adjoyned) and separate the Integers, if any, from the Deci-
mals, or supply the void places with Cyphers, and the work is done.

2.
A Lesser by a
Greater, 3
wayes.

1.
By adjoyning
Cyphers.

As if 21 l. were to be divided among 24 Men, I adjoyn 000 (which are as many as
need here) to 21, and divide by 24, the Quotient is 875, and by Substraction of the
Indices appears to be (3) for each Mans Share, which reduced into Godæticals is
17 s. 6 d.

Example.

(3)

0

(3)

21,000

24

24

24

(3)

Second Branch in dividing by the Bones, supposeth Cyphers to be adjoyned as be-
fore, but otherwise taketh as many of the dividing Figures to the Left Hand as are suf-
ficient for the first Divisor, and divideth the given Number thereby, then for every
particular subsequent Division lessen the Divisor, cutting off from the Right Hand of
the Divisor each time one Figure, having respect to the Tens arising in the next Right
Hand place (as was taught before in Multiplication) and so proceed in the Division till a
Quotient be gotten as large as is desired.

2.
By contracting
the Divisor.

As to divide 467023 by 357,09264, the first Divisor is but 357093, the next
35709, &c. as is plain by the work it self subscribed, and so the Quotient 130785
negative.

Example.

Divisor

357,09264

357,093

357,09

357,0

357

35

3

Quotient

1307,85

Multiples.

35709264

587031

357093

107127

0000

2800

286

17

Third

3.
By both.
Example.

Third Branch both adjoyneth Cyphers, and contracteth the Divisor, as afore said. As if 34 were to be divided by 4,326481, and 5 Decimals only required in the Quotient: After 5 Cyphers are adjoyned to 34, I cut off 1 from the Divisor, and divide first by 4,32648, and then by 4,3264, &c. The rest of the work needs no further Explanation.

| | | |
|---------|--|------------|
| | $\begin{array}{r} \overline{1} \\ 36 \\ 252 \\ 3713 \\ 25345 \\ 371463 \end{array}$ | |
| Divisor | $\begin{array}{r} 4,326481 \\ 34,000000 \\ 4,32648 \\ 4,3264 \\ 4,326 \\ 4,32 \\ 4,3 \\ 4 \end{array}$ | Quotient |
| | $\begin{array}{r} 7,85858 \\ 3028537 \\ 346118 \\ 21632 \\ 3461 \\ 216 \\ 35 \end{array}$ | Multiples. |

Some in the Divisions of this and the second Branch, use not to subscribe the Divisor as above, but instead thereof the Multiples, and so make Subtraction, applying the Divisor in a Loose Paper, as was shewed in the *First Book*, among the varieties of Division of *Integers*.

Compound
divided.

Compound Decimals convert into Simple, by Addition or Subtraction, if they may, and then divide them as before. Otherwise divide the Numbers of the Dividend by the Numbers of the Divisor, like the Division of *Geodætics*, in the *Third* and *Fourth Cases*, *Book 2. Part 1. Chap. 6.* as they happen. The *Index* of the Quotienary Numbers are got as above. And the signs that are like give +, and unlike —, as in the precedent *Chapter* of *Multiplication*.

Example.

As if 20,06 — 16,60 + 3,36 were to be divided by 3,4 — 1,2, being turned into Simple Decimals they are 6,82 and 2,2, and Division ended as at *A.* the Quotient is 3,1. If otherwise, divided as *Geodætics* at *B.* after 34, gotten by the first setting down of the Divisor, and the Divisor multiplied thereby, the Product is 20,06 — 9,52, which subtracted from the Dividend leaves — 7,08 + 3,36, which is the Product of — 12 the next Quotient Figures multiplied into the Divisor.

| | | | | |
|--|---|--|--|--|
| | A. | | B. | |
| $\begin{array}{r} + 20,06 \\ 3,36 \\ \hline + 23,42 \\ - 16,60 \\ \hline + 6,82 \end{array}$ | $\begin{array}{r} + 3,4 \\ - 1,2 \\ \hline + 2,2 \end{array}$ | $\begin{array}{r} (1) \quad 2 \\ (2) \quad 6,82 \\ (1) \quad 2,2 \\ (1) \quad 2,2 \end{array}$ | $\begin{array}{r} - 7,08 \\ + 3,36 \\ \hline - 3,72 \end{array}$ | $\begin{array}{r} 20,06 \\ - 9,52 \\ \hline 10,54 \end{array}$ |
| | | $\begin{array}{r} 3,1 \\ 236 \\ 177 \\ \hline 20,06 - 9,52 \end{array}$ | | $\begin{array}{r} 20,06 - 9,52 \\ \hline 10,54 \end{array}$ |
| | | $\begin{array}{r} 118 \\ 59 \\ \hline - 7,08 + 3,36 \end{array}$ | | $\begin{array}{r} 118 \\ 59 \\ \hline - 7,08 + 3,36 \end{array}$ |

Proof of Deci-
mal Division.

Division of Decimals is to be proved by Multiplication, as Multiplication by Division counterchanged; so as the Simple Multiplication of Decimals shall be proved by their Simple Division, and Compound by Compound accordingly, and on the contrary their Division by their Multiplication.

Also those Divisions that set down the Multiples of the Divisors underneath to subtract, may be proved by Addition of those Multiples, which will with the Remains, if any, return the Dividend. As in the last Example at *B.* like *Geodætics* of the fourth Case before noted, and the Divisions of the second and third Branches in this *Chapter*, thus to be placed.

| Second Branch. | Remain added. | Third Branch. |
|----------------|--------------------|---------------|
| (0 | | (1 |
| 17 | | 35 |
| 286 | | 216 |
| 2500 | | 3461 |
| 00000 | | 21632 |
| 107127 | | 346118 |
| 357093 | | 3028537 |
| 467023 | Dividend returned. | 34,00000 |

And besides this let the Decimals be turned into Geodæticals, and accordingly divided, and the Quotients of both works, when done without Error, will clearly correspond. As in the last Example at A.

| Decimals. | Geodæticals. |
|----------------|--------------------|
| 6,82 Dividend. | 6—16 $\frac{2}{5}$ |
| 2,2 Divisor. | 2—4 |
| 3,1 Quotient. | 3—2 |

$$\begin{array}{r} 4 \\ 2-4 \overline{) 6-16 \frac{2}{5}} \\ 3-2 \quad 6-12 \\ \hline 6-12 \quad 4-20 \text{ or abbreviated } \frac{2}{5} \\ 4-20 \end{array}$$

Or if the Geodæticals be all reduced into Shillings, then must 136 $\frac{2}{5}$ s. be divided by 44 s. the Quotient of which Division will be 3 l. and the 4 $\frac{2}{5}$ left if reduced into an Improper Fraction multiplied by 20, and as a Fraction divided by 44, the Quotient will be 2 s. as before.

C H A P. VIII.

Figuration of Decimals.

TO produce *Figurate Decimals* hath no difficulty therein ; for by multiplying any Decimal Simple or Compound into it self, the Square Decimal is produced. And the Square multiplied by the Root, produceth the Cube, &c. as *Figural Numbers* before in the *Second Part* of the *Second Book* were declared to be commonly produced. And because the same is done by Multiplication, the *Indices* of such Figured Decimals are found by adding together the *Indices* of both the Factors, and the signs of the Product known, as in Multiplication of Decimals before was shewed.

| Examples. | Pure Simple | Decimals | Mixt. Compound. | Examples. |
|---------------|-------------|--------------------------|-----------------|-----------|
| (2) 0,15 Root | | 1,2 —0,5 | | Root |
| (2) 0,15 | | 1,2 —0,5 | | |
| 075 | | 24 10 | | |
| 015 | | 12 5 | | |
| 0,0225 Square | | 1,44 —,60 | | |
| (2) ,15 | | —,60 +,25 | | |
| 01125 | | 1,44 —1,20 +,25 | | Square |
| 00225 | | 1,2 —0,5 | | |
| 0,003375 Cube | | 288 240 50 | | |
| &c. | | 144 120 25 | | |
| | | 1728 —1440 +300 | | |
| | | — 720 +600 — 125 | | |
| | | 1,728 —2,160 +,900 —,125 | | Cube |
| | | &c. | | |

M m m

If

THE SECOND PART OF THE THIRD BOOK.

CHAP. I.

Of ASTRONOMICALS.

THE second sort of Numbers specially Contract are *Astronomicals*, sometime called *Astronomical Fractions*, and other while *Geodeticals* of Time and Motion, which indeed they are ; but having some Operations more peculiar to them, with other Contract Numbers, then common to other Fractions, I reserved the handling of them to this place.

Astronomicals the next sort of Numbers specially Contract. How called.

Time is measured by Years, Dayes, Hours, &c. And the Cœlestial Motions by Circles, Signs, Degrees, &c.

Time and Motion how measured.

Annus, or a Year, was by the *Egyptians* in their *Hieroglyphicks*, emblem'd out by a Snake biting her Tail in her Mouth, and from *Anguis* therefore some conceive proceeded *Annus* ; but others, and more truly, from *Annulus*, a Ring or a Circle, which returns again into it self.

Year how called and Emblemed.

Annus, whence the word.

Neither the beginning of the Year, nor the length thereof, hath had the hap alwayes in all Nations to be alike ; nor is the former yet with us in *England* identified For the *Commonalty* begin the Year the First Day of *January*, the *Lawyers* the Five and Twentieth Day of *March*, and the *Astronomers* when the *Sun* enters the first Scruple of *Aries*.

Divers beginnings of the Year.

The length of the Year is now generally agreed to be that space of Time in which the *Sun* is running his course through the *Zodiack*, beginning at the first point of *Aries*, until his revolution or return thither again.

Length of the Year.

This space of Time is divided into Twelve parts, called Moneths, but very unequally in our Calendars or Almanacks, 7 of them having 31 Dayes, 4 but 30, and one but 28. More evenly ordered by *Julius Caesar*, as some say, but distorted in Honour of *Augustus*, to equal the Dayes in the Moneths bearing their Names.

Year how divided.

Moneths now unequal, and by whom so made.

Some think the Year at first was, or ought to be divided by the course of the *Moon*, who gave being to the Name *Moneth* : And by the Law of *England*, a Moneth is but 28 Dayes, agreeing to the visibility of the *Moon*, there being some time for her both before and after Conjunction with the *Sun* to be hidden under his Beams or Rayes. But neither hath this afforded a *Basis* for the exact limitation of a Moneth ; since besides the time of the *Moons* visibility, called the *Apparition Moneth*, and the time when most proper to administer *Physick*, called the *Medicinal Moneth*, containing 26 Dayes, 12 Hours, according to *Johannes de Sacra Bosco* : There is the *Periodical Moneth*, and the *Synodical Moneth* ; the former of which called also the Moneth of *Paragratiō*, is the space in which she makes her Revolution, and returns to the place she made her last \odot with \odot , and the latter called by some the Moneth of *Consecution*, the whole space from her last Conjunction with the *Sun*, unto her next : Both which by reason of the sometime swift, and sometime slow Motion of both Luminaries, take up more or less time accordingly, then can be comprised in any precise Number of Dayes without surplussage. The *Periodical* containing 27 Dayes, 7 Hours, 43 Minutes, 7 Seconds ; and the *Synodical* 29 Dayes, 12 Hours, 44 Minutes, 4 Seconds, by common Computation.

Moneth whence the word.

Sorts of Moneths.

Apparition Moneth.

Medicinal Moneth.

Periodical or Moneth of Paragratiō.

Synodical or Moneth of Consecution.

The length of the Periodical and Synodical Moneths.

Moneth omitted by some, by others reckoned at 30 Dayes.

Moneth of the Jews.

Legal Moneth explained.

Week the context.

Legal Moneths in the Year.

Year certainly divided.

Length of the Tropical Year.

Bissextile or Leap Year.

Day Natural and Artificial.

Hours how subdivided.

Of Motion.

Circle how divided.

Degrees what part thereof how called.

How many to a Sign.

Degrees subdivided.

Minutes and Miles how agree.

The reason of the Subdivisions by 60.

Denominator certain and omitted.

Difference of Astronomicals from Decimals.

Astronomicals why so called.

Sometime called Physical Fractions.

Physical Signs what.

Signs with Astronomers.

Because of these differences about the Moneth, some omit them among the certain Denominations, others in their Operations accompt 30 Dayes to a Moneth, which some conceive to be the Old Accompt of the Jews; for comparing *Gen. 7. 11.* with *Gen. 8. 3, 4* the space from the seventeenth day of the second Moneth, to the seventeenth day of the seventh Moneth, is reckoned 150 Dayes, which being 5 Moneths, allowes 30 Dayes for every Moneth. And others use neither them nor the Calendar Moneths, but the Legal Moneth of 28 Dayes, as aforesaid, which divided into 4 parts called Weeks, makes every Week 7 Dayes, and by this accompt 13 Moneths in the Year. The Year then for certainty is subdivided into Dayes, and containeth commonly 365 Dayes, and almost 6 Hours, but the true Tropical Year by late Writers is not above 365 Dayes, 5 Hours, 49 Minutes, 4 Seconds and 21 Thirds; for which odd time in *Arithmetick*, 6 Hours is usually taken, which in every Fourth Year called *Bissextile* or Leap-Year, maketh another Day, and it is added to the end of the Moneth of February, which then hath 29 Dayes, or else in the Common Years but 28; so hath the Leap Year 366 Dayes.

Dies, or a Day Natural, is in *Astronomers* accompt that space of Time run out while the *Sun* passeth from the *Meridian*, ere he can come thither again, and is divided into 24 equal parts called Hours, and comprehendeth both Day and Night. But a Day Artificial is sometime limited between ☉ and ☉, or *Sun* Rising and *Sun* Setting, or to a quantity of Hours, as a Working-Day 12 Hours, from 6 at Morning till 6 at Night.

Every of these Hours is divided into 60 parts, called Minutes, and every Minute into 60 lesser divisions, called Seconds; every Second into 60 Thirds, and so lower into Fourths, Fifths, Sixths, &c. by 60.

Touching Motion, it is to be remembred, That the Circumference of the Heavens, as also every Circle, is parted into 360 divisions, called Degrees or Grades from the *Latin Gradus*, 30 of which make one Sign, so 12 Signs make the Circle; for 12 times 30 is 360, answerable to the division of Time into 30 Dayes for a Moneth, and 12 Moneths to the Year.

Every Degree contains in Longitude or Length 60 Minutes, which are not to be taken for Minutes of Time, but Minutes of Measure, called also Scruples, and sometime Primes. Every of these Minutes of Measure in the Heavens is generally accompted to answer to one Mile on the Earth. Nevertheless Mr. *Oughtred* in his *Circles of Proportion* is of Opinion, that one Degree or 60 Minutes answer more truly to $60\frac{1}{4}$ Miles.

These Minutes, as well as Minutes of Time, are subdivided into 60 Seconds, the Second into 60 Thirds, and so into Fourths, Fifths, Sixths, &c. of which 60 lesser are comprehended in one of the next greater.

That an exact Measure both of Time and Motion might be had, the Antients have thought fit to divide one Hour of Time, and one Degree of Motion into 60, and so downwards, unto very small subdivisions, as aforesaid. Forasmuch as alwayes an whole Year, Moneth or Day, &c. is not the subject of the Question, neither the moving of the Coelestial Bodies to be alwayes measured by whole Circles, Signs, Degrees, &c. but sometimes Parts or Fractions of the whole are useful. And the rather have they chosen 60 for the Integer or Denominator, because it is a Number that may receive more equal divisions than any Number under 100; for it may be parted by 2, 3, 4, 5, 6, 10, 12, 15, 20, and 30.

The Denominator 60 being certainly known is alway omitted, as the Decimal Denominators are, and the given Numerators only used. And as in Decimals the Numbers exceeding the Units place to the Left Hand pass for Integers; so these. Nevertheless here seems to be some difference in that beyond the Units place to the Left Hand (1 Day in Time, and 1 Degree in Motion being generally accompted the Integers) here are Fractions besides those at the Right Hand, as Signs in Motion, and Moneths in Operations of Time; for the utmost Integer or greatest Geodætical of Time is the whole Year, and of Motion the whole Circle, notwithstanding although sometime between the Unit and utmost Integer such Fractions be found, yet their Denominators being certain they are omitted, and accompted as Integers.

Because the use of these Divisions is most conversant about *Astronomy*, they are called *Astronomicals*, or *Astronomical Fractions*: Some Writers call them *Physical Fractions*, and with *Alphonfus* divide the Circle into 6 divisions, and then one Sign containeth 60 Degrees, and the other divisions are as before, which is certainly the most expedient way for Multiplication and Division; but *Astronomers* in all their Calculations generally reserve the Sign of 30 Degrees, and accordingly reckon all the Aspects both Old and New, Sinister and Dexter.